> *restart*

PROBLEMA DE LA CUERDA DE GUITARRA DE I METRO

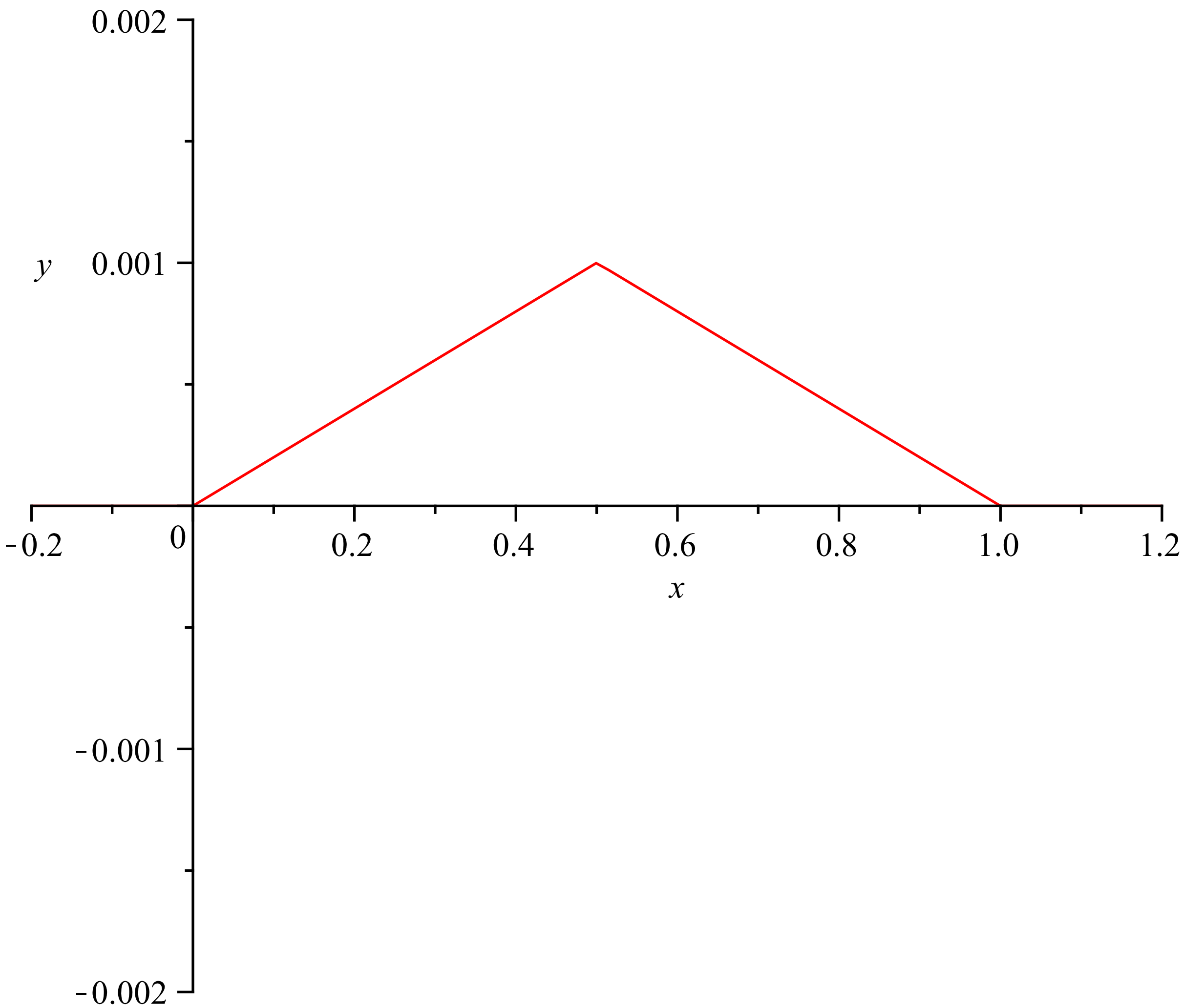
> *Ecuacion* := *diff*(*y*(*x*, *t*), *t*\$2) = *a*·2·*diff*(*y*(*x*, *t*), *x*\$2)

$$Ecuacion := \frac{\partial^2}{\partial t^2}\, y(x,t) = a^2 \left(\frac{\partial^2}{\partial x^2}\, y(x,t)\right) \tag{1}$$

> $CondicionInicialTrayectoria := f = \frac{\left(\frac{1}{1000}\right)}{\left(\frac{5}{10}\right)} \cdot x \cdot \mathrm{Heaviside}(x) - \frac{2\cdot\left(\frac{1}{1000}\right)}{\left(\frac{5}{10}\right)} \cdot \left(x - \frac{5}{10}\right) \cdot \mathrm{Heaviside}\left(x - \frac{5}{10}\right) + \frac{\left(\frac{1}{1000}\right)}{\left(\frac{5}{10}\right)} \cdot (x-1) \cdot \mathrm{Heaviside}(x-1);$

plot(*rhs*(*CondicionInicialTrayectoria*), *x* =-0.2 ..1.2, *y* =-0.002 ..0.002)

$$CondicionInicialTrayectoria := f = \frac{1}{500}\, x\, \mathrm{Heaviside}(x) - \frac{1}{250}\left(x - \frac{1}{2}\right) \mathrm{Heaviside}\left(x - \frac{1}{2}\right) + \frac{1}{500}\,(x-1)\, \mathrm{Heaviside}(x-1)$$



> $EcuacionSeparable := simplify(eval(subs(y(x,t) = F(x) \cdot G(t), a \cdot \cdot 2 = 1, Ecuacion)))$

$$EcuacionSeparable := F(x)\left(\frac{d^2}{dt^2} G(t)\right) = \left(\frac{d^2}{dx^2} F(x)\right) G(t) \qquad \textbf{(2)}$$

> $EcuacionSep := simplify\left(\frac{lhs(EcuacionSeparable)}{F(x) \cdot G(t)}\right) = simplify\left(\frac{rhs(EcuacionSeparable)}{F(x) \cdot G(t)}\right)$

$$EcuacionSep := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = \frac{\frac{d^2}{dx^2} F(x)}{F(x)} \qquad \textbf{(3)}$$

> $EcuacionX := rhs(EcuacionSep) = \mathrm{alpha}; EcuacionT := lhs(EcuacionSep) = \mathrm{alpha}$

$$EcuacionX := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \alpha$$

$$EcuacionT := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = \alpha \qquad \textbf{(4)}$$

> $CondicionesX := F(0) = 0, F(1) = 0$

$$CondicionesX := F(0) = 0, F(1) = 0 \qquad \textbf{(5)}$$

> $EcuacionXcero := subs(\mathrm{alpha} = 0, EcuacionX)$

$$EcuacionXcero := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = 0 \qquad \textbf{(6)}$$

> $SolucionXceroPart := dsolve(\{EcuacionXcero, CondicionesX\})$

$$SolucionXceroPart := F(x) = 0 \qquad \textbf{(7)}$$

> $EcuacionXpos := subs(\mathrm{alpha} = \mathrm{beta} \cdot \cdot 2, EcuacionX)$

$$EcuacionXpos := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \beta^2 \qquad \textbf{(8)}$$

> $SolucionXpos := dsolve(\{EcuacionXpos, CondicionesX\})$

$$SolucionXpos := F(x) = 0 \qquad \textbf{(9)}$$

> $EcuacionXneg := subs(\mathrm{alpha} = -\mathrm{beta} \cdot \cdot 2, EcuacionX)$

$$EcuacionXneg := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = -\beta^2 \qquad \textbf{(10)}$$

> $SolucionXneg := dsolve(EcuacionXneg)$

$$SolucionXneg := F(x) = \_C1 \sin(\beta x) + \_C2 \cos(\beta x) \qquad \textbf{(11)}$$

> $Parametro := simplify(subs(x = 0, rhs(SolucionXneg) = 0))$

$$Parametro := \_C2 = 0 \qquad \textbf{(12)}$$

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> $SolucionXnegBis := subs(\_C2 = rhs(Parametro), SolucionXneg)$

$$SolucionXnegBis := F(x) = \_C1 \sin(\beta x) \qquad \textbf{(13)}$$

> beta := $n \cdot$Pi

$$\beta := n\,\pi \tag{14}$$

> $SolucionXnegPart := SolucionXnegBis$

$$SolucionXnegPart := F(x) = \_C1\,\sin(n\,\pi\,x) \tag{15}$$

> $SolucionTneg := dsolve(subs(\text{alpha} = -\text{beta}\cdot\cdot 2, EcuacionT))$

$$SolucionTneg := G(t) = \_C1\,\sin(n\,\pi\,t) + \_C2\,\cos(n\,\pi\,t) \tag{16}$$

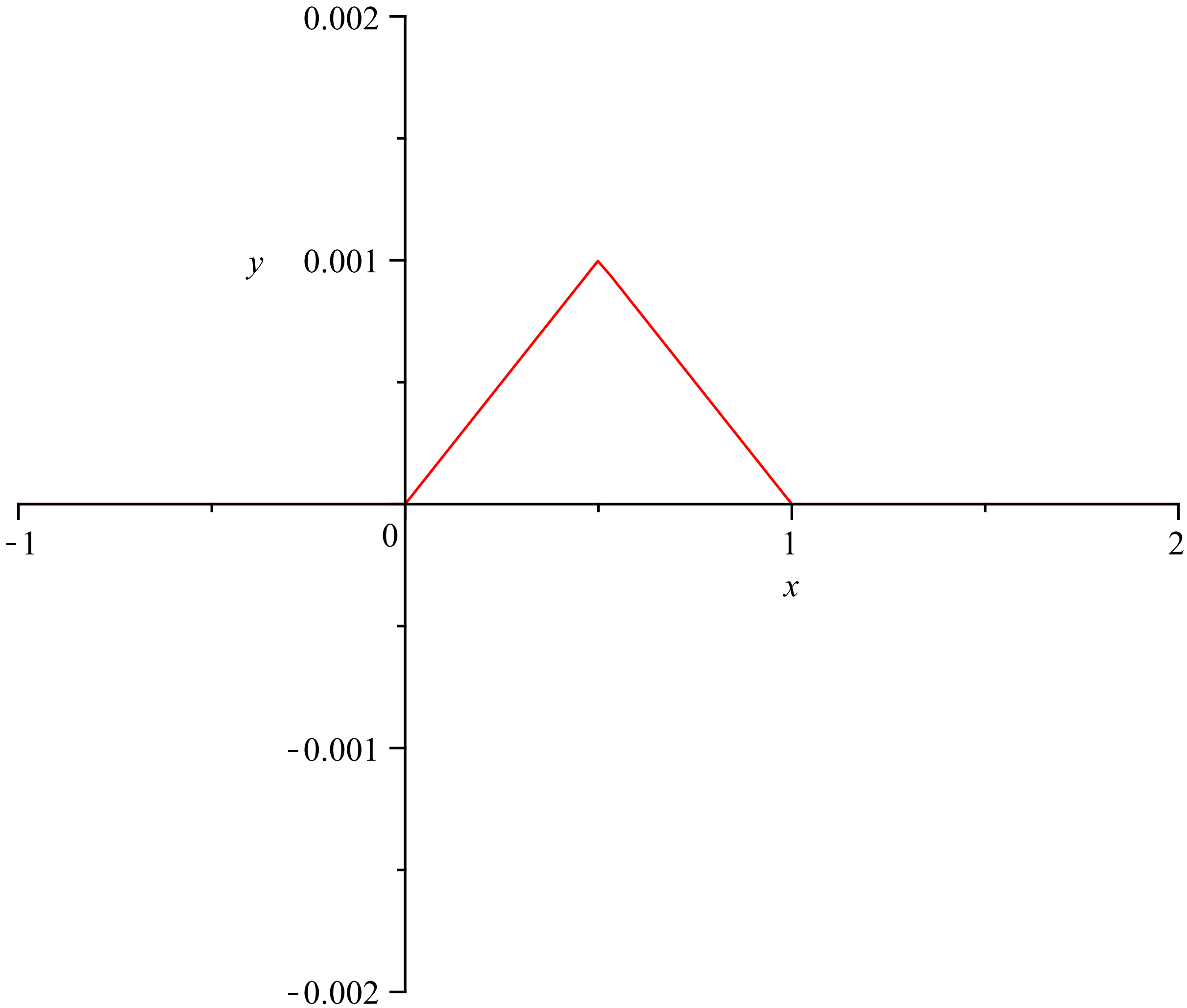
> $SolucionUno := y(x,t) = subs(\_C1 = 1, rhs(SolucionXnegPart))\cdot rhs(SolucionTneg)$

$$SolucionUno := y(x,t) = \sin(n\,\pi\,x)\,\left(\_C1\,\sin(n\,\pi\,t) + \_C2\,\cos(n\,\pi\,t)\right) \tag{17}$$

> $SolucionGeneral := y(x,t) = Sum\big(subs(\_C1 = 1, rhs(SolucionXnegPart))\cdot subs\big(\_C2 = b_n,$
$\_C1 = a_n, rhs(SolucionTneg)\big), n = 1..infinity\big)$

$$SolucionGeneral := y(x,t) = \sum_{n=1}^{\infty} \sin(n\,\pi\,x)\,\left(a_n\,\sin(n\,\pi\,t) + b_n\,\cos(n\,\pi\,t)\right) \tag{18}$$

> $plot(rhs(CondicionInicialTrayectoria), x = -1..2, y = -0.002..0.002)$



> $L := \frac{5}{10};\ b_n := \left(\frac{1}{L}\right)\cdot int(rhs(CondicionInicialTrayectoria)\cdot\sin(n\cdot\text{Pi}\cdot x), x = 0..1);\ a_n := 0$

$$L := \frac{1}{2}$$

$$b_n := \frac{1}{250} \frac{-\sin(n\pi) + 2\sin\left(\frac{1}{2} n\pi\right)}{n^2 \pi^2}$$

$$a_n := 0 \tag{19}$$

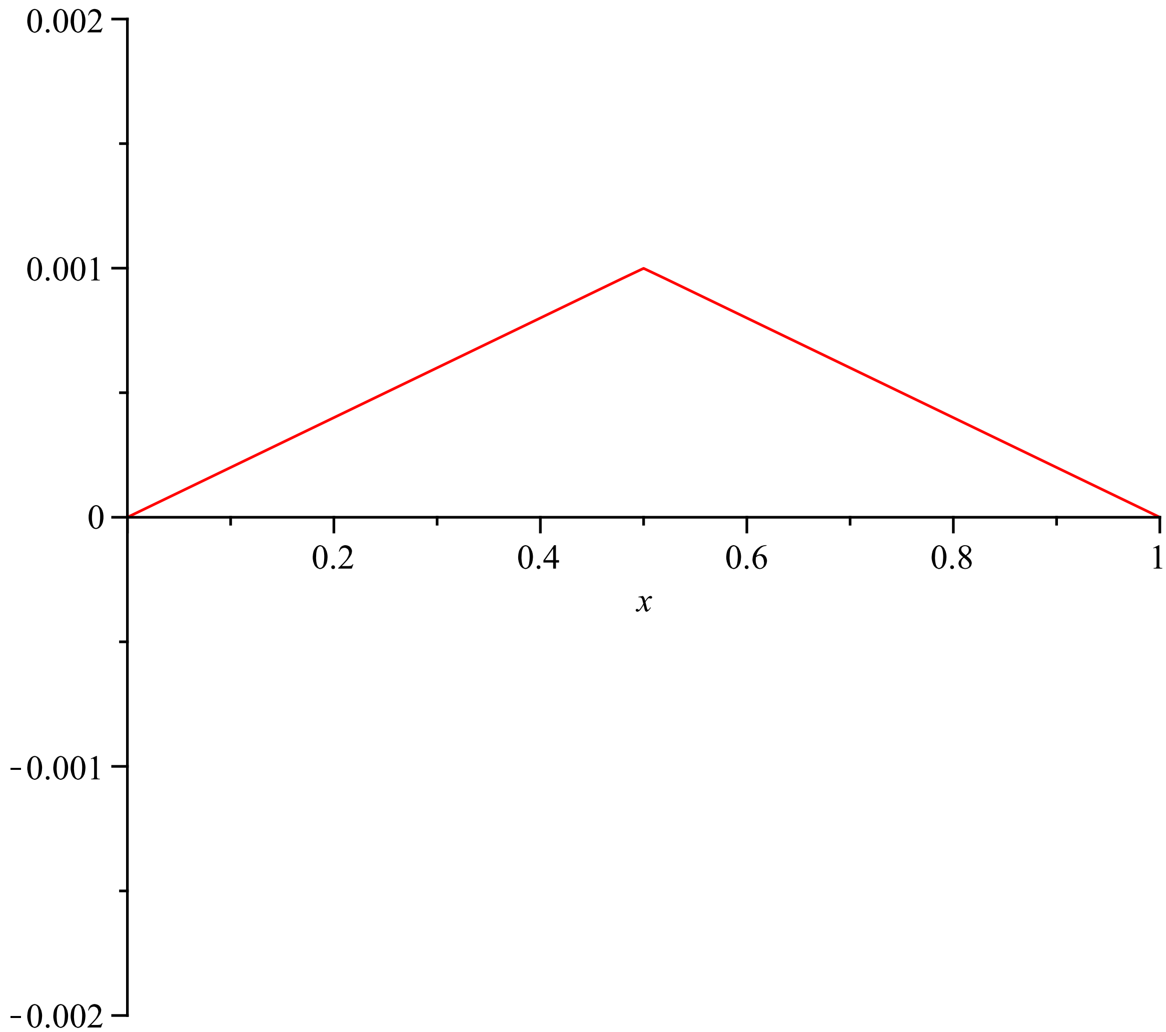
> *SolucionParticular* := *SolucionGeneral*

$$SolucionParticular := y(x, t) = \sum_{n=1}^{\infty} \frac{1}{250} \frac{\sin(n\pi x) \left(-\sin(n\pi) + 2\sin\left(\frac{1}{2} n\pi\right)\right) \cos(n\pi t)}{n^2 \pi^2} \tag{20}$$

> $Solucion_{500} := y(x, t) = \sum_{n=1}^{500} \frac{1}{250} \frac{\sin(n\pi x) \left(-\sin(n\pi) + 2\sin\left(\frac{1}{2} n\pi\right)\right) \cos(n\pi t)}{n^2 \pi^2}$:

> *with*(*plots*) :

> *animate*(*rhs*($Solucion_{500}$), $x = 0 ..1$, $t = 0 ..4$, *frames* = 150, *view* = [0 ..1, -0.002 ..0.002])



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