> *restart*

MODELO DE LA CUERDA DE GUITARRA

> $Ecuacion := diff(y(x,t), t\$2) = c\cdot 2\cdot diff(y(x,t), x\$2)$

$$Ecuacion := \frac{\partial^2}{\partial t^2}\, y(x,t) = c^2\left(\frac{\partial^2}{\partial x^2}\, y(x,t)\right) \quad \textbf{(1)}$$

> $EcuacionSeparable := eval(subs(y(x,t) = F(x)\cdot G(t), Ecuacion));$

$$EcuacionSeparable := F(x)\left(\frac{d^2}{dt^2}\, G(t)\right) = c^2\left(\frac{d^2}{dx^2}\, F(x)\right) G(t) \quad \textbf{(2)}$$

> $EcuacionSeparada := \frac{lhs(EcuacionSeparable)}{F(x)\cdot G(t)} = \frac{rhs(EcuacionSeparable)}{F(x)\cdot G(t)};$

$$EcuacionSeparada := \frac{\frac{d^2}{dt^2}\, G(t)}{G(t)} = \frac{c^2\left(\frac{d^2}{dx^2}\, F(x)\right)}{F(x)} \quad \textbf{(3)}$$

> $EcuacionX := rhs(EcuacionSeparada) = \mathrm{alpha}; EcuacionT := lhs(EcuacionSeparada) = \mathrm{alpha};$

$$EcuacionX := \frac{c^2\left(\frac{d^2}{dx^2}\, F(x)\right)}{F(x)} = \alpha$$

$$EcuacionT := \frac{\frac{d^2}{dt^2}\, G(t)}{G(t)} = \alpha \quad \textbf{(4)}$$

PARA ALFA IGUAL A CERO

> $SolucionXcero := dsolve(subs(\mathrm{alpha} = 0, EcuacionX));$

$$SolucionXcero := F(x) = \_C1\, x + \_C2 \quad \textbf{(5)}$$

> $sistemita := subs(x = 0, rhs(SolucionXcero) = 0), subs(x = 1, rhs(SolucionXcero) = 0);$

$$sistemita := \_C2 = 0, \_C1 + \_C2 = 0 \quad \textbf{(6)}$$

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> $parametrito := solve(\{sistemita\}, \{\_C1, \_C2\});$

$$parametrito := \{\_C1 = 0, \_C2 = 0\} \quad \textbf{(7)}$$

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PARA ALFA POSITIVA

> $SolucionXpositiva := dsolve(subs(\mathrm{alpha} = \mathrm{beta}\cdot 2, EcuacionX));$

$$SolucionXpositiva := F(x) = \_C1\, e^{\frac{\beta x}{c}} + \_C2\, e^{-\frac{\beta x}{c}} \quad \textbf{(8)}$$

> $sistemaPositivo := subs(x = 0, rhs(SolucionXpositiva) = 0), subs(x = 1, rhs(SolucionXpositiva) = 0)$

$$sistemaPositivo := \_C1\, e^0 + \_C2\, e^0 = 0, \_C1\, e^{\frac{\beta}{c}} + \_C2\, e^{-\frac{\beta}{c}} = 0 \quad \textbf{(9)}$$

> $parametro := solve(\{sistemaPositivo\}, \{\_C1, \_C2\});$

$$parametro := \{\_C1 = 0, \_C2 = 0\} \quad \textbf{(10)}$$

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PARA ALFA NEGATIVA

> $SolucionXnegativa := dsolve(subs(\alpha = -\beta \cdot 2, EcuacionX));$

$$SolucionXnegativa := F(x) = \_C1 \sin\left(\frac{\beta x}{c}\right) + \_C2 \cos\left(\frac{\beta x}{c}\right) \quad \textbf{(11)}$$

> $SolucionXnegativa2 := subs(\_C2 = 0, \beta = n \cdot \mathrm{Pi}, SolucionXnegativa)$

$$SolucionXnegativa2 := F(x) = \_C1 \sin\left(\frac{n \pi x}{c}\right) \quad \textbf{(12)}$$

> $SolucionTnegativa := dsolve(subs(\alpha = -(n \cdot \mathrm{Pi}) \cdot 2, EcuacionT));$

$$SolucionTnegativa := G(t) = \_C1 \sin(n \pi t) + \_C2 \cos(n \pi t) \quad \textbf{(13)}$$

> $SolucionParticular := y(x, t) = subs(\_C1 = 1, rhs(SolucionXnegativa2)) \cdot subs(\_C1 = a_n, \_C2 = b_n, rhs(SolucionTnegativa));$

$$SolucionParticular := y(x, t) = \sin\left(\frac{n \pi x}{c}\right) \left(a_n \sin(n \pi t) + b_n \cos(n \pi t)\right) \quad \textbf{(14)}$$

> $f(x) = Sum(eval(subs(t = 0, rhs(SolucionParticular))), n = 1..infinity);$

$$f(x) = \sum_{n=1}^{\infty} \sin\left(\frac{n \pi x}{c}\right) b_n \quad \textbf{(15)}$$

> $$Parametrote := simplify\left(\left(\frac{1}{\left(\frac{5}{10}\right)}\right) \cdot int\left(\left(\frac{\left(\frac{5}{1000}\right)}{\left(\frac{5}{10}\right)} \cdot x\right) \cdot \sin(n \cdot \mathrm{Pi} \cdot x), x = 0..\frac{5}{10}\right) + \left(\frac{1}{\left(\frac{5}{10}\right)}\right) \cdot int\left(\left(\frac{1}{100} - \frac{\left(\frac{5}{1000}\right)}{\left(\frac{5}{10}\right)} \cdot x\right) \cdot \sin(n \cdot \mathrm{Pi} \cdot x), x = \frac{5}{10}..1\right)\right)$$

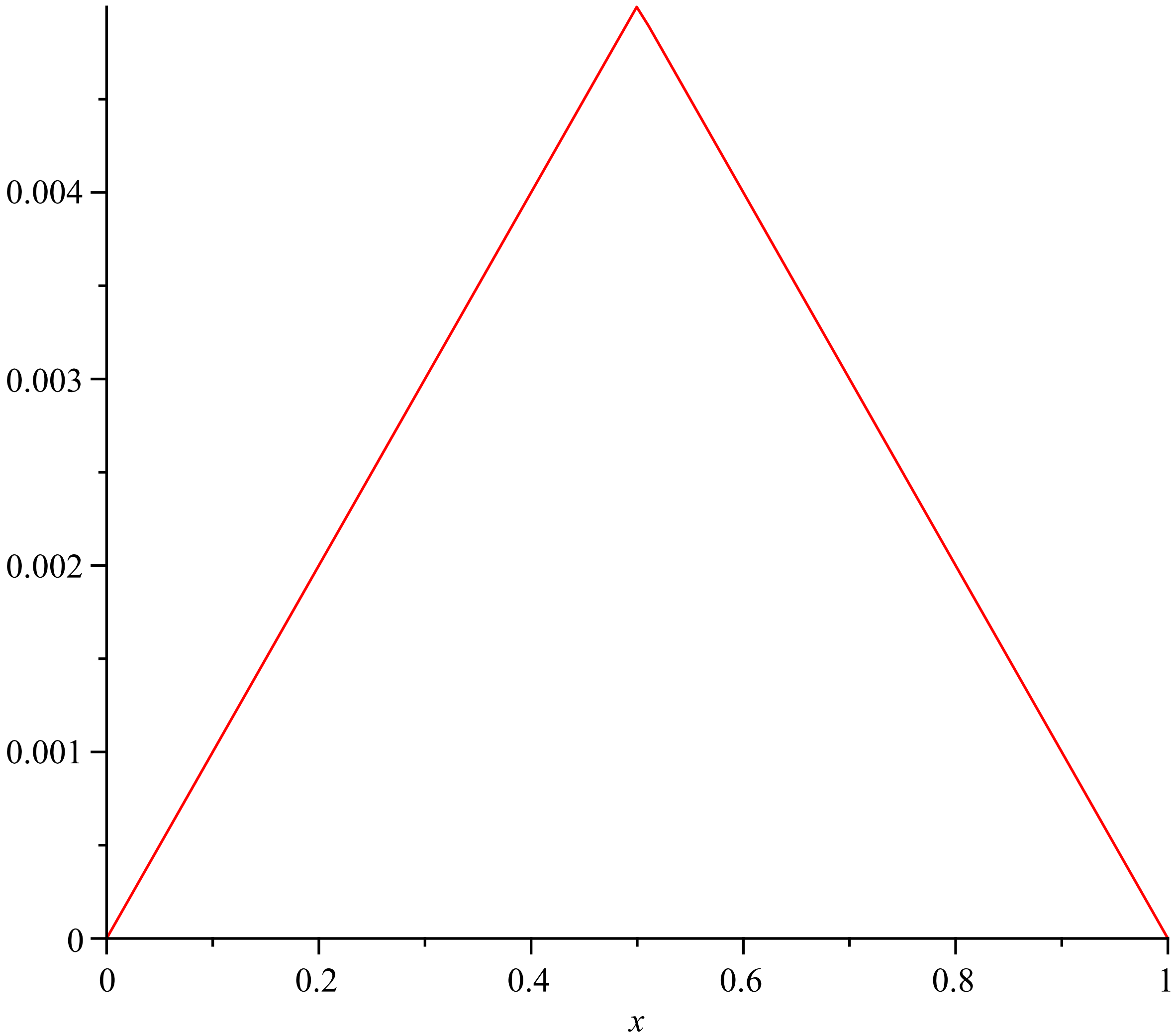
$$Parametrote := \frac{1}{50} \frac{2 \sin\left(\frac{1}{2} n \pi\right) - \sin(n \pi)}{n^2 \pi^2} \quad \textbf{(16)}$$

> $SolucionFinal := y(x, t)_{STF} = subs(b_n = Parametrote, a_n = 0, Sum(rhs(SolucionParticular), n = 1..infinity));$

$$SolucionFinal := y(x, t)_{STF} = \sum_{n=1}^{\infty} \frac{1}{50} \frac{\sin\left(\frac{n \pi x}{c}\right) \left(2 \sin\left(\frac{1}{2} n \pi\right) - \sin(n \pi)\right) \cos(n \pi t)}{n^2 \pi^2} \quad \textbf{(17)}$$

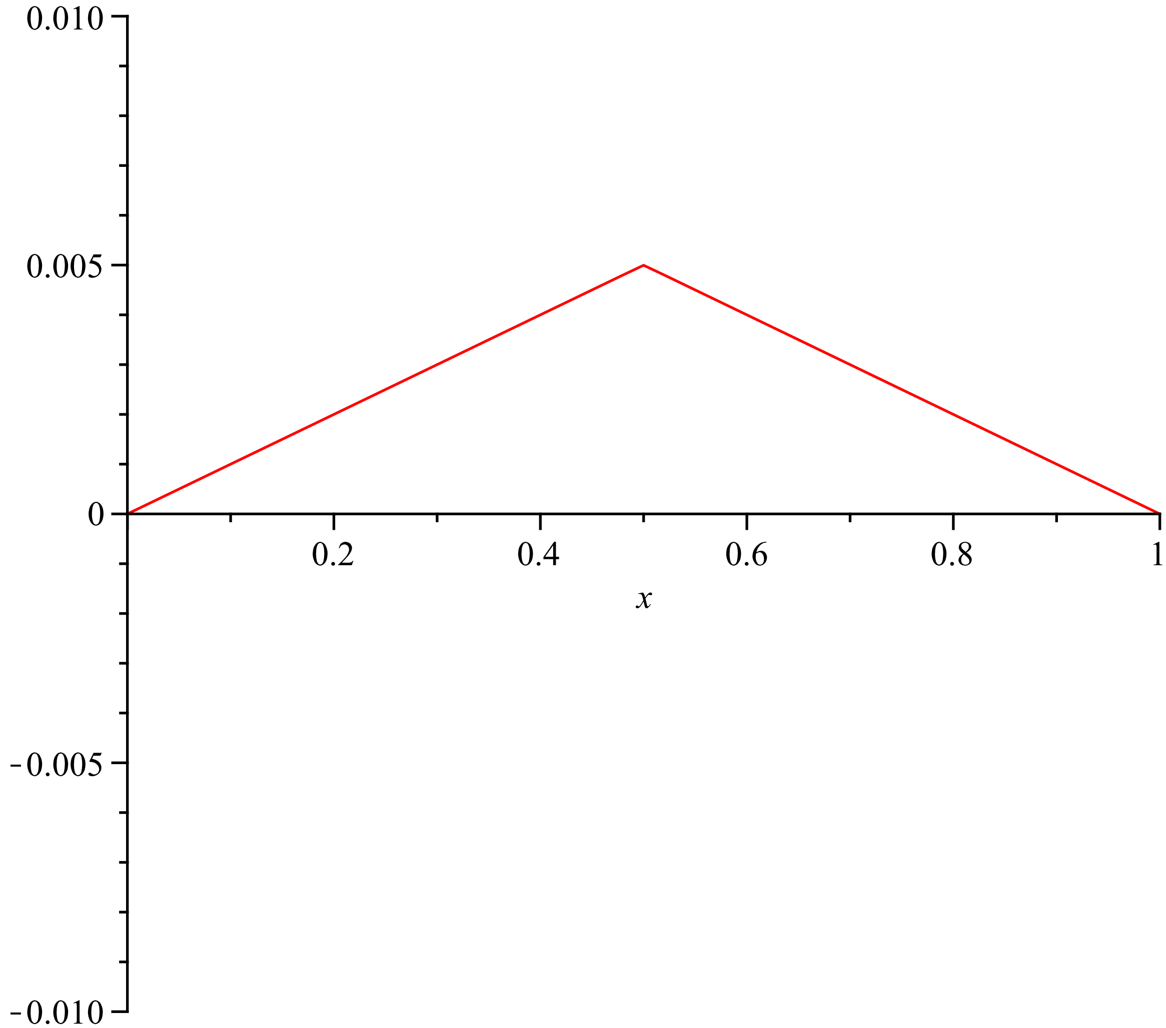
> $SolucionAproximada_{500} := y(x, t)_{500} = subs(b_n = Parametrote, a_n = 0, Sum(rhs(SolucionParticular), n = 1..500)):$

> $plot(subs(c = 1, t = 0, rhs(SolucionAproximada_{500})), x = 0..1);$



> $with(plots):$

> $animate\left(subs\left(c=1, rhs\left(SolucionAproximada_{500}\right)\right), x=0..1, t=0..4, frames=150, view=[0..1, -0.01..0.01]\right)$


0.010
0.005
0
-0.005
-0.010
0.2
0.4
0.6
0.8
1
x

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