

> restart

> Ecuacion := diff(f(x, y), x\$2) = a·2·diff(f(x, y), y)

$$Ecuacion := \frac{\partial^2}{\partial x^2} f(x, y) = a^2 \left(\frac{\partial}{\partial y} f(x, y) \right) \quad (1)$$

> Solucion := pdsolve(Ecuacion)

$$Solucion := (f(x, y) = _F1(x) _F2(y)) \&where \left[\left\{ \frac{d^2}{dx^2} _F1(x) = _c1 _F1(x), \frac{d}{dy} _F2(y) = \frac{_c1 _F2(y)}{a^2} \right\} \right] \quad (2)$$

> with(PDEtools);

[CanonicalCoordinates, ChangeSymmetry, CharacteristicQ, CharacteristicQInvariants, ConservedCurrentTest, ConservedCurrents, ConsistencyTest, D_Dx, DeterminingPDE, Eta_k, Euler, FromJet, InfinitesimalGenerator, Infinitesimals, IntegratingFactorTest, IntegratingFactors, InvariantSolutions, InvariantTransformation, Invariants, Laplace, Library, PDEplot, PolynomialSolutions, ReducedForm, SimilaritySolutions, SimilarityTransformation, SymmetrySolutions, SymmetryTest, SymmetryTransformation, TWSolutions, ToJet, build, casesplit, charstrip, dchange, dcoeffs, declare, diff_table, difforder, dpolyform, dsubs, mapde, separability, splitstrip, splitsys, undeclare] (3)

> SolucionFinal := build(Solucion)

$$SolucionFinal := f(x, y) = e^{\sqrt{-c_1} x} _C3 e^{\frac{-c_1 y}{a^2}} _C1 + \frac{_C3 e^{\frac{-c_1 y}{a^2}} _C2}{e^{\sqrt{-c_1} x}} \quad (4)$$

> Ecuacion

$$\frac{\partial^2}{\partial x^2} f(x, y) = a^2 \left(\frac{\partial}{\partial y} f(x, y) \right) \quad (5)$$

> EcuacionSeparable := simplify(eval(subs(f(x, y) = F(x)·G(y), Ecuacion)))

$$EcuacionSeparable := \left(\frac{d^2}{dx^2} F(x) \right) G(y) = a^2 F(x) \left(\frac{d}{dy} G(y) \right) \quad (6)$$

> EcuacionSeparada := $\frac{lhs(EcuacionSeparable)}{a \cdot 2 \cdot F(x) \cdot G(y)} = \frac{rhs(EcuacionSeparable)}{a \cdot 2 \cdot F(x) \cdot G(y)}$

$$EcuacionSeparada := \frac{\frac{d^2}{dx^2} F(x)}{a^2 F(x)} = \frac{\frac{d}{dy} G(y)}{G(y)} \quad (7)$$

> EcuacionX := lhs(EcuacionSeparada) = alpha; EcuacionY := rhs(EcuacionSeparada) = alpha

$$EcuacionX := \frac{\frac{d^2}{dx^2} F(x)}{a^2 F(x)} = \alpha$$

(8)

$$EcuacionY := \frac{\frac{d}{dy} G(y)}{G(y)} = \alpha \quad (8)$$

> $EcuacionXcero := \text{subs}(\alpha = 0, EcuacionX); EcuacionYcero := \text{subs}(\alpha = 0, EcuacionY)$

$$EcuacionXcero := \frac{\frac{d^2}{dx^2} F(x)}{a^2 F(x)} = 0$$

$$EcuacionYcero := \frac{\frac{d}{dy} G(y)}{G(y)} = 0 \quad (9)$$

> $SolucionXcero := \text{dsolve}(EcuacionXcero); SolucionYcero := \text{dsolve}(EcuacionYcero)$

$$SolucionXcero := F(x) = _C1 x + _C2$$

$$SolucionYcero := G(y) = _C1 \quad (10)$$

>

C1

> $SolucionCero := f(x, y) = \text{rhs}(SolucionXcero) \cdot \text{subs}(_C1 = _C3, \text{rhs}(SolucionYcero))$

$$SolucionCero := f(x, y) = (_C1 x + _C2) _C3 \quad (11)$$

> $Comprobacion_1 := \text{simplify}(\text{eval}(\text{subs}(f(x, y) = \text{rhs}(SolucionCero), Ecuacion)))$

$$Comprobacion_1 := 0 = 0 \quad (12)$$

> $EcuacionXpos := \text{subs}(\alpha = \beta \cdot 2, EcuacionX); EcuacionYpos := \text{subs}(\alpha = \beta \cdot 2, EcuacionY)$

$$EcuacionXpos := \frac{\frac{d^2}{dx^2} F(x)}{a^2 F(x)} = \beta^2$$

$$EcuacionYpos := \frac{\frac{d}{dy} G(y)}{G(y)} = \beta^2 \quad (13)$$

> $SolucionXpos := \text{dsolve}(EcuacionXpos); SolucionYpos := \text{dsolve}(EcuacionYpos)$

$$SolucionXpos := F(x) = _C1 e^{-\beta a x} + _C2 e^{\beta a x}$$

$$SolucionYpos := G(y) = _C1 e^{\beta^2 y} \quad (14)$$

> $SolucionPositiva := f(x, y) = \text{rhs}(SolucionXpos) \cdot \text{subs}(_C1 = _C3, \text{rhs}(SolucionYpos))$

$$SolucionPositiva := f(x, y) = (_C1 e^{-\beta a x} + _C2 e^{\beta a x}) _C3 e^{\beta^2 y} \quad (15)$$

> $Comprobacion_2 := \text{simplify}(\text{eval}(\text{subs}(f(x, y) = \text{rhs}(SolucionPositiva), \text{lhs}(Ecuacion) - \text{rhs}(Ecuacion) = 0)))$

$$Comprobacion_2 := 0 = 0 \quad (16)$$

> $EcuacionXneg := \text{subs}(\alpha = -\beta \cdot 2, EcuacionX); EcuacionYneg := \text{subs}(\alpha = -\beta \cdot 2, EcuacionY)$

$$EcuacionXneg := \frac{\frac{d^2}{dx^2} F(x)}{a^2 F(x)} = -\beta^2$$

$$EcuacionYneg := \frac{\frac{d}{dy} G(y)}{G(y)} = -\beta^2 \quad (17)$$

$$\begin{aligned} &> SolucionXneg := dsolve(EcuacionXneg); SolucionYneg := dsolve(EcuacionYneg) \\ &SolucionXneg := F(x) = _C1 \sin(\beta a x) + _C2 \cos(\beta a x) \end{aligned}$$

$$SolucionYneg := G(y) = _C1 e^{-\beta^2 y} \quad (18)$$

$$> SolucionNegativa := f(x, y) = rhs(SolucionXneg) \cdot subs(_C1 = _C3, rhs(SolucionYneg))$$

$$SolucionNegativa := f(x, y) = (_C1 \sin(\beta a x) + _C2 \cos(\beta a x)) _C3 e^{-\beta^2 y} \quad (19)$$

$$\begin{aligned} &> Comprobacion_3 := simplify(eval(subs(f(x, y) = rhs(SolucionNegativa), lhs(Ecuacion) \\ &\quad - rhs(Ecuacion) = 0))) \end{aligned}$$

$$Comprobacion_3 := 0 = 0 \quad (20)$$

> restart

$$> Ecuacion := diff(z(x, y), x$2) + diff(z(x, y), y) = z(x, y)$$

$$Ecuacion := \frac{\partial^2}{\partial x^2} z(x, y) + \frac{\partial}{\partial y} z(x, y) = z(x, y) \quad (21)$$

$$> EcuacionSep := simplify(eval(subs(z(x, y) = F(x) \cdot G(y), Ecuacion)))$$

$$EcuacionSep := \left(\frac{d^2}{dx^2} F(x) \right) G(y) + F(x) \left(\frac{d}{dy} G(y) \right) = F(x) G(y) \quad (22)$$

$$\begin{aligned} &> EcuacionDos := lhs(EcuacionSep) - F(x) \left(\frac{d}{dy} G(y) \right) = rhs(EcuacionSep) \\ &\quad - F(x) \left(\frac{d}{dy} G(y) \right) \end{aligned}$$

$$EcuacionDos := \left(\frac{d^2}{dx^2} F(x) \right) G(y) = F(x) G(y) - F(x) \left(\frac{d}{dy} G(y) \right) \quad (23)$$

$$> EcuacionSepUno := \frac{lhs(EcuacionDos)}{F(x) \cdot G(y)} = simplify\left(\frac{rhs(EcuacionDos)}{F(x) \cdot G(y)}\right)$$

$$EcuacionSepUno := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \frac{G(y) - \left(\frac{d}{dy} G(y) \right)}{G(y)} \quad (24)$$

$$> EcuacionTres := lhs(EcuacionDos) - F(x) \cdot G(y) = rhs(EcuacionDos) - F(x) \cdot G(y)$$

$$EcuacionTres := \left(\frac{d^2}{dx^2} F(x) \right) G(y) - F(x) G(y) = -F(x) \left(\frac{d}{dy} G(y) \right) \quad (25)$$

$$> EcuacionSepDos := simplify\left(\frac{lhs(EcuacionTres)}{F(x) \cdot G(y)}\right) = \frac{rhs(EcuacionTres)}{F(x) \cdot G(y)}$$

$$EcuacionSepDos := \frac{\frac{d^2}{dx^2} F(x) - F(x)}{F(x)} = -\frac{\frac{d}{dy} G(y)}{G(y)} \quad (26)$$

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