ECUACIÓN DIFERENCIAL ORDINARIA
PRIMER ORDEN NO-LINEAL NO-EXACTA.

$$M(x,y) + N(x,y)\frac{dy}{dx} = 0$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

MÉTODO FACTOR INTEGRANTE

$$\mu(x,y)\,M(x,y) + \mu(x,y)\,N(x,y)\frac{dy}{dx} = 0$$

EDO(1) NL - EXACTA.

$$\frac{\partial}{\partial y}\left(\mu M\right) = \frac{\partial}{\partial x}\left(\mu N\right)$$

$\mu(x,y)$
incógnita

$$M\frac{\partial \mu}{\partial y} + \mu\frac{\partial M}{\partial y} = N\frac{\partial \mu}{\partial x} + \mu\frac{\partial N}{\partial x}$$

$$\mu\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) + M\frac{\partial \mu}{\partial y} - N\frac{\partial \mu}{\partial x} = 0$$

$$M\frac{\partial \mu}{\partial y}+\mu\frac{\partial M}{\partial y}=N\frac{\partial \mu}{\partial x}+\mu\frac{\partial N}{\partial x}$$

Hipótesis: Supongamos que $\mu \Rightarrow \mu(x)$

$$\mu\frac{\partial M}{\partial y}=N\frac{d\mu}{dx}+\mu\frac{\partial N}{\partial x}$$

$$N\frac{d\mu}{dx}=\mu\frac{\partial M}{\partial y}-\mu\frac{\partial N}{\partial x}$$

$$N\frac{d\mu}{dx}=\mu\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)$$

$$\frac{d\mu}{dx}=\mu\left(\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N}\right)$$

$$d\mu=\mu\left(\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N}\right)dx$$

$$\frac{d\mu}{\mu}=\left(\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N}\right)dx$$

$$\frac{d\mu}{\mu}=P(x)\,dx$$

$$\int\frac{d\mu}{\mu}=\int P(x)\,dx+C_1$$

$$\ell(\mu)=\int P(x)\,dx+C_1$$

$$\mu(x)=e^{\int P(x)dx+C_1}$$

$$\mu(x)=e^{C_1}e^{\int P(x)dx}$$

$$\boxed{\mu(x)=C_{10}\,e^{\int P(x)dx}}$$

$$\frac{dy}{dx} + p(x)y = 0$$

$$\underbrace{\left(p(x)y\right)}_{M(x,y)} + \underbrace{(1)}_{N(x,y)}\frac{dy}{dx} = 0$$

$$\frac{\partial M}{\partial y} = p(x) \qquad \frac{\partial N}{\partial x} = 0 \qquad \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

NO-EXACTA.

$$\left(\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}\right) \Rightarrow \left(\frac{p(x) - 0}{1}\right) \Rightarrow p(x)$$

$$\frac{d\mu}{\mu} = p(x)\,dx \qquad \int \frac{d\mu}{\mu} = \int p(x)\,dx$$

$$\ln\mu = \int p(x)\,dx$$

$$\boxed{\mu = e^{\int p(x)dx}}$$

$$\underbrace{e^{\int p(x)dx}\,p(x)y}_{MM} + \underbrace{e^{\int p(x)dx}}_{NN}\frac{dy}{dx} = 0$$

$$\frac{\partial MM}{\partial y} = e^{\int p(x)dx}p(x) \qquad \frac{\partial NN}{\partial x} = e^{\int p(x)dx}p(x)$$

EXACTA $\qquad \dfrac{\partial MM}{\partial y} = \dfrac{\partial NN}{\partial x}$

Sol. GRAL $\Rightarrow \displaystyle\int NN\,dy + \int\left[MM - \frac{\partial}{\partial x}\int NN\,dy\right]dx = C_1$

$$\int e^{\int p(x)dx}dy = e^{\int p(x)dx}\int dy$$

$$= y e^{\int p(x)dx}$$

$$\frac{\partial}{\partial x}\int e^{\int p(x)dx}dy = y e^{\int p(x)dx}p(x)$$

$$\left(MM - \frac{\partial}{\partial x}\int e^{\int p(x)dx}dy\right) = \cancel{y e^{\int p(x)dx}p(x)} - \cancel{y e^{\int p(x)dx}p(x)}$$

$$S_G = y e^{\int p(x)dx} = C_1 \rightarrow y = C_1 e^{-\int p(x)dx}$$

$$\underbrace{2y^3+32x^2y^2+18xy^5}_{M}+(\underbrace{3xy^2+16x^3y+30x^2y^4}_{N})\frac{dy}{dx}=0$$

$$\frac{\partial M}{\partial y}=6y^2+64x^2y+90xy^4$$

$$\frac{\partial N}{\partial x}=3y^2+48x^2y+60xy^4$$

$$\left(\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N}\right)=\left(\frac{6y^2+64x^2y+90xy^4-3y^2-48x^2y-60xy^4}{3xy^2+16x^3y+30x^2y^4}\right)$$

$$=\frac{3y^2+16x^2y+30xy^4}{3xy^2+16x^3y+30x^2y^4}$$

$$=\frac{\cancel{(3y^2+16x^2y+30xy^4)}}{x\,\cancel{(3y^2+16x^2y+30xy^4)}}$$

$$\left(\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N}\right)=\frac{1}{x}$$

$$\frac{d\mu}{\mu}=\frac{dx}{x}\qquad\int\frac{d\mu}{\mu}=\int\frac{dx}{x}$$

$$\ln\mu=\ln x$$

$$\mu=x$$

$$M\frac{\partial \mu}{\partial y}+\mu\frac{\partial M}{\partial y}=N\frac{\partial \mu}{\partial x}+\mu\frac{\partial N}{\partial x}$$

$$\mu \to \mu(y)$$

$$M\frac{d\mu}{dy}+\mu\frac{\partial M}{\partial y}=\mu\frac{\partial N}{\partial x}$$

$$M\frac{d\mu}{dy}=\mu\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)$$

$$\frac{d\mu}{\mu}=\left(\frac{\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}}{M}\right)dy$$

$$\frac{d\mu}{\mu}=Q(y)\,dy$$

$$2xy^2+32x^3y+18x^2y^4+\left(3x^2y+16x^4+30x^3y^3\right)\frac{dy}{dx}=0$$

MM NN

$$\frac{\partial MM}{\partial y}=4xy+32x^3+72x^2y^3$$

$$\frac{\partial NN}{\partial x}=6xy+64x^3+90x^2y^3$$

$$\left(\frac{\frac{\partial NN}{\partial x}-\frac{\partial MM}{\partial y}}{M}\right)=\frac{6xy+64x^3+90x^2y^3-4xy-32x^3-72x^2y^3}{2xy^2+32x^3y+18x^2y^4}$$

$$=\frac{2xy+\cancel{32}x^3+18x^2y^3}{y\left(2xy+\cancel{32}x^3+18x^2y^3\right)}$$

$$\left(\frac{\frac{\partial NN}{\partial x}-\frac{\partial MM}{\partial y}}{M}\right)=\frac{1}{y}$$

$$\frac{du}{u}=\frac{dy}{y}$$

$$\int\frac{du}{u}=\int\frac{dy}{y}$$

$$Lu=Ly$$

$$u=y$$