Propierties of Laplace Transform

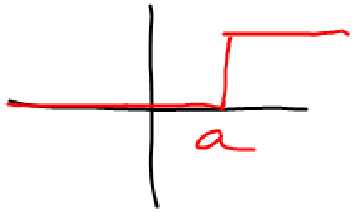
1.- $\mathcal{L}\{af+bg\} = aF(s) - bG(s)$

$a,b \in \mathbb{R}$

2.- $\mathcal{L}\{f(at)\} = \frac{1}{a}F\left(\frac{s}{a}\right)$

3.- $\mathcal{L}\{f^{(n)}(t)\} = s^{n}F(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \dots - f^{(n-1)}(0)$

$$u(t-a) = \begin{cases} 0 ; & t < a \\ 1 ; & t > a \end{cases}$$



$$\mathcal{L}\{u(t-a)\} = \int_0^\infty u(t-a)\, e^{-st}\, dt$$

step function

$$= \cancel{\int_0^a (0)\, e^{-st}\, dt}^{\,0} + \int_a^\infty (1)\, e^{-st}\, dt$$

$$\mathcal{L}\{u(t-a)\} = \left[\int e^{-st}\, dt\right]_a^\infty \Rightarrow \left[-\frac{1}{s}\int e^{-st}(-s\,dt)\right]_a^\infty$$

$$= -\frac{1}{s}\left(e^{-st}\right)_a^\infty \Rightarrow -\frac{1}{s}\left(\cancel{\lim_{t\to\infty} e^{-st}}^{\,0} - e^{-as}\right)$$

$$\mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s} \qquad \mathcal{L}\{1\} = \frac{1}{s}$$

④

$$\mathcal{L}^{-1}\{F'(s)\} = -t f(t)$$ ✓

$$\mathcal{L}^{-1}\{F^{(n)}(s)\} = (-1)^n t^n f(t)$$

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⑤ $$\mathcal{L}^{-1}\left\{\frac{F(s)}{s}\right\} = \int_0^t f(z)\,dz$$

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⑥ $$\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(v)\,dv$$

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⑦ $$\mathcal{L}^{-1}\{e^{-bs}F(s)\} = f(t-b)\cdot u(t-b)$$ ✓

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⑧ $$\mathcal{L}\{e^{at}f(t)\} = F(s-a)$$ ✓

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⑨ $$\mathcal{L}^{-1}\{F(s)G(s)\} = f(t) * g(t)$$

convolution operator

$$f(t) * g(t) = \int_0^t f(z)\,g(t-z)\,dz.$$ ✓✓

$$\mathcal{L}\{1\} = \frac{1}{s} \qquad \mathcal{L}\{e^{3t}(1)\} = \frac{1}{(s-3)}$$

$$\mathcal{L}\{te^{3t}\} = \frac{1}{(s-3)^2} \qquad \mathcal{L}\{t\} = \frac{1}{s^2}$$

$$\mathcal{L}\{\cos(4t)\} = \frac{s}{s^2+(4)^2}$$

$$\mathcal{L}\{e^{2t}\cdot\cos(4t)\} = \frac{(s-2)}{(s-2)^2+16}$$

$$\mathcal{L}\{\sin(6t)\} = \frac{6}{s^2+36}$$

$$\mathcal{L}\{e^{-3t}\sin(6t)\} = \frac{6}{(s+3)^2+36}$$

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$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+2s+2}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{(s^2+2s)+2}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{s}{(s^2+2s+1)+2-1}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{(s+1)-1}{(s+1)^2+1}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{(s+1)}{(s+1)^2+1}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+1}\right\}$$

$$= e^{-t}\cos(t) - e^{-t}\sin(t)$$

$$\mathcal{L}^{-1}\left\{\frac{e^{-4s}}{(s-9)^3}\right\} = \frac{1}{2}e^{9(t-4)}(t-4)^2 \cdot u(t-4)$$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-9)^3}\right\} = \frac{1}{2}e^{9t}t^2$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\} = \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{2!}{s^3}\right\} \Rightarrow \frac{1}{2}t^2$$

$$f(t) = \frac{1}{2}e^{9(t-4)}(t-4)^2 \cdot u(t-4)$$

$$\begin{cases} 0 \ ; \ t < 4 \\ \frac{1}{2}e^{9(t-4)}(t-4)^2 \ ; \ t > 4 \end{cases}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{(s^2+4)^2}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\cdot\frac{1}{s^2+4}\right\}$$

$$= \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\cdot\frac{2}{s^2+4}\right\}$$

$$= \frac{1}{2}\cos(2t) * \sin(2t)$$

convolution operator

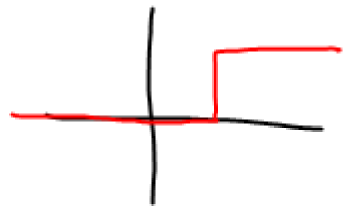
$$\cos(2t) * \sin(2t) = \int_0^t \cos(2\tau)\cdot\sin(2(t-\tau))\,d\tau$$

$$\mathcal{L}^{-1}\left\{\frac{s}{(s^2+4)^2}\right\} = \frac{1}{2}\left(\frac{1}{2}t\sin(2t)\right)$$

$$\mathcal{L}^{-1}\left\{\frac{s}{(s^2+4)^2}\right\} = \frac{1}{4}t\sin(2t)$$

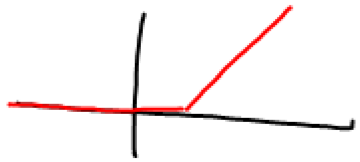
step

$$u(t-a) = \begin{cases} 0 & ; t < a \\ 1 & ; t > a \end{cases}$$



slope

$$r(t-a) = \begin{cases} 0 & ; t < a \\ (t-a) & ; t > a \end{cases}$$



delta Dirac.

$$\delta(t-a) \begin{cases} 0 & ; t \neq a \\ \int_{-\infty}^{\infty} \delta \, dt = 1. \end{cases}$$