



+ MÉTODO DE COEFICIENTES HOMOGÊNEOS

$$M(x, y) + N(x, y) \frac{dy}{dx} = 0$$

$$\left. \begin{aligned} M(\lambda x, \lambda y) &= \lambda^m M(x, y) \\ N(\lambda x, \lambda y) &= \lambda^n N(x, y) \end{aligned} \right\} m = n$$

EDO(1) NL-CH.

$$M(x, y) + N(x, y) \frac{dy}{dx} = 0$$

1ª opción

$$y(x) = u(x) \cdot x$$

$$\frac{d}{dx} y(x) = u(x) \cdot (1) + x \frac{d}{dx} u(x)$$

$$M(x, ux) + N(x, ux) \left(u + x \frac{du}{dx} \right) = 0$$

EDO(1)NL VS.

$$M(x, y) \frac{dx}{dy} + N(x, y) = 0$$

2ª opción

$$x(y) = v(y) \cdot y$$

$$\frac{dx}{dy} = v(y) \cdot (1) + y \frac{dv}{dy}$$

$$M(vy, y) \left(v + y \frac{dv}{dy} \right) + N(vy, y) = 0$$

EDO(1)NL VS

$$M(x, ux) + N(x, ux) \left(u + x \frac{du}{dx} \right) = 0$$

$$\rightarrow P(x) Q(u) + R(x) S(u) \frac{du}{dx} = 0$$

$$SG \Rightarrow \int \frac{P(x)}{R(x)} dx + \int \frac{S(u)}{Q(u)} du = C_1$$

$$F(x, u) = C_1 \quad y = u \cdot x \quad u = \frac{y}{x}$$

$$\boxed{G(x, y) = C_1}$$

149. $y' = \frac{2xy}{3x^2 - y^2}$

$$\underbrace{-2xy}_{M} + \underbrace{(3x^2 - y^2)}_N \frac{dy}{dx} = 0$$

$$M(\lambda x, \lambda y) = -2(\lambda x)(\lambda y) \Rightarrow \lambda^2(-2xy) \quad m=2$$

$$N(\lambda x, \lambda y) = 3(\lambda x)^2 - (\lambda y)^2 \Rightarrow \lambda^2(3x^2 - y^2) \quad n=2$$

EDO(1) NL - CH.

$$M=N$$

$$y = u \cdot x \quad \frac{dy}{dx} = u + x \frac{du}{dx}$$

$$-2x(ux) + (3x^2 - (ux)^2) \left(u + x \frac{du}{dx} \right) = 0$$

$$-2ux^2 + 3ux^2 - u^2x^2 + (3x^3 - u^2x^3) \frac{du}{dx} = 0$$

$$(ux^2 - u^2x^2) + (3x^3 - u^2x^3) \frac{du}{dx} = 0$$

$$\underbrace{x^2}_{P} (\underbrace{u - u^3}_{Q}) + \underbrace{x^3}_{R} (\underbrace{3 - u^2}_{S}) \frac{du}{dx} = 0$$

$$SG \Rightarrow \int \frac{x^2}{x^3} dx + \int \frac{3 - u^2}{u - u^3} du = C_1$$

$$\int \frac{dx}{x} + \int \frac{3 - u^2}{u - u^3} du = C_1$$

$$r = u - u^3$$

$$dr = (1 - 3u^2) du$$

$$+ \frac{1}{3} \int \frac{3 - 3u^2}{u - u^3} du$$

$$+ \frac{1}{3} \left(\int \frac{1 - 3u^2}{u - u^3} du + \int \frac{8 du}{u - u^3} \right)$$

$$+ \frac{1}{3} \int \frac{dr}{r} + \frac{8}{3} \int \frac{du}{u - u^3}$$

$$+ \frac{8}{3} \int \frac{du}{u(1 - u^2)}$$



$$+ \frac{8}{3} \int \frac{-\sin(\theta) d\theta}{\cos(\theta) \sin^2(\theta)} \quad \frac{u}{1} = \cos(\theta)$$

$$- \frac{8}{3} \int \frac{d\theta}{\cos(\theta) \sin(\theta)} \quad \frac{\sqrt{1-u^2}}{1} = \sin(\theta)$$

$$du = -\sin(\theta) d\theta$$

$$SG \Rightarrow \frac{+x^2 y^2 - x^4}{y^3} = C_1$$

$$\frac{\partial F}{\partial x} + \frac{\partial F}{\partial y} \cdot \frac{dy}{dx} = 0$$

$$\frac{1}{y^3} (2xy^2 - 3x^2) + \left(\frac{y^3(2x^2y - 0) - (x^2y^2 - x^4)(3y^2)}{y^6} \right) \cdot \frac{dy}{dx} = 0$$

$$\frac{2xy^2 - 3x^2}{y^3} + \frac{2y^4x^2 - 3x^2y^4 + 3x^4y^2}{y^6} \frac{dy}{dx} = 0$$

$$y^3(2xy^2 - 3x^2) + (2y^4x^2 - 3x^2y^4 + 3x^4y^2) \frac{dy}{dx} = 0$$

$$+ y^2(2x^2y^2 - 3x^2y^2 + 3x^4) \frac{dy}{dx} = 0$$

$$(2xy^2 - 3x^2) + \frac{(-x^2y^2 + 3x^4)}{y} \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{(2xy^2 - 3x^2)y}{-x^2y^2 + 3x^4}$$

=

146. $xy' = y + \sqrt{y^2 - x^2}$.

$$y + \sqrt{y^2 - x^2} - x \frac{dy}{dx} = 0$$

$$\begin{aligned} M(\lambda x, \lambda y) &= \lambda y + \sqrt{(\lambda y)^2 - (\lambda x)^2} \\ &= \lambda y + \sqrt{\lambda^2 y^2 - \lambda^2 x^2} \\ &= \lambda y + \sqrt{\lambda^2 (y^2 - x^2)} \\ &= \lambda y + \lambda \sqrt{y^2 - x^2} \\ &= \lambda (y + \sqrt{y^2 - x^2}) \quad \eta = 1 \\ N(\lambda x, \lambda y) &= -(\lambda x) \\ &= \lambda(-x) \quad \eta = 1 \quad \eta = 0. \end{aligned}$$

$$\begin{aligned} u x + \sqrt{(u x)^2 - x^2} - x \left(u + x \frac{du}{dx} \right) &= 0 \\ \cancel{u x} + \sqrt{\cancel{u^2 x^2} - x^2} - \cancel{u x} - x^2 \frac{du}{dx} &= 0 \\ x \sqrt{(u^2 - 1)} - x^2 \frac{du}{dx} &= 0 \end{aligned}$$

$$\frac{x \frac{dx}{x} - \frac{du}{\sqrt{u^2 - 1}} = 0 \quad \begin{array}{c} \text{C.H.} \\ \text{u} = \sec(\theta) \\ \sqrt{u^2 - 1} = \tan(\theta) \\ du = \sec(\theta) \tan(\theta) d\theta \end{array}$$

(56)

$$\int \frac{dx}{x} - \int \frac{du}{\sqrt{u^2 - 1}} = C_1$$

$$\int \frac{dx}{x} - \int \frac{\sec(\theta) \tan(\theta) d\theta}{\tan(\theta)} = C_1$$

$$Lx - \int \sec(\theta) d\theta = C_1$$

$$-L(\sec(\theta) + \tan(\theta)) = C_1$$

$$Lx - L(u + \sqrt{u^2 - 1}) = C_1$$

$$L\left(\frac{x}{\left(\frac{y}{x}\right) + \sqrt{\left(\frac{y}{x}\right)^2 - 1}}\right) = C_1$$

$$\frac{x}{\frac{y}{x} + \sqrt{\frac{y^2 - x^2}{x^2}}} = e^{C_1}$$

$$\boxed{\frac{x^2}{y + \sqrt{y^2 - x^2}} = C_1} \quad (56)$$