

> restart

>

$$\frac{dx_1(t)}{dt} = 3x_1(t) + 4x_2(t) + 6e^{2t}; \quad x_1(0) = 2$$

$$\frac{dx_2(t)}{dt} = 2x_1(t) + 5x_2(t) + 3t^2 + 2t; \quad x_2(0) = -3$$

> Syst := diff(x₁(t), t) = 3·x₁(t) + 4·x₂(t) + 6·exp(2·t), diff(x₂(t), t) = 2·x₁(t) + 5·x₂(t) + 3·t·2 + 2·t : Syst₁; Syst₂;

$$\frac{d}{dt} x_1(t) = 3x_1(t) + 4x_2(t) + 6e^{2t}$$

$$\frac{d}{dt} x_2(t) = 2x_1(t) + 5x_2(t) + 3t^2 + 2t \quad (1)$$

> InitCond := x₁(0) = 2, x₂(0) = -3;

$$InitCond := x_1(0) = 2, x_2(0) = -3 \quad (2)$$

> with(inttrans) :

> LapTransOne := subs(InitCond, laplace(Syst₁, t, s))

$$LapTransOne := s \operatorname{laplace}(x_1(t), t, s) - 2 = 3 \operatorname{laplace}(x_1(t), t, s) + 4 \operatorname{laplace}(x_2(t), t, s) + \frac{6}{s-2} \quad (3)$$

> LapTransTwo := subs(InitCond, laplace(Syst₂, t, s))

$$LapTransTwo := s \operatorname{laplace}(x_2(t), t, s) + 3 = 2 \operatorname{laplace}(x_1(t), t, s) + 5 \operatorname{laplace}(x_2(t), t, s) + \frac{6}{s^3} + \frac{2}{s^2} \quad (4)$$

> Incog := simplify(isolate(LapTransTwo, laplace(x₁(t), t, s)))

$$Incog := \operatorname{laplace}(x_1(t), t, s) = \frac{1}{2} \frac{s^4 \operatorname{laplace}(x_2(t), t, s) + 3s^3 - 5 \operatorname{laplace}(x_2(t), t, s)s^3 - 6 - 2s}{s^3} \quad (5)$$

> LapTransSolTwo := simplify(isolate(subs(laplace(x₁(t), t, s) = rhs(Incog), LapTransOne), laplace(x₂(t), t, s)))

$$LapTransSolTwo := \operatorname{laplace}(x_2(t), t, s) = -\frac{-19s^4 + 12s^3 + 3s^5 + 4s^2 + 18s - 36}{(s-2)s^3(s^2 - 8s + 7)} \quad (6)$$

> PartSolTwo := invlaplace(LapTransSolTwo, s, t)

$$PartSolTwo := x_2(t) = -\frac{12}{5} e^{2t} + 3e^t - \frac{144}{49} t - \frac{1534}{1715} e^{7t} - \frac{9}{7} t^2 - \frac{928}{343} \quad (7)$$

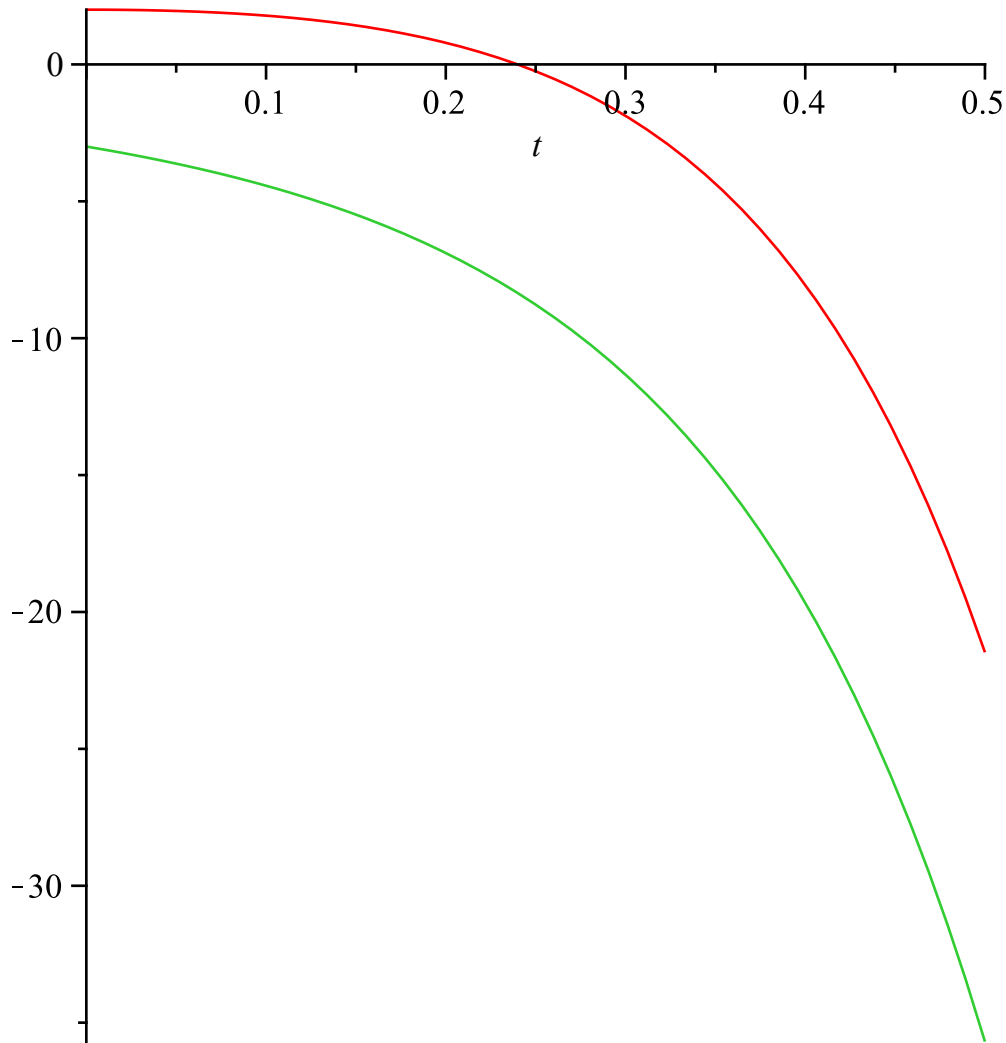
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> LapTransSolOne := simplify(subs(laplace(x2(t), t, s) = rhs(LapTransSolTwo), Incog))
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$$\text{LapTransSolOne} := \text{laplace}(x_1(t), t, s) = \frac{2(s^5 - 10s^4 + 7s^3 + 4s^2 + 4s - 24)}{(s-2)s^3(s^2 - 8s + 7)} \quad (8)$$

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> PartSolOne := simplify(invlaplace(LapTransSolOne, s, t))
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$$\text{PartSolOne} := x_1(t) = \frac{18}{5} e^{2t} - 6 e^t + \frac{248}{49} t - \frac{1534}{1715} e^{7t} + \frac{12}{7} t^2 + \frac{1816}{343} \quad (9)$$

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> plot([rhs(PartSolOne), rhs(PartSolTwo)], t = 0..0.5)
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> AA := array([[3, 4], [2, 5]])
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$$AA := \begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix} \quad (10)$$

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> II := array([[1, 0], [0, 1]])
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$$II := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (11)$$

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> TransInitExpMat := evalm(s*II - AA)
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$$\text{TransInitExpMat} := \begin{bmatrix} s-3 & -4 \\ -2 & s-5 \end{bmatrix} \quad (12)$$

> with(linalg) :

> InvTransInitExpMat := inverse(TransInitExpMat)

$$\text{InvTransInitExpMat} := \begin{bmatrix} \frac{s-5}{s^2-8s+7} & \frac{4}{s^2-8s+7} \\ \frac{2}{s^2-8s+7} & \frac{s-3}{s^2-8s+7} \end{bmatrix} \quad (13)$$

> with(inttrans) :

> ExpMat := map(convert, map(invlaplace, InvTransInitExpMat, s, t), exp)

$$\text{ExpMat} := \begin{bmatrix} \frac{2}{3} e^t + \frac{1}{3} e^{7t} & \frac{2}{3} e^{7t} - \frac{2}{3} e^t \\ \frac{1}{3} e^{7t} - \frac{1}{3} e^t & \frac{1}{3} e^t + \frac{2}{3} e^{7t} \end{bmatrix} \quad (14)$$

> Ident := map(rcurry(eval, t=0'), ExpMat)

$$\text{Ident} := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (15)$$

> ExpMatDer := map(diff, ExpMat, t)

$$\text{ExpMatDer} := \begin{bmatrix} \frac{2}{3} e^t + \frac{7}{3} e^{7t} & \frac{14}{3} e^{7t} - \frac{2}{3} e^t \\ \frac{7}{3} e^{7t} - \frac{1}{3} e^t & \frac{1}{3} e^t + \frac{14}{3} e^{7t} \end{bmatrix} \quad (16)$$

> ZeroMat := evalm(ExpMatDer - evalm(AA &* ExpMat))

$$\text{ZeroMat} := \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (17)$$

> OrigAA := map(rcurry(eval, t=0'), ExpMatDer)

$$\text{OrigAA} := \begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix} \quad (18)$$

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