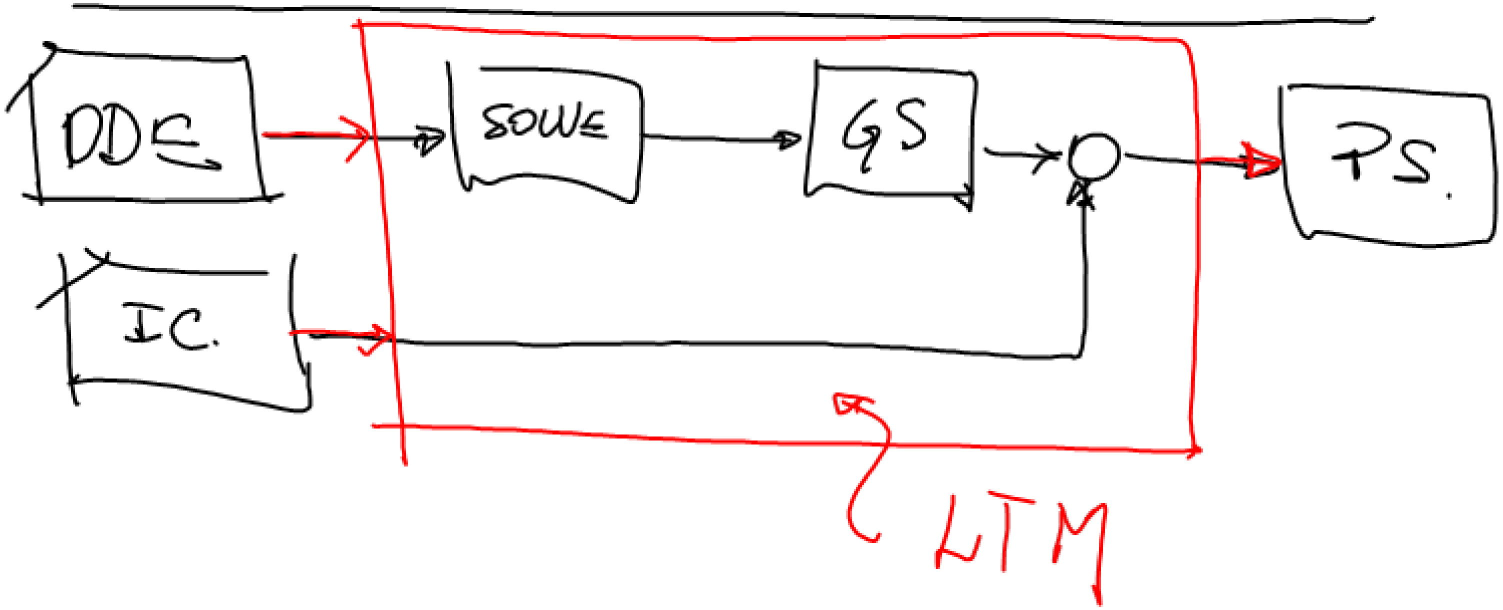
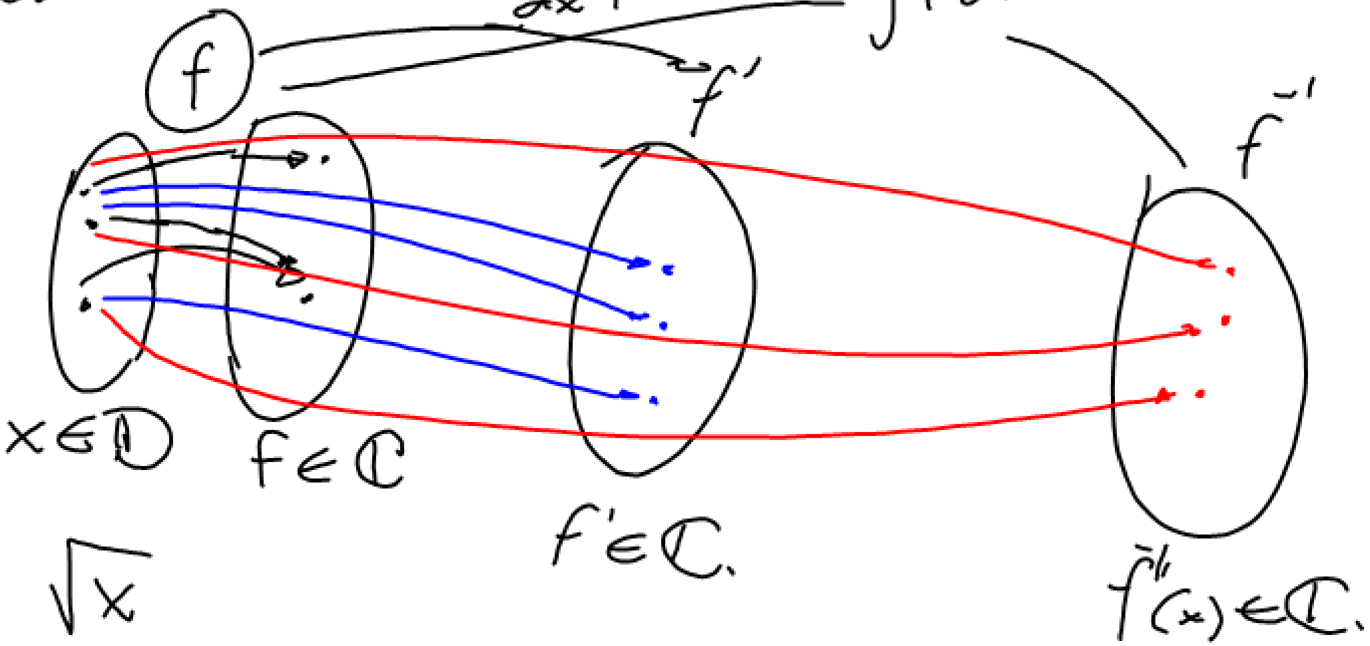
Chapter IV :- Laplace Transform as the best method for solve initial conditions problems



# Calculus

$\frac{d}{dx} f$ $\qquad$ $\int f \, dx$




$\sqrt{x}$

$x^3 \quad \frac{d}{dx} \quad 3x^2$

$\int dx \quad \frac{x^4}{4}$

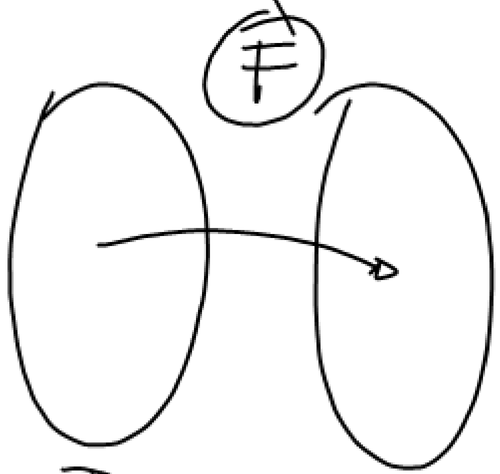
$f(x)$

$T\{f\} = F$

$f$

$F(s)$

$x \in \mathbb{D}$ $f \in \mathbb{C}$

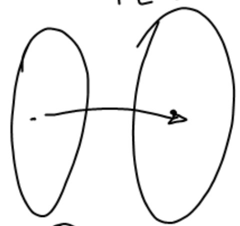



$s \in \mathbb{D}$ $F \in \mathbb{C}$

$\mathcal{L}\{f(x)\} = F(s)$

$f(x)$ $F(s)$

$x \in \mathbb{R}$ $f \in \mathbb{R}$



$s \in \mathbb{C}$ $F \in \mathbb{R}$

Laplace Transform. complex Real

$f(x)$ $g(x)$ $F(s)$ $G(s)$

$$\boxed{a \cdot f + b \cdot g} \xrightarrow{\mathcal{L}\{\ \}} \boxed{aF + bG}$$

$a, b \in \mathbb{R}$ $a, b \in \mathbb{R}$

Linear over
add and subtract
and multiplication by constant.

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$$f'(x) \xrightarrow{\mathcal{L}\{\ \}} sF(s) - f(0)$$

$$\int f(x)\,dx \xleftarrow{\mathcal{L}^{-1}\{\ \}} \frac{F(s)}{s}$$

$\mathcal{L}\{f(x)\} = F(s)$ is unique

tabla $\mathcal{L}^{-1}\{F(s)\} = f(x)$ this is not unique

| f | F |
|---|---|
| ⋮ | ⋮ |

$$F(s) = \int_0^\infty f(t)e^{-st}\,dt$$

$$\mathcal{L}\{e^{5t}\} = \int_0^\infty e^{5t}e^{-st}\,dt$$

$$= \int_0^\infty e^{-(s-5)t}\,dt$$

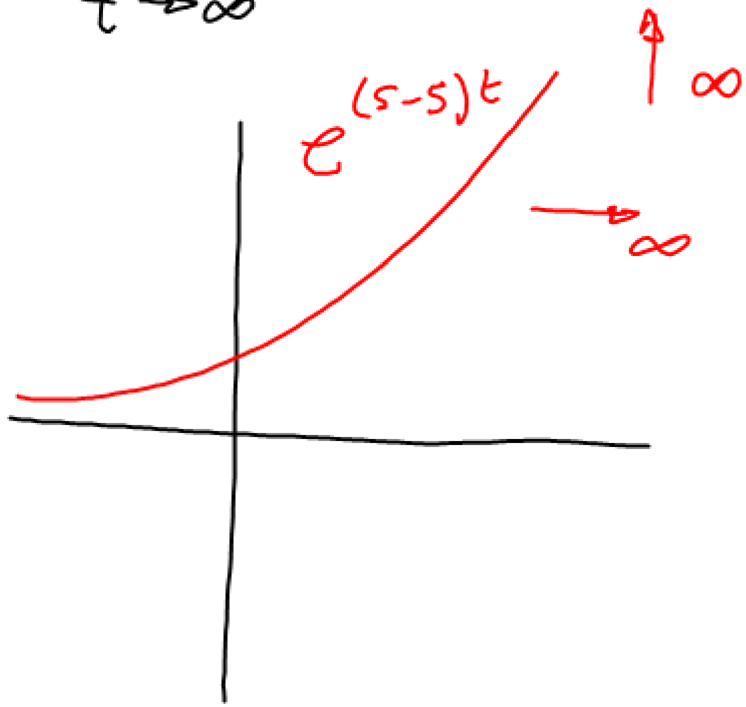
$$= \left[\frac{-1}{(s-5)}\int e^{-(s-5)t}(-(s-5))\,dt\right]_0^\infty$$

$$= \frac{-1}{s-5}\left(e^{-(s-5)t}\right)_0^\infty$$

$$= \frac{-1}{s-5}\left(\lim_{t\to\infty} e^{-(s-5)t} - 1\right)$$

$$\boxed{\mathcal{L}\{e^{5t}\} = \frac{1}{s-5}} \qquad \mathcal{L}\{e^{at}\} = \frac{1}{s-a}$$

$$\lim_{t \to \infty} \frac{1}{e^{(5-5)t}} = \lim_{b \to \infty} \frac{1}{b} = 0$$



$$f(t) = 8$$

$$\mathcal{L}\{8\} = 8\mathcal{L}\{1\}$$

$$\mathcal{L}\{1\} = \int_0^\infty (1)e^{-st}dt$$

$$= \frac{-1}{s}\left(\int e^{-st}(-s)dt\right)_0^\infty$$

$$= \frac{-1}{s}\left(e^{-st}\right)_0^\infty$$

$$= \frac{-1}{s}\left(\lim_{b\to\infty} e^{-sb} - (1)\right)$$

$$\boxed{\mathcal{L}\{1\} = \frac{1}{s}} \qquad \mathcal{L}\{8\} = \frac{8}{s}$$

$$f(t) = t$$

$$\mathcal{L}\{t\} = \int_0^\infty t e^{-st}\,dt$$

$$= \left( \int t e^{-st}\,dt \right)_0^\infty \qquad \int u\,dv = uv - \int v\,du$$

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$$\int t e^{-st}\,dt \qquad \begin{cases} u = t & dv = e^{-st}\,dt \\ du = dt & v = \dfrac{e^{-st}}{-s} \end{cases}$$

$$\int t e^{-st}\,dt = \frac{t e^{-st}}{-s} - \left(\frac{1}{-s}\right) \int e^{-st}\,dt$$

$$= \frac{t e^{-st}}{-s} - \frac{1}{s^2} e^{-st}$$

$$= -\frac{1}{s}\left(t e^{-st}\right) - \frac{1}{s^2}\left(e^{-st}\right)$$

$$\mathcal{L}\{t\} = -\frac{1}{s}\left(te^{-st}\right)_0^{\infty} - \frac{1}{s^2}\left(e^{-st}\right)_0^{\infty}$$

$$= -\frac{1}{s}\left(\lim_{t\to\infty} \overset{0}{t e^{-st}} - (0)\right) - \frac{1}{s^2}\left(\lim_{t\to\infty} \overset{0}{e^{-st}} - 1\right)$$

$$\lim_{t\to\infty} \frac{t}{e^{+st}} = \lim_{t\to\infty} \overset{0}{(\infty)\cdot(0)}$$

$$\boxed{\mathcal{L}\{t\} = \frac{1}{s^2}} \qquad \mathcal{L}\{1\} = \frac{1}{s}$$

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$$

Existence of Laplace Transform.

If we have f(t) we need
is A class for existence of L.T.

A class function

a) exponential class.

$$|f| \leq Me^{At} \quad M, A \in \mathbb{R}$$

b) sectional continous function

$$e^{t^n} \quad n>1 \qquad |t| \leq Me^{At}$$