

> restart

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$$154. \quad y^3 dx + 2(x^2 - xy^2) dy = 0.$$

> Ecuacion := $y(x) \cdot 3 + 2 \cdot (x \cdot 2 - x \cdot y(x) \cdot 2) \cdot \text{diff}(y(x), x) = 0$

$$\text{Ecuacion} := y(x)^3 + 2 (x^2 - x y(x)^2) \left(\frac{d}{dx} y(x) \right) = 0 \quad (1)$$

> with(DEtools) :

> odeadvisor(Ecuacion)

$$[[\text{homogeneous}, \text{class G}], \text{rational}] \quad (2)$$

> EcuacionDos := simplify(isolate(expand(eval(subs(y(x) = u(x) \cdot x, Ecuacion))), diff(u(x), x)))

$$\text{EcuacionDos} := \frac{d}{dx} u(x) = -\frac{1}{2} \frac{u(x) (u(x)^2 x - 2)}{x (-1 + u(x)^2 x)} \quad (3)$$

> odeadvisor(EcuacionDos)

$$[[\text{homogeneous}, \text{class G}], \text{rational}] \quad (4)$$

> intfactor(Ecuacion)

$$\frac{1}{y(x) x^2} \quad (5)$$

> M := y \cdot 3; N := 2 \cdot (x \cdot 2 - x \cdot y \cdot 2)

$$M := y^3$$

$$N := 2 x^2 - 2 x y^2 \quad (6)$$

> FactInt := $\frac{1}{x \cdot 2 \cdot y}$

$$\text{FactInt} := \frac{1}{y x^2} \quad (7)$$

> MM := simplify(M \cdot FactInt)

$$MM := \frac{y^2}{x^2} \quad (8)$$

> NN := expand(N \cdot FactInt)

$$NN := \frac{2}{y} - \frac{2 y}{x} \quad (9)$$

> comprobacion := simplify(diff(MM, y) - diff(NN, x)) = 0

$$\text{comprobacion} := 0 = 0 \quad (10)$$

> IntMMx := int(MM, x)

$$\text{IntMMx} := -\frac{y^2}{x} \quad (11)$$

> SolucionGeneral := IntMMx + int((NN - diff(IntMMx, y)), y) = C_1

$$\text{SolucionGeneral} := -\frac{y^2}{x} + 2 \ln(y) = C_1 \quad (12)$$

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147.  $4x^2 - xy + y^2 + y'(x^2 - xy + 4y^2) = 0$ .
> Ecuacion := 4*x^2 - x*y(x) + y(x)^2 + (x^2 - x*y(x) + 4*y(x)^2)*diff(y(x), x) = 0
      Ecuacion := 4 x^2 - x y(x) + y(x)^2 + (x^2 - x y(x) + 4 y(x)^2) \left( \frac{d}{dx} y(x) \right) = 0
                                         (13)
> with(DEtools):
> odeadvisor(Ecuacion)
      [ _homogeneous, class A ], _rational, _dAlembert
                                         (14)
> EcuacionDos := factor(isolate(simplify(eval(subs(y(x) = u(x)*x, Ecuacion))), diff(u(x), x)))
      EcuacionDos := \frac{d}{dx} u(x) = - \frac{4 (u(x) + 1) (u(x)^2 - u(x) + 1)}{x (1 - u(x) + 4 u(x)^2)}
                                         (15)
> odeadvisor(EcuacionDos)
      [ _separable ]
                                         (16)
> P := x; Q := \frac{4 (u + 1) (u^2 - u + 1)}{(1 - u + 4 u^2)}
      P := x
      Q := \frac{4 (u + 1) (u^2 - u + 1)}{1 - u + 4 u^2}
                                         (17)
> Solucion := int\left(\frac{1}{P}, x\right) + int\left(\frac{1}{Q}, u\right) = C_1
      Solucion := \ln(x) + \frac{1}{2} \ln(u + 1) + \frac{1}{4} \ln(u^2 - u + 1) = C_1
                                         (18)
> SolucionPrima := lhs(Solucion) * 4 = rhs(Solucion) * 4
      SolucionPrima := 4 \ln(x) + 2 \ln(u + 1) + \ln(u^2 - u + 1) = 4 C_1
                                         (19)
> SolucionBiPrima := expand(exp(lhs(SolucionPrima))) = C_1
      SolucionBiPrima := x^4 u^4 + x^4 u^3 + x^4 u + x^4 = C_1
                                         (20)
> SolucionFinal := subs\left(u = \frac{y}{x}, SolucionBiPrima\right)
      SolucionFinal := y^4 + x y^3 + x^3 y + x^4 = C_1
                                         (21)
> SolucionDerivable := y(x)^4 + x y(x)^3 + x^3 y(x) + x^4 = C_1
      SolucionDerivable := y(x)^4 + x y(x)^3 + x^3 y(x) + x^4 = C_1
                                         (22)
> EcuacionSegunda := simplify(isolate(diff(SolucionDerivable, x), diff(y(x), x)))
      EcuacionSegunda := \frac{d}{dx} y(x) = - \frac{y(x)^2 - x y(x) + 4 x^2}{x^2 - x y(x) + 4 y(x)^2}
                                         (23)
> EcuacionOriginal := isolate(Ecuacion, diff(y(x), x))
                                         (24)

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$$EcuacionOriginal := \frac{d}{dx} y(x) = \frac{-4x^2 + xy(x) - y(x)^2}{x^2 - xy(x) + 4y(x)^2} \quad (24)$$

$$> comprobacionDos := simplify(rhs(EcuacionSegunda) - rhs(EcuacionOriginal)) = 0 \\ comprobacionDos := 0 = 0 \quad (25)$$

$$> Ecuacion \\ 4x^2 - xy(x) + y(x)^2 + (x^2 - xy(x) + 4y(x)^2) \left(\frac{d}{dx} y(x) \right) = 0 \quad (26)$$

$$> intfactor(Ecuacion) \\ \frac{1}{(y(x)^2 - xy(x) + x^2)(y(x) + x)} \quad (27)$$

$$> FactInt := \frac{1}{(y^2 - xy + x^2)(y + x)} \\ FactInt := \frac{1}{(x^2 - xy + y^2)(x + y)} \quad (28)$$

$$> M := 4x^2 - xy + y^2; N := x^2 - xy + 4y^2 \\ M := 4x^2 - xy + y^2 \\ N := x^2 - xy + 4y^2 \quad (29)$$

$$> MM := simplify(M \cdot FactInt) \\ MM := \frac{4x^2 - xy + y^2}{(x^2 - xy + y^2)(x + y)} \quad (30)$$

$$> NN := simplify(N \cdot FactInt) \\ NN := \frac{x^2 - xy + 4y^2}{(x + y)(x^2 - xy + y^2)} \quad (31)$$

$$> comprobacionTres := simplify(diff(MM, y) - diff(NN, x)) = 0 \\ comprobacionTres := 0 = 0 \quad (32)$$

$$> SolucionExacta := int(MM, x) + int((NN - diff(int(MM, x), y)), y) = C_1 \\ SolucionExacta := 2\ln(x + y) + \ln(x^2 - xy + y^2) = C_1 \quad (33)$$

$$> SolucionMuyExacta := expand(exp(lhs(SolucionExacta))) = C_1 \\ SolucionMuyExacta := y^4 + xy^3 + x^3y + x^4 = C_1 \quad (34)$$

$$> SolucionFinal \\ y^4 + xy^3 + x^3y + x^4 = C_1 \quad (35)$$

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