LINEAL

$$\frac{dy}{dx} + p(x)y = q(x) \Longleftarrow$$

$$\frac{dy}{dx} + p(x)y = 0 \rightarrow y_g = ce^{-\int p\,dx}$$

$$y_g = ce^{-\int p\,dx} + e^{-\int p\,dx}\int e^{\int p\,dx} q(x)\,dx$$

$$y_{g/n-h} = y_{g/h} + y_{p/q(x)}$$

$$y_{g/nh} = Ce^{-\int p\,dx} + e^{-\int p\,dx}\int e^{\int p(x)dx} q(x)\,dx$$

$$y_{g/n\text{-}h} = \left(\underbrace{C + \int e^{\int p\,dx} q(x)\,dx}_{A(x)}\right) e^{-\int p(x)dx}$$

$$y_{g/n\text{-}h} = A(x)\,e^{-\int p(x)dx}$$

Método
Parámetro
Variable

$$y_{g/h} = C_1 e^{-\int p(x)dx}$$

$$\frac{dy}{dx} - 3y = 4e^{2x}$$

$$\boxed{\begin{array}{l} \dfrac{dy}{dx} - 3y = 0 \\ p(x) = -3 \\ e^{-\int p(x)dx} \Rightarrow e^{-(-3\int dx)} \Rightarrow e^{3x} \\ y_{g/h} = Ce^{3x} \end{array}}$$

$$y_{g/n-h} = A(x)e^{3x}$$

$$\frac{dy}{dx} = 3A(x)e^{3x} + e^{3x}A'(x)$$

$$\underbrace{\left(3A(x)e^{3x} + A'(x)e^{3x}\right)}_{\frac{dy}{dx}} - 3\underbrace{\left(A(x)e^{3x}\right)}_{y} = 4e^{2x}$$

$$\cancel{3A(x)}e^{3x} - \cancel{3A(x)}e^{3x} + A'(x)e^{3x} = 4e^{2x}$$

$$A'(x)e^{3x} = 4e^{2x}$$

$$A'(x) = 4e^{2x}e^{-3x}$$

$$A'(x) = 4e^{-x}$$

$$A'(x) = 4e^{-x}$$

$$A(x) = 4\int e^{-x}dx$$

$$A(x) = 4\left(\frac{e^{-x}}{-1}\right) + C_1$$

$$y_{g/n.h} = \left(-4e^{-x} + C_1\right)e^{3x}$$

$$\boxed{y = C_1e^{3x} - 4e^{2x}}$$

$$\frac{dy}{dx} - 3y = 4e^{2x}$$

$$y_g = \underbrace{C_1 e^{3x} + C_2 e^{-3x}} + \underbrace{4xe^{3x} + 2e^{2x}}$$

EDO(2) Lcc NH

$$\boxed{\begin{gathered} y_{g/h} = C_1 e^{3x} + C_2 e^{-3x} \\ (m-3)(m+3)=0 \\ m^2 - 9 = 0 \\ \frac{d^2y}{dx^2} - 9y = 0 \end{gathered}}$$

$m_1 = 3$

$m_2 = -3$

$m_1 \neq m_2 \in \mathbb{R}$

CASO I

$$\frac{d^2y}{dx^2} - 9y = Q(x)$$

$$y_p = 4xe^{3x} + 2e^{2x}$$

$$\frac{dy}{dx} = 12xe^{3x} + 4e^{3x} + 4e^{2x}$$

$$\frac{d^2y}{dx^2} = 36xe^{3x} + 12e^{3x} + 12e^{3x} + 8e^{2x}$$

$$\frac{d^2y}{dx^2} = 36xe^{3x} + 24e^{3x} + 8e^{2x}$$

$$Q(x) = \left(\cancel{36xe^{3x}} + 24e^{3x} + 8e^{2x}\right) - \cancel{36xe^{3x}} - 18e^{2x}$$

$$\boxed{Q(x) = 24e^{3x} - 10e^{2x}}$$

$$y = C_1 e^{3x} + C_2 e^{-3x} + 4x e^{3x} + 2e^{2x}$$

$$\frac{d^2y}{dx^2} - 9y = 24e^{3x} - 10e^{2x}$$

$$\frac{d^2y}{dx^2} - 9y = 0 \Rightarrow y_h = C_1 e^{3x} + C_2 e^{-3x}$$

$$m^2 - 9 = 0$$

$$(m-3)(m+3) = 0$$

$$m_1 = 3 \quad m_2 = -3$$

$$y_{p-h} = A(x)e^{3x} + B(x)e^{-3x}$$

$$y = A(x)e^{3x} + B(x)e^{-3x}$$

$$\frac{dy}{dx} = 3A(x)e^{3x} - 3B(x)e^{-3x} + \underbrace{\left(A'(x)e^{3x} + B'(x)e^{-3x}\right)}_{=0}$$

$$\frac{dy}{dx} = 3A(x)e^{3x} - 3B(x)e^{-3x} + (0)$$

$$\frac{d^2y}{dx^2} = 9A(x)e^{3x} + 9B(x)e^{-3x} + \underbrace{\left(3A'(x)e^{3x} - 3B'(x)e^{-3x}\right)}_{=Q(x)}$$

$$\frac{d^2y}{dx^2} = 9A(x)e^{3x} + 9B(x)e^{-3x} + Q(x)$$

$$\frac{d^2y}{dx^2} \longleftrightarrow 9A(x)e^{3x} + 9B(x)e^{-3x} + Q(x)$$

$$+$$

$$-9y \longleftrightarrow -9A(x)e^{3x} - 9B(x)e^{-3x}$$

$$=$$

$$4xe^{3x} + 2e^{2x} = (0)e^{3x} + (0)e^{-3x} + Q(x)$$

$$A'(x)e^{3x}+B'(x)e^{-3x}=0$$

$$3A'(x)e^{3x}-3B'(x)e^{-3x}=4xe^{3x}+2e^{2x}$$

$$\begin{bmatrix} e^{3x} & e^{-3x} \\ 3e^{3x} & -3e^{-3x} \end{bmatrix}\begin{bmatrix} A'(x) \\ B'(x) \end{bmatrix}=\begin{bmatrix} 0 \\ 4xe^{3x}+2e^{2x} \end{bmatrix}$$

$$A'(x)=\frac{\begin{vmatrix} 0 & e^{-3x} \\ 4xe^{3x}+2e^{2x} & -3e^{-3x} \end{vmatrix}}{\begin{vmatrix} e^{3x} & e^{-3x} \\ 3e^{3x} & -3e^{-3x} \end{vmatrix}}=\frac{+\cancel{e^{-3x}}\left(4xe^{3x}+2e^{2x}\right)}{+6e^{3x}\cancel{e^{-3x}}}$$

$$=\frac{2}{3}x+\frac{1}{3}e^{-x}$$

$$A'(x)=\frac{2}{3}x+\frac{1}{3}e^{-x}$$

$$B'(x)=\frac{\begin{vmatrix} e^{3x} & 0 \\ 3e^{3x} & 4xe^{3x}+2e^{2x} \end{vmatrix}}{-6e^{3x}e^{-3x}}=\frac{\cancel{e^{3x}}\left(4xe^{3x}+2e^{2x}\right)}{-6\cancel{e^{3x}}e^{-3x}}$$

$$B'(x)=-\frac{2}{3}xe^{6x}-\frac{1}{3}e^{5x}$$

$$A(x) = \frac{2}{3}\int x\,dx + \frac{1}{3}\int e^{-x}\,dx$$

$$= \frac{2}{3}\cdot\left(\frac{x^2}{2}\right) + \frac{1}{3}\left(\frac{e^{-x}}{-1}\right) + C_1$$

$$B(x) = -\frac{2}{3}\int x e^{6x}\,dx - \frac{1}{3}\int e^{5x}\,dx$$