CAP. III: Sistemas EDOL(1) cc $\begin{cases} H \\ NH \end{cases}$

$$\frac{dx}{dt} = 3x + 4y \qquad x(t)$$

$$\frac{dy}{dt} = 2x + 5y \qquad y(t)$$

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método de sustitución.

$$4y = \frac{dx}{dt} - 3x$$

$$y = \frac{1}{4}\frac{dx}{dt} - \frac{3}{4}x$$

$$\xrightarrow{\frac{d}{dt}} \quad \frac{dy}{dt} = \frac{1}{4}\frac{d^2x}{dt^2} - \frac{3}{4}\frac{dx}{dt}$$

$$\left(\frac{1}{4}\frac{d^2x}{dt^2} - \frac{3}{4}\frac{dx}{dt}\right) = 2x + 5\left(\frac{1}{4}\frac{dx}{dt} - \frac{3}{4}x\right)$$

$$\frac{d^2x}{dt^2} - 3\frac{dx}{dt} = 8x + 5\frac{dx}{dt} - 15x$$

$$\frac{d^2x}{dt^2} - 3\frac{dx}{dt} - 5\frac{dx}{dt} - 8x + 15x = 0$$

$$\left.\begin{aligned} \frac{dx}{dt} &= 3x + 4y \\ \frac{dy}{dt} &= 2x + 5y \end{aligned}\right\} \Leftrightarrow \left\{ \frac{d^2x}{dt^2} - 8\frac{dx}{dt} + 7x = 0 \right\}$$

$$m^2 - 8m + 7 = 0$$

$$(m-1)(m-7) = 0$$

$$\boxed{x(t) = c_1 e^{t} + c_2 e^{7t}}$$

$$\frac{dx}{dt} = c_1 e^{t} + 7c_2 e^{7t}$$

$$y = \frac{1}{4}\left(c_1 e^{t} + 7c_2 e^{7t}\right) - \frac{3}{4}\left(c_1 e^{t} + c_2 e^{7t}\right)$$

$$\boxed{y(t) = -\frac{c_1}{2} e^{t} + c_2 e^{7t}}$$

$$\frac{dx}{dt} = 3x + 4y$$

$$\frac{dy}{dt} = 2x + 5y$$

$$\frac{dx}{dt} \quad c_1 e^{t} + 7c_2 e^{7t}$$

$$=$$

$$3x \quad 3c_1 e^{t} + 3c_2 e^{7t}$$

$$+$$

$$4y \quad -2c_1 e^{t} + 4c_2 e^{7t}$$

$$\frac{dy}{dt} \quad -\frac{c_1}{2} e^{t} + 7c_2 e^{7t}$$

$$=$$

$$2x \quad 2c_1 e^{t} + 2c_2 e^{7t}$$

$$+$$

$$5y \quad -\frac{5}{2} c_1 e^{t} + 5c_2 e^{7t}$$

$$x = c_1 e^{t} + c_2 e^{7t}$$

$$y = -\frac{c_1}{2} e^{t} + c_2 e^{7t}$$

$$\frac{dx}{dt} = c_1 e^{t} + 7c_2 e^{7t}$$

$$\frac{dy}{dt} = -\frac{c_1}{2} e^{t} + 7c_2 e^{7t}$$

$$\frac{dx}{dt} = 3x + 4y \qquad x(0) = 4$$

$$\frac{dy}{dt} = 2x + 5y \qquad y(0) = -5.$$

$$x = c_1 e^{t} + c_2 e^{7t} \qquad 4 = c_1(1) + c_2(1)$$

$$y = -\frac{c_1}{2} e^{t} + c_2 e^{7t} \qquad -5 = -\frac{c_1}{2}(1) + c_2(1)$$

$$c_1 + c_2 = 4 \qquad c_1 = 4 - c_2$$

$$-\frac{c_1}{2} + c_2 = -5 \qquad -\frac{(4-c_2)}{2} + c_2 = -5$$

$$c_1 = 4 - (-2) \qquad (4 - c_2) - 2c_2 = 10$$

$$\boxed{c_1 = 6} \qquad 4 - 3c_2 = 10$$

$$-3c_2 = 10 - 4$$

$$x = 6e^{t} - 2e^{7t} \qquad -3c_2 = 6$$

$$y = -3e^{t} - 2e^{7t} \qquad \boxed{c_2 = -2}$$

testada

$$\bar{x}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} \qquad \frac{d}{dt}\bar{x}(t) = \begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{bmatrix} \qquad \bar{x}(0) = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} \frac{dx}{dt} = 3x + 4y \\ \frac{dy}{dt} = 2x + 5y \end{bmatrix} \qquad \begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix} \times \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

A

$$\frac{d}{dt}\bar{x} = A\vec{x} \qquad \bar{x}(t) = e^{At} \times \bar{x}(0)$$

$$\left[e^{At}\right]^{-1} = \left[e^{A(-t)}\right] \qquad e^{-t} = \frac{1}{e^{t}}$$

$$\frac{d}{dt}\left[e^{At}\right] = A\left[e^{At}\right] \qquad \frac{d}{dt}e^{at} = a e^{at}$$

$$\left[e^{At}\right]_{t=0} = I \qquad e^{(0)} = 1.$$