

```

> restart
> AA := array([ [1, 1], [1, -1] ])

```

$$AA := \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad (1)$$

```

> Sistema := diff(x1(t), t) = x1(t) + x2(t), diff(x2(t), t) = x1(t) - x2(t) : Sistema1; Sistema2

```

$$\frac{d}{dt} x_1(t) = x_1(t) + x_2(t)$$

$$\frac{d}{dt} x_2(t) = x_1(t) - x_2(t) \quad (2)$$

```

> with(linalg) :
> MatExp := simplify(exponential(AA, t)) : map(evalf, MatExp, 2)

```

$$\begin{bmatrix} 0.85 e^{1.4t} + 0.15 e^{-1.4t} & -0.35 e^{-1.4t} + 0.35 e^{1.4t} \\ -0.35 e^{-1.4t} + 0.35 e^{1.4t} & 0.15 e^{1.4t} + 0.85 e^{-1.4t} \end{bmatrix} \quad (3)$$

```

> AAA := simplify(map(rcurry(eval, t=0'), map(diff, MatExp, t)))

```

$$AAA := \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad (4)$$

```

> MatExpInv := map(rcurry(eval, t=-t'), MatExp) : map(evalf, MatExpInv, 2)

```

$$\begin{bmatrix} 0.15 e^{1.4t} + 0.85 e^{-1.4t} & 0.35 e^{-1.4t} - 0.35 e^{1.4t} \\ 0.35 e^{-1.4t} - 0.35 e^{1.4t} & 0.85 e^{1.4t} + 0.15 e^{-1.4t} \end{bmatrix} \quad (5)$$

```

> AAAA := simplify(evalm( MatExpInv &* map(diff, MatExp, t) ))

```

$$AAAA := \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad (6)$$

```

> restart
> AA := array([ [0, 1], [3, -5] ])

```

$$AA := \begin{bmatrix} 0 & 1 \\ 3 & -5 \end{bmatrix} \quad (7)$$

```

> BB := array([0, 5 exp(-t)])

```

$$BB := \begin{bmatrix} 0 & 5 e^{-t} \end{bmatrix} \quad (8)$$

```

>
> with(linalg) :
> MatExp := exponential(AA, t) : map(evalf, MatExp, 2)

```

$$\begin{bmatrix} 0.09 e^{-5.5t} + 0.91 e^{0.55t} & 0.16 e^{0.55t} - 0.16 e^{-5.5t} \\ 0.49 e^{0.55t} - 0.49 e^{-5.5t} & 0.91 e^{-5.5t} + 0.09 e^{0.55t} \end{bmatrix} \quad (9)$$

```

> Condiciones := y1(0) = K1, y2(0) = K2 :
> Xcero := array([K1, K2])

```

(10)

$$X_{cero} := \begin{bmatrix} K_1 & K_2 \end{bmatrix} \quad (10)$$

> SOL := evalm(MatExp &* Xcero) :

$$\begin{aligned} > \text{MatExpTau} := \text{map}(\text{rcurry}(\text{eval}, t \Rightarrow t - \tau), \text{MatExp}) : \text{map}(\text{evalf}, \text{MatExpTau}, 2) \\ & \begin{bmatrix} 0.09 e^{-5.5t + 5.5\tau} + 0.91 e^{0.55t - 0.55\tau} & 0.16 e^{0.55t - 0.55\tau} - 0.16 e^{-5.5t + 5.5\tau} \\ 0.49 e^{0.55t - 0.55\tau} - 0.49 e^{-5.5t + 5.5\tau} & 0.91 e^{-5.5t + 5.5\tau} + 0.09 e^{0.55t - 0.55\tau} \end{bmatrix} \end{aligned} \quad (11)$$

> BBtau := map(rcurry(eval, t => tau'), BB)

$$BBtau := \begin{bmatrix} 0 & 5 e^{-\tau} \end{bmatrix} \quad (12)$$

> MatProdBB := evalm(MatExpTau &* BBtau) :

$$\begin{aligned} > \text{SolNoHom} := \text{map}(\text{int}, \text{MatProdBB}, \tau = 0 .. t) : \text{map}(\text{evalf}, \text{SolNoHom}, 2) \\ & \begin{bmatrix} 0.0097 (55. e^{1.5t} + 19. e^{-4.5t} - 74.) e^{-1.t} & 0.0097 (-100. e^{-4.5t} + 30. e^{1.5t} + 74.) e^{-1.t} \end{bmatrix} \end{aligned} \quad (13)$$

>

> SolUno := y₁(t) = evalm(SOL₁ + SolNoHom₁) : SolDos := y₂(t) = evalm(SOL₂ + SolNoHom₂) : evalf(SolUno, 2); evalf(SolDos, 2)

$$\begin{aligned} y_1(t) &= (0.09 e^{-5.5t} + 0.91 e^{0.55t}) K_1 + (0.16 e^{0.55t} - 0.16 e^{-5.5t}) K_2 + 0.0097 (55. e^{1.5t} \\ & \quad + 19. e^{-4.5t} - 74.) e^{-1.t} \\ y_2(t) &= (0.49 e^{0.55t} - 0.49 e^{-5.5t}) K_1 + (0.91 e^{-5.5t} + 0.09 e^{0.55t}) K_2 + 0.0097 (-100. e^{-4.5t} \\ & \quad + 30. e^{1.5t} + 74.) e^{-1.t} \end{aligned} \quad (14)$$

> Ecuacion := diff(y(t), t\$2) + 5·diff(y(t), t) - 3·y(t) = 5·exp(-t)

$$Ecuacion := \frac{d^2}{dt^2} y(t) + 5 \left(\frac{d}{dt} y(t) \right) - 3 y(t) = 5 e^{-t} \quad (15)$$

> comprobacion := simplify(eval(subs(y(t) = rhs(SolUno), lhs(Ecuacion) - rhs(Ecuacion) = 0)))

$$comprobacion := 0 = 0 \quad (16)$$

> comprobacion₂ := simplify(rhs(SolDos) - rhs(diff(SolUno, t))) = 0

$$comprobacion_2 := 0 = 0 \quad (17)$$

> restart

> SolGral := y(x) = C₁·exp(-x) + C₂·exp(-x)·cos(2x) + C₃·exp(-x)·sin(2x) + 5·exp(x)

$$SolGral := y(x) = C_1 e^{-x} + C_2 e^{-x} \cos(2x) + C_3 e^{-x} \sin(2x) + 5 e^x \quad (18)$$

> SolNoHom := y(x) = 5·exp(x)

$$SolNoHom := y(x) = 5 e^x \quad (19)$$

> EcuaCarac := expand((m + 1)·((m + 1)·2 - (2·I)·2)) = 0

$$EcuaCarac := m^3 + 3 m^2 + 7 m + 5 = 0 \quad (20)$$

> EcuaHom := y''' + 3 y'' + 7 y' + 5 y = 0

$$EcuaHom := \frac{d^3}{dx^3} y(x) + 3 \left(\frac{d^2}{dx^2} y(x) \right) + 7 \left(\frac{d}{dx} y(x) \right) + 5 y(x) = 0 \quad (21)$$

> Q := simplify(eval(subs(y(x) = rhs(SolNoHom), lhs(EcuaHom))))

(22)

$$Q := 80 e^x \quad (22)$$

$$> QQ := \text{simplify}(\text{eval}(\text{subs}(y(x) = \text{rhs}(\text{SolGral}), \text{lhs}(\text{EcuaHom})))$$

$$QQ := 80 e^x \quad (23)$$

$$> \text{EcuaacionOriginal} := \text{lhs}(\text{EcuaHom}) = Q$$

$$\text{EcuaacionOriginal} := \frac{d^3}{dx^3} y(x) + 3 \left(\frac{d^2}{dx^2} y(x) \right) + 7 \left(\frac{d}{dx} y(x) \right) + 5 y(x) = 80 e^x \quad (24)$$

$$> \text{SolucionGeneral} := \text{dsolve}(\text{EcuaacionOriginal})$$

$$\text{SolucionGeneral} := y(x) = 5 e^x + _C1 e^{-x} + _C2 e^{-x} \cos(2 x) + _C3 e^{-x} \sin(2 x) \quad (25)$$

$$> \text{SolGral}$$

$$y(x) = C_1 e^{-x} + C_2 e^{-x} \cos(2 x) + C_3 e^{-x} \sin(2 x) + 5 e^x \quad (26)$$

$$> \text{Sistema} := \text{diff}(\text{SolGral}, x), \text{diff}(\text{SolGral}, x\$2), \text{diff}(\text{SolGral}, x\$3) : \text{Sistema}_1; \text{Sistema}_2; \text{Sistema}_3$$

$$\frac{d}{dx} y(x) = -C_1 e^{-x} - C_2 e^{-x} \cos(2 x) - 2 C_2 e^{-x} \sin(2 x) - C_3 e^{-x} \sin(2 x) + 2 C_3 e^{-x} \cos(2 x) + 5 e^x$$

$$\frac{d^2}{dx^2} y(x) = C_1 e^{-x} - 3 C_2 e^{-x} \cos(2 x) + 4 C_2 e^{-x} \sin(2 x) - 3 C_3 e^{-x} \sin(2 x) - 4 C_3 e^{-x} \cos(2 x) + 5 e^x$$

$$\frac{d^3}{dx^3} y(x) = -C_1 e^{-x} + 11 C_2 e^{-x} \cos(2 x) + 2 C_2 e^{-x} \sin(2 x) + 11 C_3 e^{-x} \sin(2 x) - 2 C_3 e^{-x} \cos(2 x) + 5 e^x \quad (27)$$

$$> \text{Parametro} := \text{simplify}(\text{solve}(\{\text{Sistema}\}, \{C_1, C_2, C_3\})) : \text{Parametro}_1; \text{Parametro}_2; \text{Parametro}_3$$

$$\begin{aligned} C_1 &= \frac{1}{4} \left(- \left(\frac{d^3}{dx^3} y(x) \right) - 5 \left(\frac{d}{dx} y(x) \right) - 2 \left(\frac{d^2}{dx^2} y(x) \right) + 40 e^x \right) e^x \\ C_2 &= -\frac{1}{20} e^x \left(-2 \sin(2 x) \left(\frac{d^3}{dx^3} y(x) \right) - 4 \sin(2 x) \left(\frac{d}{dx} y(x) \right) - 6 \left(\frac{d^2}{dx^2} y(x) \right) \sin(2 x) \right. \\ &\quad \left. + 60 e^x \sin(2 x) - \cos(2 x) \left(\frac{d^3}{dx^3} y(x) \right) + 3 \cos(2 x) \left(\frac{d}{dx} y(x) \right) \right. \\ &\quad \left. + 2 \left(\frac{d^2}{dx^2} y(x) \right) \cos(2 x) - 20 e^x \cos(2 x) \right) \\ C_3 &= \frac{1}{20} e^x \left(\sin(2 x) \left(\frac{d^3}{dx^3} y(x) \right) - 3 \sin(2 x) \left(\frac{d}{dx} y(x) \right) - 2 \cos(2 x) \left(\frac{d^3}{dx^3} y(x) \right) \right. \\ &\quad \left. - 4 \cos(2 x) \left(\frac{d}{dx} y(x) \right) - 2 \left(\frac{d^2}{dx^2} y(x) \right) \sin(2 x) - 6 \left(\frac{d^2}{dx^2} y(x) \right) \cos(2 x) \right. \\ &\quad \left. + 20 e^x \sin(2 x) + 60 e^x \cos(2 x) \right) \end{aligned} \quad (28)$$

> *EcuacionIntermedia* := expand(simplify(subs($C_1 = rhs(Parametro_1)$, $C_2 = rhs(Parametro_2)$,
 $C_3 = rhs(Parametro_3)$, SolGral)))

$$EcuacionIntermedia := y(x) = -\frac{1}{5} \frac{d^3}{dx^3} y(x) - \frac{7}{5} \frac{d}{dx} y(x) - \frac{3}{5} \frac{d^2}{dx^2} y(x) + 16 e^x \quad (29)$$

> *EcuacionPrimitiva* := lhs(*EcuacionIntermedia*) · 5 − rhs(*EcuacionIntermedia*) · 5 + 80 e^x = 0
+ 80 e^x

$$EcuacionPrimitiva := \frac{d^3}{dx^3} y(x) + 3 \left(\frac{d^2}{dx^2} y(x) \right) + 7 \left(\frac{d}{dx} y(x) \right) + 5 y(x) = 80 e^x \quad (30)$$

> *EcuacionOriginal*;

$$\frac{d^3}{dx^3} y(x) + 3 \left(\frac{d^2}{dx^2} y(x) \right) + 7 \left(\frac{d}{dx} y(x) \right) + 5 y(x) = 80 e^x \quad (31)$$

>

>