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[> restart
> AA := array([ [0, 1], [-1, 0] ])
                                     
$$AA := \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \quad (1)$$

> Xcero := array([2, -3])
                                     
$$Xcero := \begin{bmatrix} 2 & -3 \end{bmatrix} \quad (2)$$

> B := array([3·exp(2·t), 4·cos(t)])
                                     
$$B := \begin{bmatrix} 3 e^{2t} & 4 \cos(t) \end{bmatrix} \quad (3)$$

> with(linalg):
> MatExp := exponential(AA, t)
                                     
$$MatExp := \begin{bmatrix} \cos(t) & \sin(t) \\ -\sin(t) & \cos(t) \end{bmatrix} \quad (4)$$

> SolHom := evalm( MatExp &* Xcero ) : SolHom1; SolHom2;
                                     
$$\begin{matrix} 2 \cos(t) - 3 \sin(t) \\ -2 \sin(t) - 3 \cos(t) \end{matrix} \quad (5)$$

> MatExpTau := map(rcurry(eval, t='t - tau'), MatExp)
                                     
$$MatExpTau := \begin{bmatrix} \cos(t - \tau) & \sin(t - \tau) \\ -\sin(t - \tau) & \cos(t - \tau) \end{bmatrix} \quad (6)$$

> Btau := map(rcurry(eval, t='tau'), B)
                                     
$$Btau := \begin{bmatrix} 3 e^{2\tau} & 4 \cos(\tau) \end{bmatrix} \quad (7)$$

> ProdMatExpTauBtau := evalm(MatExpTau &* Btau)
ProdMatExpTauBtau :=
                                     
$$\begin{bmatrix} 3 \cos(t - \tau) e^{2\tau} + 4 \sin(t - \tau) \cos(\tau) & -3 \sin(t - \tau) e^{2\tau} + 4 \cos(t - \tau) \cos(\tau) \end{bmatrix} \quad (8)$$

> IntProd := map(int, ProdMatExpTauBtau, tau = 0 ..t)
IntProd :=
                                     
$$\begin{bmatrix} -\frac{6}{5} \cos(t) + \frac{3}{5} \sin(t) + \frac{6}{5} e^{2t} + 2 \sin(t) t, \frac{3}{5} \cos(t) + \frac{16}{5} \sin(t) - \frac{3}{5} e^{2t} \\ + 2 \cos(t) t \end{bmatrix} \quad (9)$$

> IntProd1; IntProd2
                                     
$$\begin{matrix} -\frac{6}{5} \cos(t) + \frac{3}{5} \sin(t) + \frac{6}{5} e^{2t} + 2 \sin(t) t \\ \frac{3}{5} \cos(t) + \frac{16}{5} \sin(t) - \frac{3}{5} e^{2t} + 2 \cos(t) t \end{matrix} \quad (10)$$

> Comprobacion1 := map(rcurry(eval, t='0'), IntProd)
                                     
$$Comprobacion_1 := \begin{bmatrix} 0 & 0 \end{bmatrix} \quad (11)$$


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> $SolUno := x_1(t) = SolHom_1 + IntProd_1; SolDos := x_2(t) = SolHom_2 + IntProd_2$

$$SolUno := x_1(t) = \frac{4}{5} \cos(t) - \frac{12}{5} \sin(t) + \frac{6}{5} e^{2t} + 2 \sin(t) t$$

$$SolDos := x_2(t) = \frac{6}{5} \sin(t) - \frac{12}{5} \cos(t) - \frac{3}{5} e^{2t} + 2 \cos(t) t \quad (12)$$

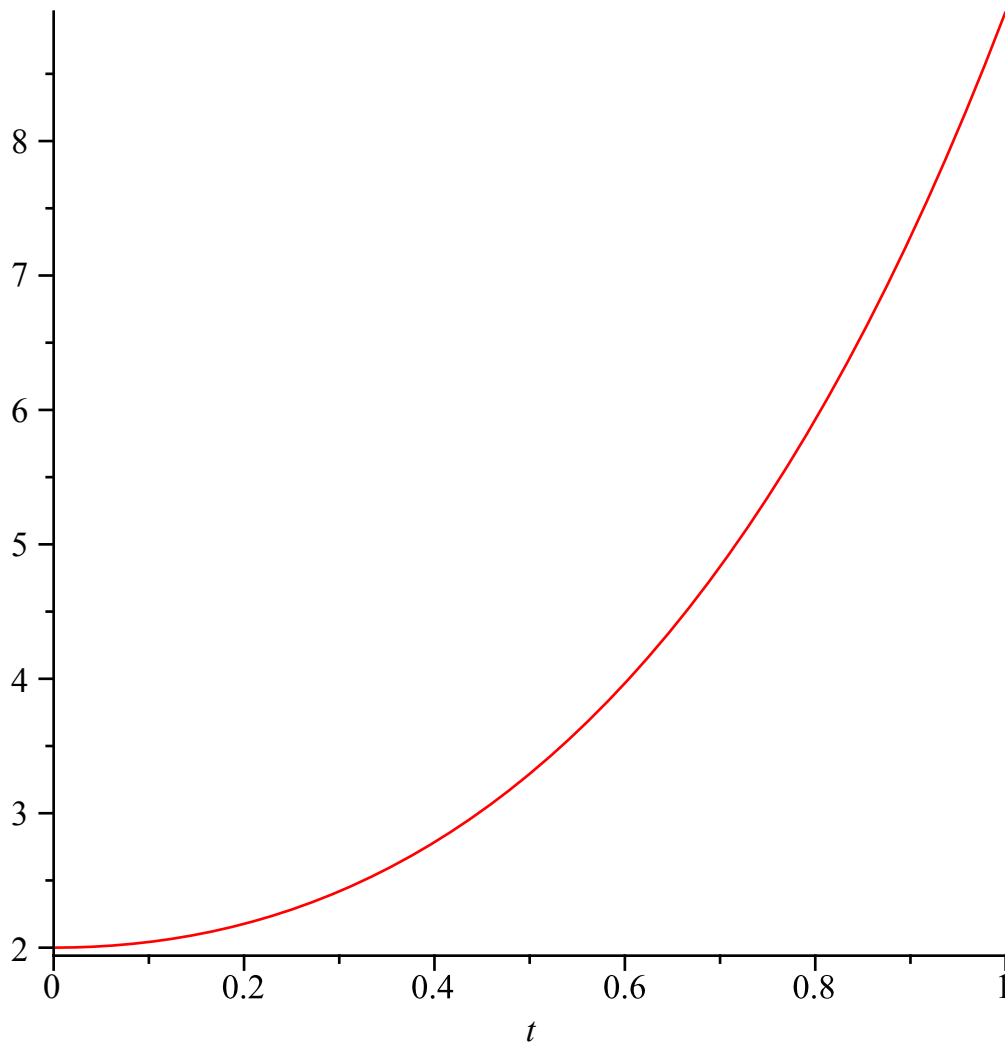
> $Comprobacion_2 := simplify(subs(t=0, SolUno))$

$$Comprobacion_2 := x_1(0) = 2 \quad (13)$$

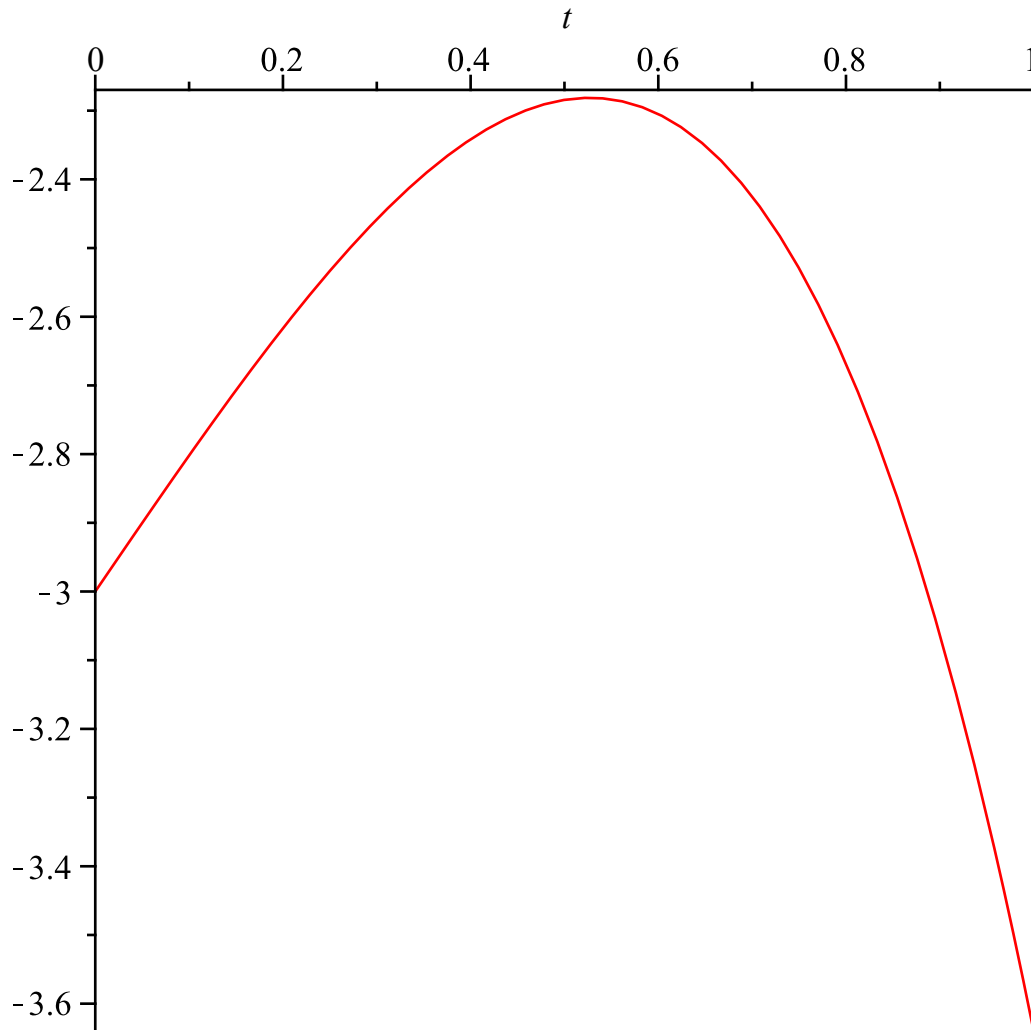
> $Comprobacion_3 := simplify(subs(t=0, SolDos))$

$$Comprobacion_3 := x_2(0) = -3 \quad (14)$$

> $plot(rhs(SolUno), t=0..1)$



> $plot(rhs(SolDos), t=0..1)$



> $Sistema := \text{diff}(x_1(t), t) = x_2(t) + 3 \cdot \exp(2 \cdot t), \text{diff}(x_2(t), t) = -x_1(t) + 4 \cdot \cos(t) : Sistema_1;$
 $Sistema_2$

$$\frac{d}{dt} x_1(t) = x_2(t) + 3 e^{2t}$$

$$\frac{d}{dt} x_2(t) = -x_1(t) + 4 \cos(t) \quad (15)$$

> $Condiciones := x_1(0) = 2, x_2(0) = -3$

$$Condiciones := x_1(0) = 2, x_2(0) = -3 \quad (16)$$

> $Comprobacion_4 := \text{simplify}(\text{eval}(\text{subs}(x_1(t) = rhs(SolUno), x_2(t) = rhs(SolDos),$
 $lhs(Sistema_1) - rhs(Sistema_1) = 0)))$

$$Comprobacion_4 := 0 = 0 \quad (17)$$

> $Comprobacion_5 := \text{simplify}(\text{eval}(\text{subs}(x_1(t) = rhs(SolUno), x_2(t) = rhs(SolDos),$
 $lhs(Sistema_2) - rhs(Sistema_2) = 0)))$

$$Comprobacion_5 := 0 = 0 \quad (18)$$

> $Solucion := \text{dsolve}(\{Sistema, Condiciones\}) : Solucion_1; Solucion_2$

$$\begin{aligned}x_1(t) &= \frac{4}{5} \cos(t) - \frac{12}{5} \sin(t) + \frac{6}{5} e^{2t} + 2 \sin(t) t \\x_2(t) &= \frac{6}{5} \sin(t) - \frac{12}{5} \cos(t) - \frac{3}{5} e^{2t} + 2 \cos(t) t\end{aligned}\tag{19}$$

> SolUno; SolDos

$$\begin{aligned}x_1(t) &= \frac{4}{5} \cos(t) - \frac{12}{5} \sin(t) + \frac{6}{5} e^{2t} + 2 \sin(t) t \\x_2(t) &= \frac{6}{5} \sin(t) - \frac{12}{5} \cos(t) - \frac{3}{5} e^{2t} + 2 \cos(t) t\end{aligned}\tag{20}$$

> restart

> AA := array([[3,-2], [-5, 4]])

$$AA := \begin{bmatrix} 3 & -2 \\ -5 & 4 \end{bmatrix}\tag{21}$$

> Xinicial := array([10,-12])

$$Xinicial := \begin{bmatrix} 10 & -12 \end{bmatrix}\tag{22}$$

> BB := array([t·2 + 4·t, 2·exp(-t) + 5·cos(3·t)])

$$BB := \begin{bmatrix} t^2 + 4t & 2e^{-t} + 5\cos(3t) \end{bmatrix}\tag{23}$$

> with(linalg) :

> MatExp := exponential(AA, t) : evalf(MatExp[1, 1], 3)

$$0.578 e^{0.300t} + 0.422 e^{6.70t}\tag{24}$$

> map(evalf, MatExp, 3)

$$MatExpVista := \begin{bmatrix} 0.578 e^{0.300t} + 0.422 e^{6.70t} & -0.312 e^{6.70t} + 0.312 e^{0.300t} \\ -0.781 e^{6.70t} + 0.781 e^{0.300t} & 0.422 e^{0.300t} + 0.578 e^{6.70t} \end{bmatrix}\tag{25}$$

> MatExpDesplazada := map(rcurry(eval, t=t-10'), MatExp) :

> map(evalf, MatExpDesplazada, 3)

$$\begin{bmatrix} 0.578 e^{0.300t-3.00} + 0.422 e^{6.70t-67.0} & -0.312 e^{6.70t-67.0} + 0.312 e^{0.300t-3.00} \\ -0.781 e^{6.70t-67.0} + 0.781 e^{0.300t-3.00} & 0.422 e^{0.300t-3.00} + 0.578 e^{6.70t-67.0} \end{bmatrix}\tag{26}$$

> SolHom := evalm(MatExpDesplazada &* Xinicial) :

> map(evalf, SolHom₁, 3); map(evalf, SolHom₂, 3)

$$\begin{aligned} & 2.04 e^{0.300t-3.00} + 7.96 e^{6.70t-67.0} \\ & -14.77 e^{6.70t-67.0} + 2.77 e^{0.300t-3.00} \end{aligned}\tag{27}$$

> MatExpTau := map(rcurry(eval, t=t-tau'), MatExp) :

> map(evalf, MatExpTau, 3)

$$\begin{bmatrix} 0.578 e^{0.300t-0.300\tau} + 0.422 e^{6.70t-6.70\tau} & -0.312 e^{6.70t-6.70\tau} + 0.312 e^{0.300t-0.300\tau} \\ -0.781 e^{6.70t-6.70\tau} + 0.781 e^{0.300t-0.300\tau} & 0.422 e^{0.300t-0.300\tau} + 0.578 e^{6.70t-6.70\tau} \end{bmatrix}\tag{28}$$

> BBtau := map(rcurry(eval, t=tau'), BB) :

> map(evalf, BBtau, 3)

$$\left[\tau^2 + 4. \tau \cdot 2. e^{-1. \tau} + 5. \cos(3. \tau) \right] \quad (29)$$

> *Prod* := simplify(evalm(MatExpTau &* BBtau)) :

> map(evalf, Prod, 3)

$$\begin{aligned} & [0.578 e^{0.300 t - 0.300 \tau} \tau^2 + 2.31 e^{0.300 t - 0.300 \tau} \tau + 0.422 e^{6.70 t - 6.70 \tau} \tau^2 + 1.69 e^{6.70 t - 6.70 \tau} \tau \\ & - 0.625 e^{6.70 t - 7.70 \tau} - 1.56 e^{6.70 t - 6.70 \tau} \cos(3. \tau) + 0.625 e^{0.30 t - 1.30 \tau} \\ & + 1.56 e^{0.300 t - 0.300 \tau} \cos(3. \tau), -0.781 e^{6.70 t - 6.70 \tau} \tau^2 - 3.12 e^{6.70 t - 6.70 \tau} \tau \\ & + 0.781 e^{0.300 t - 0.300 \tau} \tau^2 + 3.12 e^{0.300 t - 0.300 \tau} \tau + 0.844 e^{0.30 t - 1.30 \tau} \\ & + 2.11 e^{0.300 t - 0.300 \tau} \cos(3. \tau) + 1.16 e^{6.70 t - 7.70 \tau} + 2.89 e^{6.70 t - 6.70 \tau} \cos(3. \tau)] \end{aligned} \quad (30)$$

> IntProd := map(int, Prod, tau = 10 ..t) :

> map(evalf, IntProd, 3)

$$\begin{aligned} & [0.000174 (2.70 10^6 e^{1.30 t + 29.0} + 60000. e^{7.70 t - 35.} - 460. e^{7.70 t - 45.} + 2760. e^{1.30 t + 19.0} \\ & + 9840. e^{32.0 + t} \sin(t) \cos(t)^2 - 2460. e^{32.0 + t} \sin(t) + 3280. e^{32.0 + t} \cos(t)^3 \\ & - 2460. e^{32.0 + t} \cos(t) - 1.82 10^{17} - 1.21 10^5 e^{32.0 + t} t - 3.99 10^5 e^{32.0 + t} \\ & - 11500. e^{32.0 + t} t^2) e^{-32.0 - 1. t}, -0.0000871 (-7.29 10^6 e^{1.30 t + 29.0} + 1.9 10^5 e^{7.70 t - 35.} \\ & - 1720. e^{7.70 t - 45.} - 7460. e^{1.30 t + 19.0} - 39400. e^{32.0 + t} \sin(t) \cos(t)^2 \\ & + 9840. e^{32.0 + t} \sin(t) + 19700. e^{32.0 + t} \cos(t)^3 - 14800. e^{32.0 + t} \cos(t) + 7.25 10^{17} \\ & + 3.16 10^5 e^{32.0 + t} t + 1.08 10^6 e^{32.0 + t} + 28700. e^{32.0 + t} t^2) e^{-32.0 - 1. t}] \end{aligned} \quad (31)$$

> Comprobacion₁ := simplify(map(rcurry(eval, t=10'), IntProd))

$$Comprobacion_1 := \begin{bmatrix} 0 & 0 \end{bmatrix} \quad (32)$$

> Solucion := evalm(SolHom + IntProd) :

> SolUno := x(t) = Solucion₁ : SolDos := y(t) = Solucion₂ :

> evalf(SolUno, 3); evalf(SolDos, 3)

$$\begin{aligned} x(t) &= 2.04 e^{0.300 t - 3.00} + 7.96 e^{6.70 t - 67.0} + 0.000174 (2.70 10^6 e^{1.30 t + 29.0} + 60000. e^{7.70 t - 35.} \\ & - 460. e^{7.70 t - 45.} + 2760. e^{1.30 t + 19.0} + 9840. e^{32.0 + t} \sin(t) \cos(t)^2 - 2460. e^{32.0 + t} \sin(t) \\ & + 3280. e^{32.0 + t} \cos(t)^3 - 2460. e^{32.0 + t} \cos(t) - 1.82 10^{17} - 1.21 10^5 e^{32.0 + t} t \\ & - 3.99 10^5 e^{32.0 + t} - 11500. e^{32.0 + t} t^2) e^{-32.0 - 1. t} \\ y(t) &= -14.8 e^{6.70 t - 67.0} + 2.77 e^{0.300 t - 3.00} - 0.0000871 (-7.29 10^6 e^{1.30 t + 29.0} \\ & + 1.9 10^5 e^{7.70 t - 35.} - 1720. e^{7.70 t - 45.} - 7460. e^{1.30 t + 19.0} - 39400. e^{32.0 + t} \sin(t) \cos(t)^2 \\ & + 9840. e^{32.0 + t} \sin(t) + 19700. e^{32.0 + t} \cos(t)^3 - 14800. e^{32.0 + t} \cos(t) + 7.25 10^{17} \\ & + 3.16 10^5 e^{32.0 + t} t + 1.08 10^6 e^{32.0 + t} + 28700. e^{32.0 + t} t^2) e^{-32.0 - 1. t} \end{aligned} \quad (33)$$

> Comprobacion₁₂ := simplify(eval(subs(t = 10, SolUno)))

$$Comprobacion_{12} := x(10) = 10 \quad (34)$$

> Comprobacion₁₃ := simplify(eval(subs(t = 10, SolDos)))

$$Comprobacion_{13} := y(10) = -12 \quad (35)$$

> plot([rhs(SolUno), rhs(SolDos)], t = 7 ..10.2)

