Serie trigonométrica de FOURIER

$$\text{funciones} \begin{cases} \text{simétricas} \begin{cases} \text{PAR} \\ \text{IMPAR} \end{cases} \\ \text{no-simétricas} \quad \left[e^{at} \right] \end{cases}$$

$$y = x^2 \longrightarrow \text{PAR} \longrightarrow \quad f(-x) = f(x)$$

$$a \leq x \leq b$$

b^2 $\quad$ b^2

$-b$ $\quad$ b

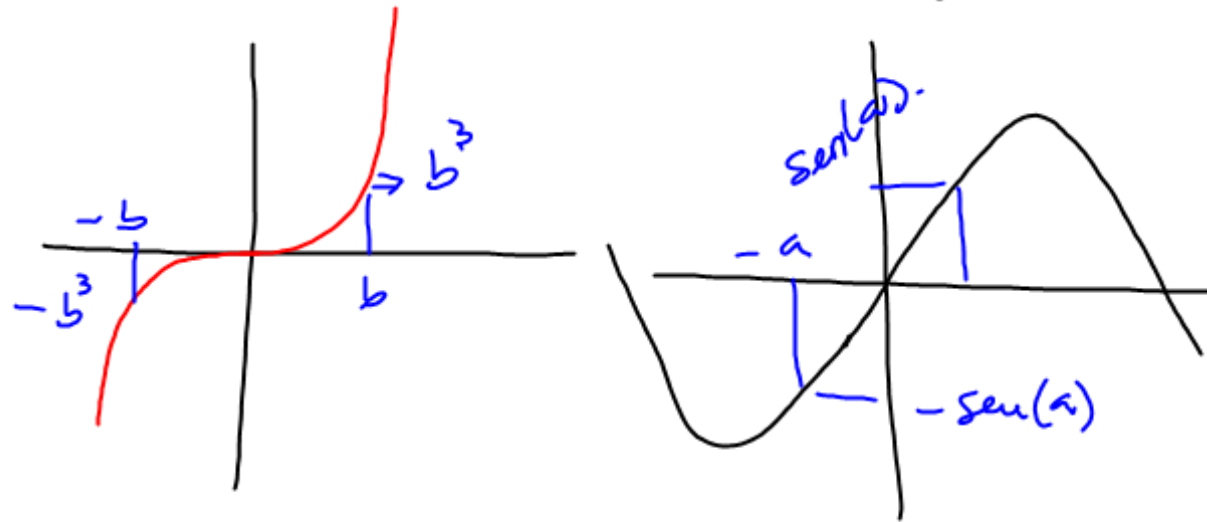$\cos(-a)$ $\qquad$ $\cos(a)$

$-a$ $\quad$ a
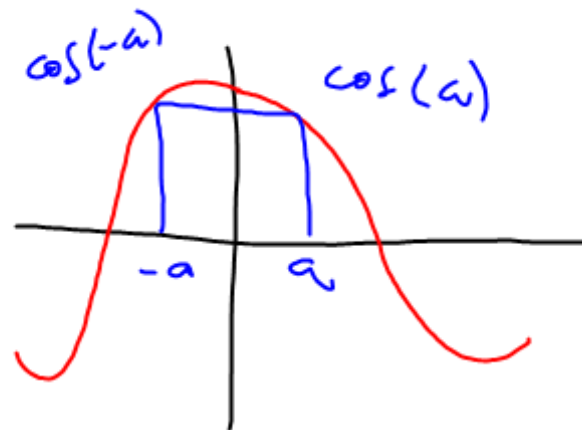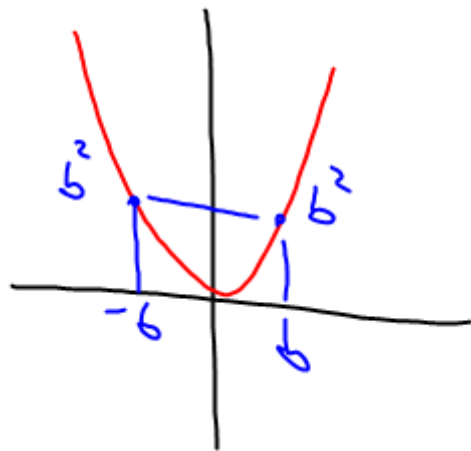
$$\cos(bx)$$

$$y = x^3 \longrightarrow \text{IMPAR} \qquad f(-x) = -f(x)$$

$$a \leq x \leq b$$

$$\text{PAR} \longrightarrow \cos(Lx), x^2, C, |x|, x^4, x^6 \ldots$$

$$\text{IMPAR} \longrightarrow \text{sen}(bx), x^3, x, x^5, x^7, \ldots$$

PAR (+)

IMPAR (-)

$$+ \quad * \quad + \quad \rightarrow \quad +$$
$$- \quad * \quad - \quad \rightarrow \quad +$$
$$+ \quad * \quad - \quad \rightarrow \quad -$$
$$- \quad * \quad + \quad \rightarrow \quad -$$

$$\langle PAR \rangle * \langle PAR \rangle \longrightarrow \langle PAR \rangle$$

$$\langle IMPAR \rangle * \langle IMPAR \rangle \longrightarrow \langle PAR \rangle$$

$$\langle PAR \rangle * \langle IMAR \rangle \longrightarrow \langle IMPAR \rangle$$

$$\langle IMPAR \rangle * \langle PAR \rangle \longrightarrow \langle IMPAR \rangle$$

$$\int_{-L}^{L} \langle IMPAR \rangle = 0.$$

$$\int_{-L}^{L} \langle PAR \rangle \implies 2 \int_{0}^{L} \langle PAR \rangle \neq 0.$$

STF $\langle par \rangle$

$$C = \frac{a_0}{2} \qquad a_0 = \frac{1}{L} \int_{-L}^{L} \langle PAR \rangle = \frac{2}{L} \int_{0}^{L} \langle par \rangle \neq 0$$

$$a_N = \frac{1}{L} \int_{-L}^{L} \langle par \rangle \cos \left(\frac{n\pi}{L} x \right) dx = \frac{2}{L} \int_{0}^{L} \langle par \rangle \cos \left(\frac{n\pi}{L} x \right) dy \neq 0.$$

$$b_N = \frac{1}{L} \int_{-L}^{L} \langle par \rangle \, sen \left(\frac{n\pi}{L} x \right) dx = 0$$

$$STF (par) = C + \sum_{n=1}^{\infty} a_n \cos \left(\frac{n\pi}{L} x \right) \qquad SERIE \, COSENO$$
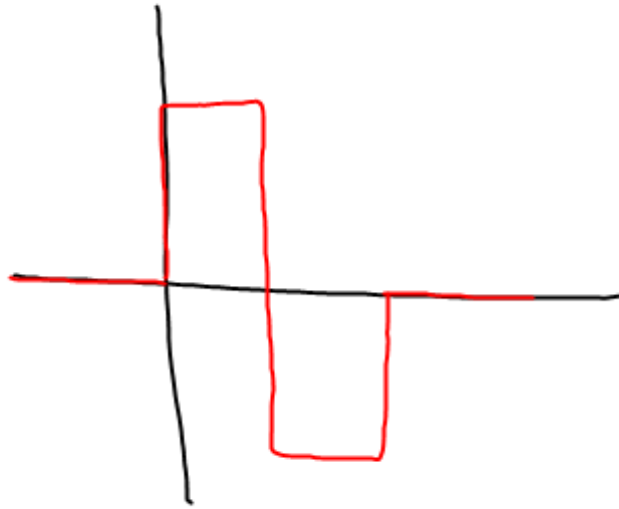
$STF \langle impar \rangle$

$$C = \frac{a_0}{2} \qquad a_0 = \left(\frac{1}{L}\right) \int_{-L}^{L} \langle impar \rangle = 0$$

$$C = 0.$$

$$a_n = \left(\frac{1}{L}\right) \int_{-L}^{L} \langle impar \rangle \cos\left(\frac{n\pi}{L} x\right) dx = 0$$

$$b_n = \left(\frac{1}{L}\right) \int_{-L}^{L} \langle impar \rangle \, sen\left(\frac{n\pi}{L} x\right) dx = \frac{3}{L} \int_{0}^{L} \langle impar \rangle \, sen\left(\frac{n\pi}{L} x\right) dx \neq 0$$
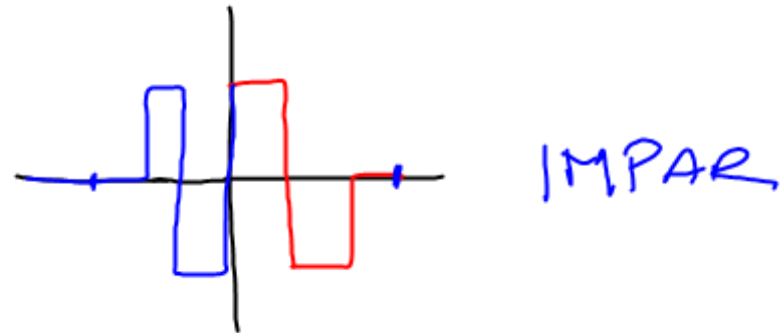
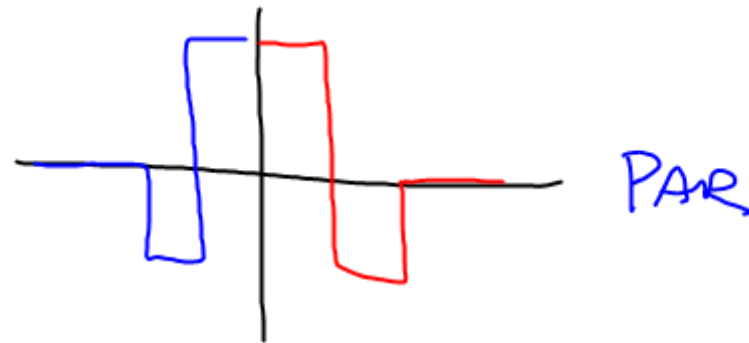$$STF(impar) = \sum_{n=1}^{\infty} b_n \, sen\left(\frac{n\pi}{L} x\right) \quad \text{SERIE SENO}$$

$$0 \leq x \leq a$$

$$L = \frac{a}{2}$$

PAR.

IMPAR

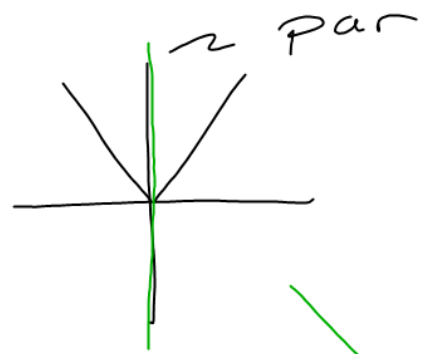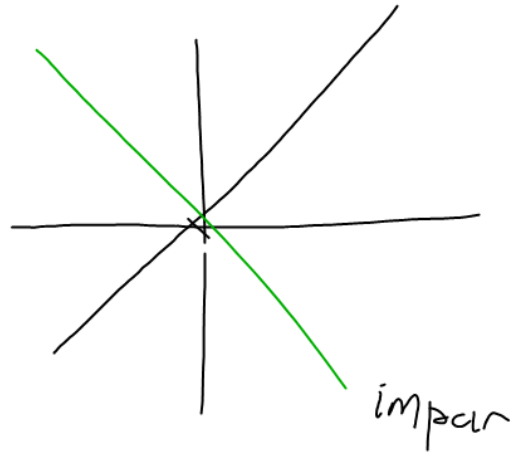$$= \sum_{n=1}^{\infty} b_n \, \text{sen}\left(\frac{n\pi}{L}x\right)$$

PAR

$$= C + \sum_{n \geq 1}^{\infty} a_n \cos\left(\frac{n\pi}{L}x\right)$$

~ par

$f = |x|$

impar

sin simetría

$$2a = \frac{2}{1000}$$

$$a = \frac{1}{1000} \, (m)$$

$$\frac{\ell}{2} = \frac{5}{10} \, [m]$$

$$\ell = 1 \, [m]$$

$$y(x,t)$$

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$$
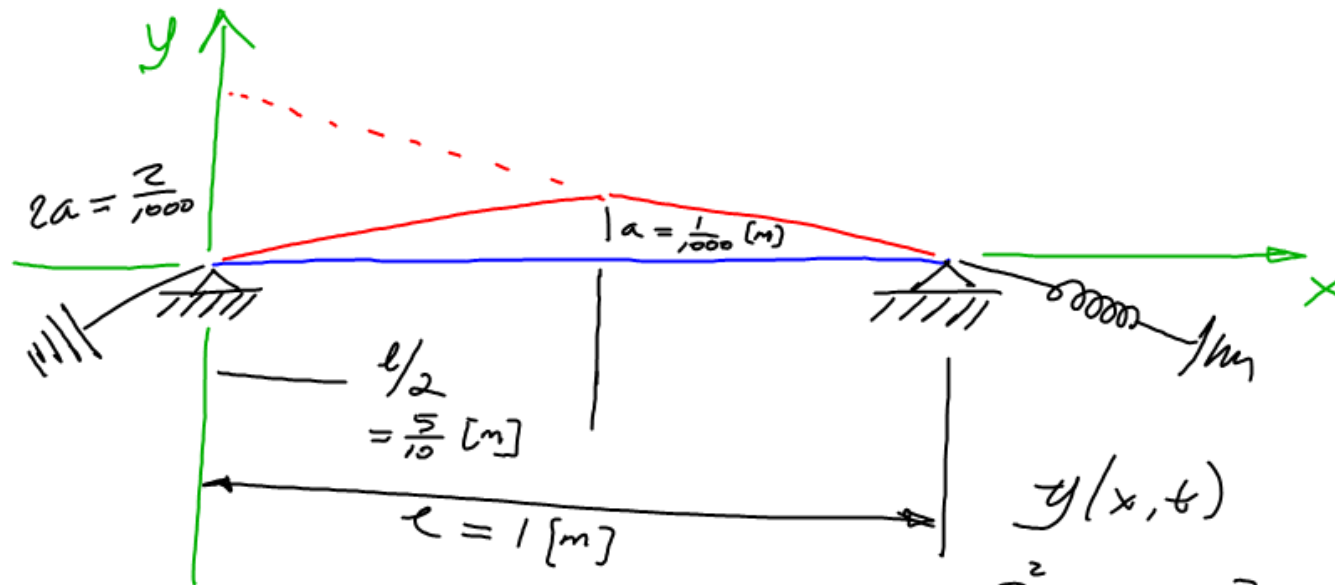
$$a^2 = \frac{\sigma}{\rho} \qquad a \neq 0 \in \mathbb{R}$$

$$a^2 = 1.$$

$$\left. \begin{array}{l} y(0,t) = 0 \\ y(1,t) = 0 \end{array} \right\} \begin{array}{l} \text{condiciones} \\ \text{de} \\ \text{frontera física} \end{array}$$

$$t = 0$$

$$y(x,0) = \begin{cases} \dfrac{\frac{1}{1000}}{\frac{5}{10}} x & ; \quad 0 \leq x \leq \frac{5}{10} \\[3mm] -\dfrac{\frac{1}{1000}}{\frac{5}{10}} x + \dfrac{2}{1000} & ; \quad \frac{5}{10} < x \leq 1 \end{cases}$$

$$\left. \frac{d}{dt} y(x,t) \right|_{t=0} = 0$$