

$$\exists \text{DOL}(z) \subset \mathbb{H}$$

$$\frac{d^2 y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0$$

$y=0$ trivial

$$y = e^{mx} \Rightarrow m^2 + a_1 m + a_2 = 0 \quad \text{Ecuación Característica}$$

CASO I.
2 raíces $\begin{cases} m_1 \\ m_2 \end{cases} \quad m_1 \neq m_2 \in \mathbb{R}$

$$y_g = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$

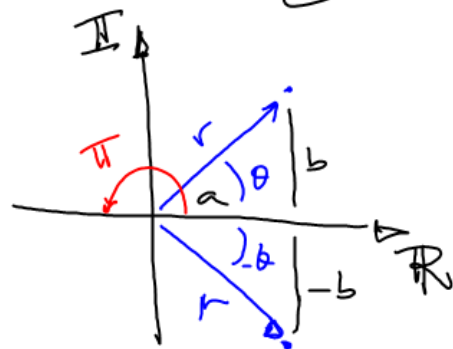
CASO II ——— $m_1 = m_2 \in \mathbb{R}$

CASO III ——— $m_1 \neq m_2 \in \mathbb{C}$

$$y_g = c_1 e^{(a+bi)x} + c_2 e^{(a-bi)x}$$

$$\forall x \in \mathbb{R} \Rightarrow y \in \mathbb{R}$$

Euler $e^{\pi i} + 1 = 0$



$$m_1 = a + bi$$

$$i = \sqrt{-1}$$

$$m_2 = a - bi$$

$$re^{\theta i} = r \cos(\theta) + r \sin(\theta) i$$

$$re^{-\theta i} = r \cos(\theta) - r \sin(\theta) i$$

$$e^{\theta i} = \cos(\theta) + \sin(\theta) i$$

$$e^{-\theta i} = \cos(\theta) - \sin(\theta) i \rightarrow \theta = \pi [\text{rad}]$$

$$e^{\pi i} = \cos(\pi) + \sin(\pi) i \quad \cos(\pi) = -1$$

$$\sin(\pi) = 0$$

$$e^{\pi i} = -1 \Rightarrow \boxed{e^{\pi i} + 1 = 0}$$

$$m^2 + a_1 m + a_2 = 0$$

$$m_1 \neq m_2 \in \mathbb{C} \quad \begin{array}{l} m_1 = a + bi \\ m_2 = a - bi \end{array} \quad \begin{array}{l} a \in \mathbb{R} \\ b \in \mathbb{R}^+ \end{array}$$

$$y_g = C_1 e^{(a+bi)x} + C_2 e^{(a-bi)x}$$

$$y_g = C_1 e^{ax} e^{bxi} + C_2 e^{ax} e^{-bxi}$$

$$y_g = e^{ax} (C_1 e^{bxi} + C_2 e^{-bxi})$$

$$y_g = e^{ax} (C_1 [\cos(bx) + \text{sen}(bx)i] + C_2 [\cos(bx) - \text{sen}(bx)i])$$

$$y_g = e^{ax} ([C_1 + C_2] \cos(bx) + [C_1 i - C_2 i] \text{sen}(bx))$$

$$y_g = \overset{C_{10}}{C_1} e^{ax} \cos(bx) + \overset{C_{20}}{C_2} e^{ax} \text{sen}(bx) \quad \begin{array}{l} \forall x \in \mathbb{R} \\ y \in \mathbb{R} \end{array}$$

$$m_1 = a + bi$$

$$m_2 = a - bi \quad m_1 \neq m_2 \in \mathbb{C}$$

$$\forall C_i \in \mathbb{R}$$

$$\frac{d^2 y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0 \quad \text{EDO}(2) \text{ LCC H.}$$

$$m^2 + a_1 m + a_2 = 0 \quad \text{E.C.}$$

$$\begin{matrix} m_1 \\ m_2 \end{matrix} \begin{cases} m_1 = m_2 \end{cases}$$

$$y_g = C_1 e^{m_1 x} + C_2 (x e^{m_1 x})$$

$$(m - m_1)^2 = 0$$

$$m_1 \neq m_2$$

$$(m - m_1)(m - m_2) = 0$$

$$(m - m_2) + (m - m_1) = 0$$

 $\frac{d}{dm}$

$$2(m - m_1) = 0$$

$$2m + a_1 = 0$$

$$m_1 = m_2$$

$$\frac{d}{dm} \begin{cases} e^{m,x} \xrightarrow{m=m_1} e^{m_1,x} \\ x e^{m,x} \xrightarrow{m=m_1} x e^{m_1,x} \end{cases}$$

$$\frac{d^2 y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0$$

$$\frac{d^2 y}{dx^2} \Rightarrow m_1^2 x e^{m_1 x} + 2m_1 e^{m_1 x}$$

$$+ a_1 \frac{dy}{dx} \Rightarrow a_1 m_1 x e^{m_1 x} + a_1 e^{m_1 x}$$

$$+ a_2 y \Rightarrow a_2 x e^{m_1 x}$$

$$0 = (\cancel{m_1^2 + a_1 m_1} + a_2) x e^{m_1 x} + (\cancel{2m_1 + a_1}) e^{m_1 x}$$

$$y = x e^{m_1 x}$$

$$\frac{dy}{dx} = m_1 x e^{m_1 x} + e^{m_1 x}$$

$$\frac{d^2 y}{dx^2} = m_1 (m_1 x e^{m_1 x} + e^{m_1 x}) + m_1 e^{m_1 x}$$

$$\frac{d^2 y}{dx^2} = m_1^2 x e^{m_1 x} + 2m_1 e^{m_1 x}$$

$$W = \begin{bmatrix} e^{m_1 x} & x e^{m_1 x} \\ m_1 e^{m_1 x} & m_1 x e^{m_1 x} + e^{m_1 x} \end{bmatrix}$$

$$|W| \neq 0.$$

$$\frac{d^2 y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0$$

CASO II

$$m^2 + a_1 m + a_2 = 0 \quad \left. \begin{matrix} m_1 \\ m_2 \end{matrix} \right\} m_1 = m_2$$

$$y_g = C_1 e^{m_1 x} + C_2 x e^{m_1 x}$$

$$\frac{d^2 y}{dx^2} = 0 \quad \text{EDO}(2) \text{ LccH}$$

$$\frac{d}{dx} \left(\frac{dy}{dx} \right) = 0 \quad \frac{dy}{dx} = C_1 \quad dy = C_1 dx$$

$$m^2 = 0 \quad \left. \begin{matrix} m_1 = 0 \\ m_2 = 0 \end{matrix} \right\} \neq \quad \int dy = C_1 \int dx$$

$$y_g = C_1 e^{m_1 x} + C_2 x e^{m_1 x}$$

$$y_g = C_1 + C_2 x$$

$$y + k_1 = C_1 (x + k_2)$$

$$y = C_1 x + (C_1 k_2 - k_1)$$

$$y = C_1 x + C_2$$