$$y = C_1 e^{2x} + C_2 e^{-2x} + C_3 e^{x} \Longleftarrow y_1$$

$$(m-2)(m+2)(m-1) = 0$$

$$(m^2-4)(m-1) = 0$$

$$m^3 - m^2 - 4m + 4 = 0$$

$$\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$$

$$\frac{dy_1}{dx} = a_{11} y_1 + a_{12} y_2 + a_{13} y_3$$

$$\frac{dy_2}{dx} = a_{21} y_1 + a_{22} y_2 + a_{23} y_3$$

$$\frac{dy_3}{dx} = a_{31} y_1 + a_{32} y_2 + a_{33} y_3$$

$$yy_1(x) = \frac{1}{6} K_1 e^{-2x} + \frac{4}{3} K_1 e^{x} - \frac{1}{2} K_1 e^{2x} + \frac{1}{4} K_2 e^{2x} - \frac{1}{4} K_2 e^{-2x} + \frac{1}{4} K_3 e^{2x} - \frac{1}{3} K_3 e^{x} + \frac{1}{12} K_3 e^{-2x}$$

$$yy_2(x) = -K_1 e^{2x} + \frac{4}{3} K_1 e^{x} - \frac{1}{3} K_1 e^{-2x} + \frac{1}{2} K_2 e^{-2x} + \frac{1}{2} K_2 e^{2x} + \frac{1}{2} K_3 e^{2x} - \frac{1}{3} K_3 e^{x} - \frac{1}{6} K_3 e^{-2x}$$

$$yy_3(x) = \frac{4}{3} K_1 e^{x} + \frac{2}{3} K_1 e^{-2x} - 2 K_1 e^{2x} + K_2 e^{2x} - K_2 e^{-2x} + \frac{1}{3} K_3 e^{-2x} + K_3 e^{2x} - \frac{1}{3} K_3 e^{x}$$

$$yy_1(x) = \left(\frac{4}{3}k_1 - \frac{1}{3}k_3\right)e^{x} + \left(-\frac{1}{2}k_1 + \frac{1}{4}k_2 + \frac{1}{4}k_3\right)e^{2x} +$$
$$+ \left(\frac{1}{6}k_1 - \frac{1}{4}k_2 + \frac{1}{12}k_3\right)e^{-2x}$$

$$C_1 = \frac{4}{3}k_1 - \frac{1}{3}k_3$$

$$C_2 = -\frac{1}{2}k_1 + \frac{1}{4}k_2 + \frac{1}{4}k_3 \qquad yy_1(x) = C_1 e^{x} + C_2 e^{2x} + C_3 e^{-2x}$$

$$C_3 = \frac{1}{6}k_1 - \frac{1}{4}k_2 + \frac{1}{12}k_3$$

$$yy_2(x) = \left(\frac{4}{3}k_1 - \frac{1}{3}k_3\right)e^{x} + \left(-k_1 + \frac{1}{2}k_2 + \frac{1}{2}k_3\right)e^{2x} + \left(-\frac{1}{3}k_1 + \frac{1}{2}k_2 - \frac{1}{6}k_3\right)e^{-2x}$$

$$yy_2(x) = C_1 e^{x} + 2C_2 e^{2x} - 2C_3 e^{-2x}$$

$$yy_3(x) = \left(\frac{4}{3}k_1 - \frac{1}{3}k_3\right)e^{x} + \left(-2k_1 + k_2 + k_3\right)e^{2x} + \left(\frac{2}{3}k_1 - k_2 + \frac{1}{3}k_3\right)e^{-2x}$$

$$yy_3(x) = C_1 e^{x} + 4C_2 e^{2x} + 4C_3 e^{-2x}$$

$$\frac{dx_1}{dt} = 4x_1 + 6x_2 + 4e^{2t} + t^3$$

$$\frac{dx_2}{dt} = 2x_1 + 3x_2 + 8e^{2t} + t + 1$$

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 4e^{2t} + t^3 \\ 8e^{2t} + t + 1 \end{bmatrix}$$

$$\frac{d}{dt}\bar{x} = A\bar{x} + \bar{b}(t) \quad \Bigg| \quad \frac{d}{dt}\bar{x} = A\bar{x} \Rightarrow \bar{x} = e^{At}\bar{x}(0)$$

$$\bar{x} = e^{At}\bar{x}(0) + \int_0^t e^{A(t-\tau)} \bar{b}(\tau)\, d\tau.$$

$$\bar{x}\Big|_{t=0} = I\bar{x}(0) + (0)$$

$A_{n\times n}$

$$e^{At} = \beta_0(t)I + \beta_1(t)A + \dots + \beta_{n-1}(t)A^{n-1}$$

$$e^{At} = \beta_0 I + \beta_1 A$$

$$e^{At} = \beta_0 I + \beta_1 A + \beta_2 A^2$$

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \qquad \begin{vmatrix} 2-\lambda & 3 \\ 1 & 4-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)(4-\lambda) - (3)(1) = 0$$

$$A^2 = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \qquad \lambda^2 - 6\lambda + 8 - 3 = 0$$

$$A^2 = \begin{bmatrix} 7 & 18 \\ 6 & 19 \end{bmatrix} \qquad \lambda^2 - 6\lambda + 5 = 0$$

$$(\lambda-1)(\lambda-5) = 0 \qquad \lambda_1 = 1, \quad \lambda_2 = 5$$

$$A^2 - 6A + 5I = [0] \qquad \text{Hamilton-Cayle.}$$

$$\begin{bmatrix} 7 & 18 \\ 6 & 19 \end{bmatrix} + \begin{bmatrix} -12 & -18 \\ -6 & -24 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$e^{At} = \beta_0 I + \beta_1 A$$

$$e^{\lambda_1 t} = \beta_0 + \lambda_1 \beta_1$$

$$e^{\lambda_2 t} = \beta_0 + \lambda_2 \beta_1$$

$$e^{t} = \beta_0 + \beta_1$$

$$e^{5t} = \beta_0 + 5\beta_1$$

$$-e^{t} = -\beta_0 - \beta_1$$

$$e^{5t} - e^{t} = 4\beta_1$$

$$\beta_1 = \frac{1}{4}\left(e^{5t} - e^{t}\right)$$

$$\beta_0 = e^{t} - \beta_1$$

$$\beta_0 = e^{t} - \left(\frac{1}{4}e^{5t} - \frac{1}{4}e^{t}\right)$$

$$\beta_0 = -\frac{1}{4}e^{5t} + \frac{3}{4}e^{t}$$

$$e^{At} = \left(-\frac{1}{4}e^{5t} + \frac{3}{4}e^{t}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \left(\frac{1}{4}e^{5t} - \frac{1}{4}e^{t}\right)\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

$$e^{At} = \Big[\ \frac{1}{4}e^{5t} + \frac{1}{4}e^{t}$$

$$\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$$

$$y(x) \Rightarrow y_1(x)$$

$$\frac{dy}{dx} \Rightarrow \frac{dy_1}{dx} = y_2(x)$$

$$\frac{d^2y}{dx^2} \Rightarrow \frac{dy_2}{dx} = y_3(x)$$

$$\frac{d^3y}{dx^3} \Rightarrow \frac{dy_3}{dx}$$

$$\frac{dy_3}{dx} - y_3 - 4y_2 + 4y_1 = 0$$

$$\frac{dy_3}{dx} = -4y_1 + 4y_2 + y_3$$

$$\frac{dy_1}{dx} = y_2$$

$$\frac{dy_2}{dx} = y_3$$

$$\frac{dy_3}{dx} = -4y_1 + 4y_2 + y_3$$

$$\frac{d}{dx}\begin{bmatrix} y_1(x) \\ y_2(x) \\ y_3(x) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & 4 & 1 \end{bmatrix}\begin{bmatrix} y_1(x) \\ y_2(x) \\ y_3(x) \end{bmatrix}$$