# Inverse Laplace Transform

$$f(t) = \frac{1}{2\pi i} \int_{a-i\infty}^{a+i\infty} e^{pt} F(p)\, dp,$$

not unique

$$\mathcal{L}^{-1}\{F(s)\} = \frac{1}{2\pi i} \int_{a-i\infty}^{a+i\infty} e^{st} F(s)\, ds \Rightarrow f(t)$$

$$\mathcal{L}\{f(t)\} = \int_{0}^{t} e^{-st} f(t)\, dt \Rightarrow F(s)$$

$$\mathcal{L}\{f(t)\} = F(s) \quad \mathcal{L}\{g(t)\} = G(s)$$

$$\mathcal{L}\{af(t) + bg(t)\} = a\mathcal{L}\{f(t)\} + b\mathcal{L}\{g(t)\}$$

$a, b \in \mathbb{R}$
$$= aF(s) + bG(s)$$

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$$\mathcal{L}\{f(at)\} = \frac{1}{a} \cdot F\left(\frac{s}{a}\right) \quad a \in \mathbb{R}$$

$$\mathcal{L}\{e^{t}\} = \frac{1}{s-1}$$

$$\mathcal{L}\{e^{-4t}\} = \left(-\frac{1}{4}\right) \cdot \left(\frac{1}{\left(\frac{s}{-4}\right) - 1}\right)$$

$$= \left(-\frac{1}{4}\right) \cdot \left(\frac{1}{\frac{s+4}{-4}}\right)$$

$$= \left(-\frac{1}{4}\right) \cdot \left(\frac{-4}{s+4}\right)$$

$$= + \frac{1}{s+4}$$

$$\mathcal{L}\{e^{-4t}\} = \frac{1}{(s-(-4))} \qquad \mathcal{L}\{e^{at}\} = \frac{1}{s-a}$$

$$\mathcal{L}\{\sin(t)\} = \frac{1}{s^2 + 1^2}$$

$$\mathcal{L}\{\cos(t)\} = \frac{s}{s^2 + 1^2}$$

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$$\mathcal{L}\{\cos(6t)\} = \frac{1}{6}\left(\frac{\frac{s}{6}}{\left(\frac{s}{6}\right)^2 + 1^2}\right)$$

$$= \frac{1}{36}\left(\frac{s}{\frac{s^2}{36} + 1}\right)$$

$$= \frac{1}{\cancel{36}}\left(\frac{s}{\frac{s^2 + 36}{\cancel{36}}}\right)$$

$$= \frac{s}{s^2 + 36}$$

$$\mathcal{L}\{\cos(6t)\} = \frac{s}{s^2 + (6)^2}$$

$$\begin{cases} \mathcal{L}\{\cos(bt)\} = \dfrac{s}{s^2 + b^2} \\ \mathcal{L}\{\sin(bt)\} = \dfrac{b}{s^2 + b^2} \end{cases}$$

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

$$\mathcal{L}\{f''(t)\} = s^2F(s) - s\cdot f(0) - f'(0)$$

$$\mathcal{L}\{f'''(t)\} = s^3F(s) - s^2f(0) - sf'(0) - f''(0)$$

$$\mathcal{L}\{f^{(n)}(t)\} = s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$$

$$\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y = 4e^t \qquad \begin{cases} y(0) = 4 \\ y'(0) = -3 \end{cases}$$

$$\mathcal{L}\left\{\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y\right\} = \mathcal{L}\{4e^t\}$$

$$\mathcal{L}\left\{\frac{d^2y}{dt^2}\right\} - 5\mathcal{L}\left\{\frac{dy}{dt}\right\} + 6\mathcal{L}\{y\} = 4\mathcal{L}\{e^t\}$$

$$\left[s^2\cancel{Y(s)} - s\cdot(4) - (-3)\right] - 5\cdot\left[s\cancel{Y(s)} - (4)\right] + 6\cancel{Y(s)} = 4\left[\frac{1}{s-1}\right]$$

$$(s^2-5s+6)\cdot Y(s) - 4s + 23 = \frac{4}{s-1}$$

$$(s^2-5s+6)\cdot Y(s) = \frac{4}{s-1} + 4s - 23$$

$$= \frac{4 + (4s-23)(s-1)}{s-1}$$

$$= \frac{4 + 4s^2 - 27s + 23}{s-1}$$

$$(s^2-5s+6)\, Y(s) = \frac{4s^2 - 27s + 27}{(s-1)}$$

$$Y(s) = \frac{4s^2 - 27s + 27}{(s-1)(s^2-5s+6)}$$

$y(t)$

$$= \frac{4s^2 - 27s + 27}{(s-1)(s-2)(s-3)}$$

$$= \frac{A}{s-1} + \frac{B}{s-2} + \frac{C}{s-3}$$

$$\frac{4s^2-27s+27}{(s-1)(s-2)(s-3)} = \frac{A}{s-1} + \frac{B}{s-2} + \frac{C}{s-3}$$

$$4s^2-27s+27 = A(s-2)(s-3)+B(s-1)(s-3)+C(s-1)(s-2)$$

IF $s=1$

$$4(1)^2-27(1)+27 = A(-1)(-2)$$

$$4 = 2A \quad \boxed{A=2}$$

IF $s=2$

$$4(2)^2-27(2)+27 = 2(0)+B(1)(-1)+C(0)$$

$$16-27 = -B$$

$$\boxed{B=11}$$

IF $s=3$

$$4(3)^2-27(3)+27 = 2(0)+11(0)+C(2)(1)$$

$$36-54 = 2C$$

$$-18 = 2C \quad \boxed{C=-9}$$

$$Y(s) = \frac{2}{s-1} + \frac{11}{s-2} - \frac{9}{s-3}$$

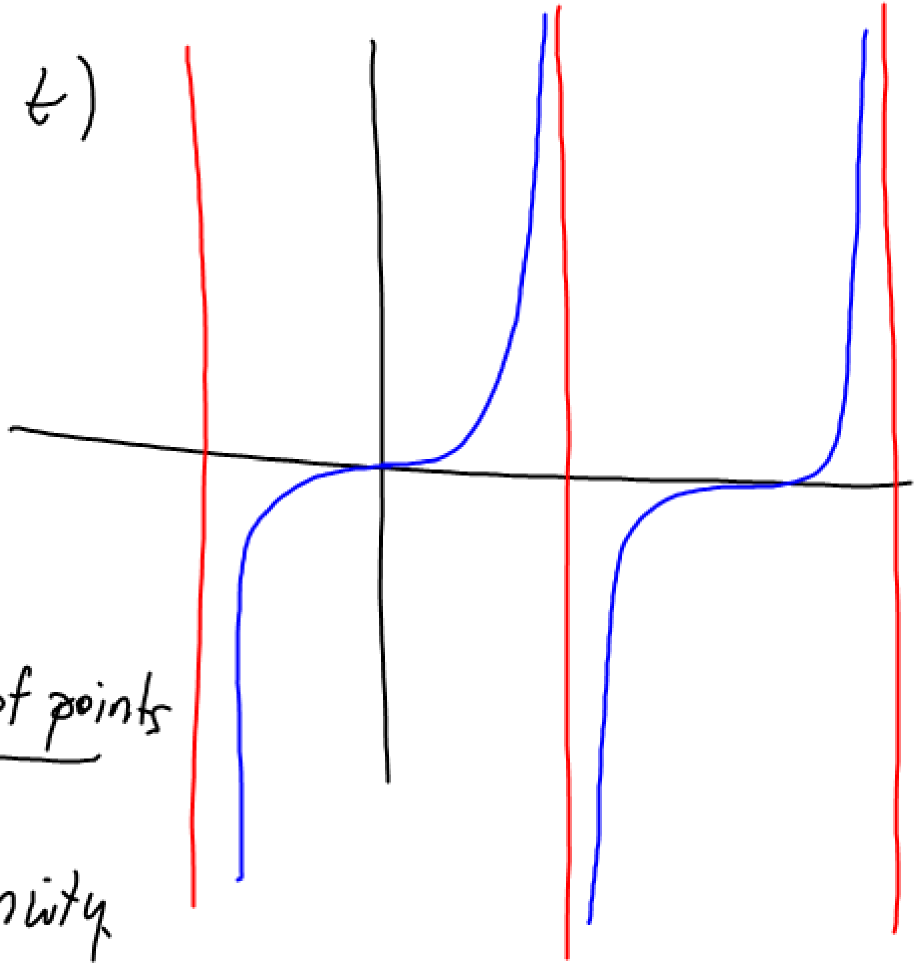
$$Y(s) = \frac{2}{s-1} + \frac{11}{s-2} - \frac{9}{s-3}$$

$$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{2}{s-1} + \frac{11}{s-2} - \frac{9}{s-3}\right\}$$

$$y(t) = 2\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} + 11\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} - 9\mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\}$$

$$y(t) = 2e^{t} + 11e^{2t} - 9e^{3t}$$

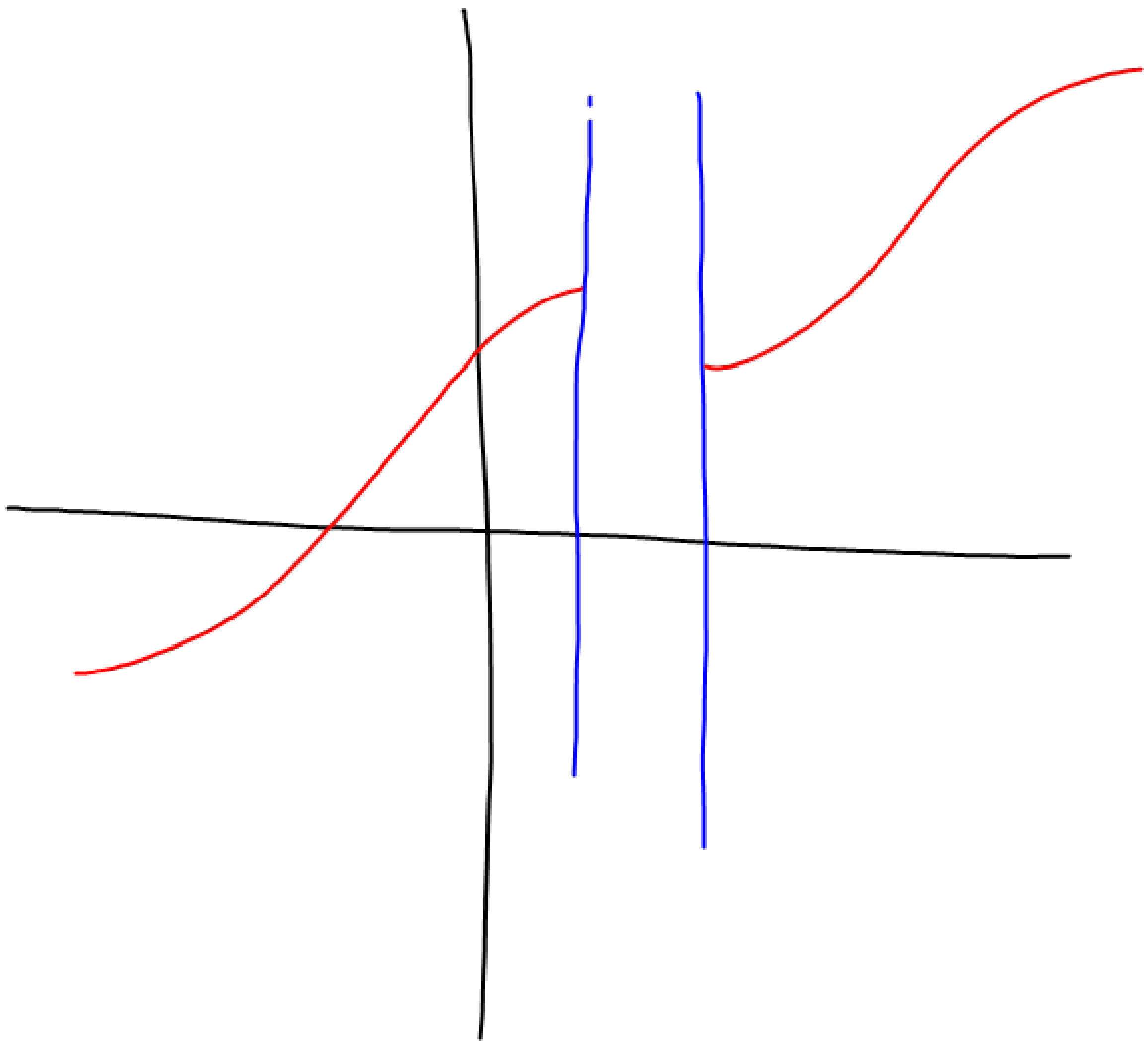
tan(t)



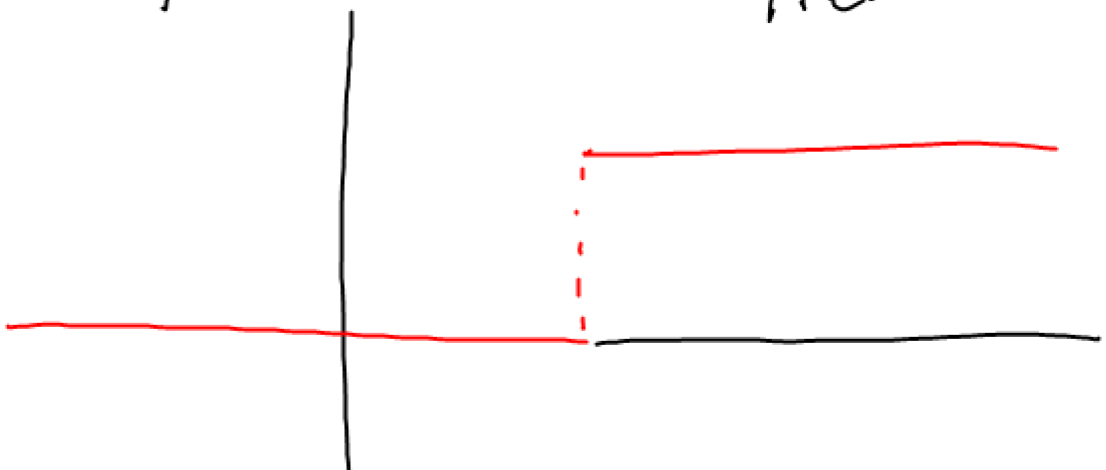
finit number of points

we have

discontinuity



step function Heaviside$(t - a)$



Dirac$(t - a)$

$$u(t-a) = \begin{cases} 0 ; & t < a \\ 1 : & t > a \end{cases}$$

$$\boxed{\mu(t-4)-\mu(t-5)}$$