> `restart`

> $SolPart_1 := y_1(x) = \exp(3 \cdot x)$

$$SolPart_1 := y_1(x) = e^{3x} \tag{1}$$

> $SolPart_2 := y_2(x) = \cos(4 \cdot x)$

$$SolPart_2 := y_2(x) = \cos(4x) \tag{2}$$

> $SolPart_3 := y_3(x) = \sin(4 \cdot x)$

$$SolPart_3 := y_3(x) = \sin(4x) \tag{3}$$

> $WW := array\big(\,[\,[\,rhs(SolPart_1), rhs(SolPart_2), rhs(SolPart_3)\,], [\,rhs(diff(SolPart_1, x)),$
$rhs(diff(SolPart_2, x)), rhs(diff(SolPart_3, x))\,], [\,rhs(diff(SolPart_1, x\$2)),$
$rhs(diff(SolPart_2, x\$2)), rhs(diff(SolPart_3, x\$2))\,]\,]\big)$

$$WW := \begin{bmatrix} e^{3x} & \cos(4x) & \sin(4x) \\ 3\,e^{3x} & -4\sin(4x) & 4\cos(4x) \\ 9\,e^{3x} & -16\cos(4x) & -16\sin(4x) \end{bmatrix} \tag{4}$$

> $with(linalg):$

> $WWW := wronskian\big(\,[\,rhs(SolPart_1), rhs(SolPart_2), rhs(SolPart_3)\,], x\big)$

$$WWW := \begin{bmatrix} e^{3x} & \cos(4x) & \sin(4x) \\ 3\,e^{3x} & -4\sin(4x) & 4\cos(4x) \\ 9\,e^{3x} & -16\cos(4x) & -16\sin(4x) \end{bmatrix} \tag{5}$$

> $comprobacion_1 := simplify(det(WWW)) \neq 0$

$$comprobacion_1 := 100\,e^{3x} \neq 0 \tag{6}$$

> $SolucionGeneral := y(x) = C_1 \cdot rhs(SolPart_1) + C_2 \cdot rhs(SolPart_2) + C_3 \cdot rhs(SolPart_3)$

$$SolucionGeneral := y(x) = C_1\,e^{3x} + C_2\cos(4x) + C_3\sin(4x) \tag{7}$$

> $Sistema := diff(SolucionGeneral, x), diff(SolucionGeneral, x\$2), diff(SolucionGeneral, x$
$\$3): Sistema_1; Sistema_2; Sistema_3;$

$$\frac{d}{dx}\,y(x) = 3\,C_1\,e^{3x} - 4\,C_2\sin(4x) + 4\,C_3\cos(4x)$$

$$\frac{d^2}{dx^2}\,y(x) = 9\,C_1\,e^{3x} - 16\,C_2\cos(4x) - 16\,C_3\sin(4x)$$

$$\frac{d^3}{dx^3}\,y(x) = 27\,C_1\,e^{3x} + 64\,C_2\sin(4x) - 64\,C_3\cos(4x) \tag{8}$$

> $Parametro := simplify(solve(\{Sistema\}, \{C_1, C_2, C_3\})): Parametro_1; Parametro_2;$
$Parametro_3;$

$$C_1 = \frac{1}{75}\left(\frac{d^3}{dx^3}\,y(x) + 16\left(\frac{d}{dx}\,y(x)\right)\right)e^{-3x}$$

$$C_2 = \frac{3}{400}\cos(4\,x)\left(\frac{d^3}{dx^3}\,y(x)\right) + \frac{3}{25}\cos(4\,x)\left(\frac{d}{dx}\,y(x)\right) - \frac{1}{16}\cos(4\,x)\left(\frac{d^2}{dx^2}\,y(x)\right)$$

$$+ \frac{1}{100}\sin(4\,x)\left(\frac{d^3}{dx^3}\,y(x)\right) - \frac{9}{100}\left(\frac{d}{dx}\,y(x)\right)\sin(4\,x)$$

$$C_3 = \frac{3}{400}\sin(4\,x)\left(\frac{d^3}{dx^3}\,y(x)\right) + \frac{3}{25}\left(\frac{d}{dx}\,y(x)\right)\sin(4\,x) - \frac{1}{100}\cos(4\,x)\left(\frac{d^3}{dx^3}\,y(x)\right) \qquad \textbf{(9)}$$

$$+ \frac{9}{100}\cos(4\,x)\left(\frac{d}{dx}\,y(x)\right) - \frac{1}{16}\left(\frac{d^2}{dx^2}\,y(x)\right)\sin(4\,x)$$

> $EcuacionInicial := simplify(subs(C_1 = rhs(Parametro_1), C_2 = rhs(Parametro_2), C_3 = rhs(Parametro_3), SolucionGeneral))$

$$EcuacionInicial := y(x) = \frac{1}{48}\frac{d^3}{dx^3}\,y(x) + \frac{1}{3}\frac{d}{dx}\,y(x) - \frac{1}{16}\frac{d^2}{dx^2}\,y(x) \qquad \textbf{(10)}$$

> $EcuacionFinal := rhs(EcuacionInicial)\cdot 48 - lhs(EcuacionInicial)\cdot 48 = 0$

$$EcuacionFinal := \frac{d^3}{dx^3}\,y(x) + 16\left(\frac{d}{dx}\,y(x)\right) - 3\left(\frac{d^2}{dx^2}\,y(x)\right) - 48\,y(x) = 0 \qquad \textbf{(11)}$$

> $SolGral := dsolve(EcuacionFinal)$

$$SolGral := y(x) = \_C1\,e^{3x} + \_C2\sin(4\,x) + \_C3\cos(4\,x) \qquad \textbf{(12)}$$

> $SolucionGeneral;$

$$y(x) = C_1\,e^{3x} + C_2\cos(4\,x) + C_3\sin(4\,x) \qquad \textbf{(13)}$$

> $Comprobacion_2 := eval(subs(y(x) = rhs(SolucionGeneral), EcuacionFinal))$

$$Comprobacion_2 := 0 = 0 \qquad \textbf{(14)}$$

> $SolPart_4 := y(x) = -11\cdot rhs(SolPart_1) + 14\cdot rhs(SolPart_2) + 25\cdot rhs(SolPart_3)$

$$SolPart_4 := y(x) = -11\,e^{3x} + 14\cos(4\,x) + 25\sin(4\,x) \qquad \textbf{(15)}$$

> $Comprobacion_3 := eval(subs(y(x) = rhs(SolPart_4), EcuacionFinal))$

$$Comprobacion_3 := 0 = 0 \qquad \textbf{(16)}$$

> $Condiciones := y(0) = 1, D(y)(0) = -5, D(D(y))(0) = 6$

$$Condiciones := y(0) = 1, D(y)(0) = -5, D^{(2)}(y)(0) = 6 \qquad \textbf{(17)}$$

> $SolPart_5 := dsolve(\{EcuacionFinal, Condiciones\})$

$$SolPart_5 := y(x) = \frac{22}{25}\,e^{3x} - \frac{191}{100}\sin(4\,x) + \frac{3}{25}\cos(4\,x) \qquad \textbf{(18)}$$

> $Sistemita := subs(x = 0, rhs(SolucionGeneral) = 1), subs(x = 0, rhs(diff(SolucionGeneral, x)) = -5), subs(x = 0, rhs(diff(SolucionGeneral, x\$2)) = 6) : Sistemita_1; Sistemita_2; Sistemita_3;$
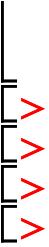
$$C_1 + C_2 = 1$$

$$3\,C_1 + 4\,C_3 = -5$$

$$9\,C_1 - 16\,C_2 = 6 \qquad \textbf{(19)}$$

> $Parametrito := solve(\{Sistemita\}, \{C_1, C_2, C_3\})$

$$\mathit{Parametrito} := \left\{ C_1 = \frac{22}{25}, C_2 = \frac{3}{25}, C_3 = -\frac{191}{100} \right\}$$

(20)

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