S(2) EDO(1) L CC NH.

Matriz Exponencial

$$\frac{dy_1}{dt} = 2y_1 + 3y_2 + 5e^{2t} \qquad y_1(0) = 3$$
$$\frac{dy_2}{dt} = y_1 + 4y_2 - 6e^{-t} \qquad y_2(0) = -2$$

$$\frac{d}{dt}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} 5e^{2t} \\ -6e^{-t} \end{bmatrix}$$

$$\bar{y} = e^{At}\bar{y}(0) + \int_0^t e^{A(t-z)} b(z)\,dz \qquad b(t) = \begin{bmatrix} 5e^{2t} \\ -6e^{-t} \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \qquad \det(A - \lambda I) = 0 \qquad \bar{y}(0) = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

$$\begin{vmatrix} 2-\lambda & 3 \\ 1 & 4-\lambda \end{vmatrix} = 0 \qquad (2-\lambda)(4-\lambda) - 3 = 0$$
$$\lambda^2 - 6\lambda + 8 - 3 = 0$$
$$\lambda^2 - 6\lambda + 5 = 0$$
$$(\lambda - 1)(\lambda - 5) = 0$$
$$\lambda_1 = 1 \quad \lambda_2 = 5 \qquad \lambda_1 \neq \lambda_2$$

$$e^{At} = B_0(t) I + B_1(t) A.$$
$$e^{\lambda_1 t} = B_0 + \lambda_1 B_1$$
$$e^{\lambda_2 t} = B_0 + \lambda_2 B_1$$

$$\ominus \quad e^t = B_0 + B_1$$
$$e^{5t} = B_0 + 5B_1$$
$$e^{5t} - e^t = 4B_1 \Rightarrow \boxed{B_1(t) = \frac{1}{4}e^{5t} - \frac{1}{4}e^t}$$
$$B_0 = e^t - B_1$$
$$B_0 = e^t - \left(\frac{1}{4}e^{5t} - \frac{1}{4}e^t\right) \rightarrow \boxed{B_0(t) = -\frac{1}{4}e^{5t} + \frac{5}{4}e^t}$$

$$e^{At} = \left(-\frac{1}{4}e^{5t} + \frac{5}{4}e^t\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \left(\frac{1}{4}e^{5t} - \frac{1}{4}e^t\right)\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$
$$e^{At} = \frac{1}{4}\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix}e^{5t} + \frac{1}{4}\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix}e^t$$

$$e^{At}\Big|_{t=0} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow e^{A(0)} = \frac{1}{4}\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\frac{d}{dt}e^{At} = Ae^{At}$$
$$\frac{d}{dt}e^{At} = \frac{1}{4}\begin{bmatrix} 5 & 15 \\ 5 & 15 \end{bmatrix}e^{5t} + \frac{1}{4}\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix}e^t$$
$$Ae^{At} = \frac{1}{4}\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix}e^{5t} + \frac{1}{4}\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix}e^t$$
$$= \frac{1}{4}\begin{bmatrix} 5 & 15 \\ 5 & 15 \end{bmatrix}e^{5t} + \frac{1}{4}\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix}e^t$$

$$e^{At} = \frac{1}{4}\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix} e^{5t} + \frac{1}{4}\begin{bmatrix} 3 & -3 \\ -1 & +1 \end{bmatrix} e^{t}$$

$$e^{At}\bar{y}(0) = \frac{1}{4}\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \end{bmatrix} e^{5t} + \frac{1}{4}\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \end{bmatrix} e^{t}$$

$$e^{At}\bar{y}(0) = \frac{1}{4}\begin{bmatrix} -3 \\ -3 \end{bmatrix} e^{5t} + \frac{1}{4}\begin{bmatrix} 15 \\ -5 \end{bmatrix} e^{t}$$

$$e^{At}\bar{y}(0) = \begin{bmatrix} -\frac{3}{4}e^{5t} + \frac{15}{4}e^{t} \\ -\frac{3}{4}e^{5t} - \frac{5}{4}e^{t} \end{bmatrix} \longrightarrow \begin{bmatrix} \frac{12}{4} \\ -\frac{8}{4} \end{bmatrix} \Rightarrow \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

$$\left(e^{At}\bar{y}(0)\right)_{t=0} = \bar{y}(0)$$

$$\int_0^t e^{A(t-\tau)} \bar{b}(\tau)\,d\tau \qquad \bar{b}(\tau) = \begin{bmatrix} 5e^{2\tau} \\ -6e^{-\tau} \end{bmatrix}$$

$$e^{A(t-\tau)} = \frac{1}{4}\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix} e^{5(t-\tau)} + \frac{1}{4}\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix} e^{(t-\tau)}$$

$$e^{A(t-\tau)} \bar{b}(\tau) = \frac{e^{5t}}{4}\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 5e^{2\tau} \\ -6e^{-\tau} \end{bmatrix} e^{-5\tau} +$$

$$+ \frac{e^{t}}{4}\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 5e^{2\tau} \\ -6e^{-\tau} \end{bmatrix} e^{-\tau}$$

$$e^{A(t-\tau)} \bar{b}(\tau) = \frac{e^{5t}}{4}\begin{bmatrix} 5e^{-3\tau} - 18e^{-6\tau} \\ 5e^{-3\tau} - 18e^{-6\tau} \end{bmatrix} + \frac{e^{t}}{4}\begin{bmatrix} 15e^{\tau} + 18e^{-2\tau} \\ -5e^{\tau} - 6e^{-2\tau} \end{bmatrix}$$

$$e^{A(t-\tau)} \bar{b}(\tau)\,d\tau = \frac{5e^{5t}}{4}\begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-3\tau} - \frac{9e^{5t}}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-6\tau} +$$

$$+ \frac{5e^{t}}{4}\begin{bmatrix} 3 \\ -1 \end{bmatrix} e^{\tau} + \frac{3e^{t}}{2}\begin{bmatrix} 3 \\ -1 \end{bmatrix} e^{-2\tau}$$

$$\int e^{A(t-\tau)} \bar{b}(\tau)\,d\tau = \frac{5}{4}e^{5t}\begin{bmatrix} 1 \\ 1 \end{bmatrix}\int e^{-3\tau}d\tau - \frac{9}{2}e^{5t}\begin{bmatrix} 1 \\ 1 \end{bmatrix}\int e^{-6\tau}d\tau +$$

$$+ \frac{5}{4}e^{t}\begin{bmatrix} 3 \\ -1 \end{bmatrix}\int e^{\tau}d\tau + \frac{3}{2}e^{t}\begin{bmatrix} 3 \\ -1 \end{bmatrix}\int e^{-2\tau}d\tau$$

$$= \frac{5}{4}e^{5t}\begin{bmatrix} 1 \\ 1 \end{bmatrix}\frac{e^{-3\tau}}{-3} - \frac{9}{2}e^{5t}\begin{bmatrix} 1 \\ 1 \end{bmatrix}\frac{e^{-6\tau}}{-6} +$$

$$+ \frac{5}{4}e^{t}\begin{bmatrix} 3 \\ -1 \end{bmatrix} e^{\tau} + \frac{3}{2}e^{t}\begin{bmatrix} 3 \\ -1 \end{bmatrix}\frac{e^{-2\tau}}{-2}$$

$$= -\frac{5}{12}e^{5t}\begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-3\tau} + \frac{9}{12}e^{5t}\begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-6\tau} +$$

$$+ \frac{5}{4}e^{t}\begin{bmatrix} 3 \\ -1 \end{bmatrix} e^{\tau} - \frac{3}{4}e^{t}\begin{bmatrix} 3 \\ -1 \end{bmatrix} e^{-2\tau}$$

$$\int_0^t e^{A(t-\tau)} \bar{b}(\tau)\,d\tau = -\frac{5}{12}e^{5t}\begin{bmatrix} 1 \\ 1 \end{bmatrix}(e^{-3t} - 1) + \frac{9}{12}e^{5t}\begin{bmatrix} 1 \\ 1 \end{bmatrix}(e^{-6t} - 1) +$$

$$+ \frac{5}{4}e^{t}\begin{bmatrix} 3 \\ -1 \end{bmatrix}(e^{t} - 1) - \frac{3}{4}e^{t}\begin{bmatrix} 3 \\ -1 \end{bmatrix}(e^{-2t} - 1)$$

$$= -\frac{5}{12}\begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} + \frac{5}{12}\begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{5t} +$$

$$+ \frac{9}{12}\begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-t} - \frac{9}{12}\begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{5t} +$$

$$+ \frac{5}{4}\begin{bmatrix} 3 \\ -1 \end{bmatrix} e^{2t} - \frac{5}{4}\begin{bmatrix} 3 \\ -1 \end{bmatrix} e^{t} -$$

$$- \frac{3}{4}\begin{bmatrix} 3 \\ -1 \end{bmatrix} e^{-t} + \frac{3}{4}\begin{bmatrix} 3 \\ -1 \end{bmatrix} e^{t}$$

$$= \begin{bmatrix} -\frac{5}{12} + \frac{15}{4} \\ -\frac{5}{12} - \frac{5}{4} \end{bmatrix} e^{2t} + \begin{bmatrix} \frac{9}{12} - \frac{9}{4} \\ \frac{9}{12} + \frac{3}{4} \end{bmatrix} e^{-t} +$$

$$+ \begin{bmatrix} \frac{5}{12} - \frac{9}{12} \\ \frac{5}{12} - \frac{9}{12} \end{bmatrix} e^{5t} + \begin{bmatrix} -\frac{15}{4} + \frac{9}{4} \\ +\frac{5}{4} - \frac{3}{4} \end{bmatrix} e^{t}$$

$$= \begin{bmatrix} \frac{10}{3} \\ -\frac{5}{3} \end{bmatrix} e^{2t} + \begin{bmatrix} -\frac{3}{2} \\ \frac{3}{2} \end{bmatrix} e^{-t} +$$

$$+ \begin{bmatrix} -\frac{1}{3} \\ -\frac{1}{3} \end{bmatrix} e^{5t} + \begin{bmatrix} -\frac{3}{2} \\ \frac{1}{2} \end{bmatrix} e^{t}$$

$$= \begin{bmatrix} \frac{10}{3} - \frac{3}{2} - \frac{1}{3} - \frac{3}{2} \\ -\frac{5}{3} + \frac{3}{2} - \frac{1}{3} + \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$y_1(t) = -\frac{3}{4}e^{5t} + \frac{15}{4}e^{t} - \frac{1}{3}e^{5t} - \frac{3}{2}e^{t} + \frac{10}{3}e^{2t} - \frac{3}{2}e^{-t}$$

$$= -\left(\frac{3}{4} + \frac{1}{3}\right)e^{5t} + \left(\frac{15}{4} - \frac{3}{2}\right)e^{t} + \frac{10}{3}e^{2t} - \frac{3}{2}e^{-t}$$

$$y_1(t) = -\frac{13}{12}e^{5t} + \frac{9}{4}e^{t} + \frac{10}{3}e^{2t} - \frac{3}{2}e^{-t}$$

$$y_1(0) = -\frac{13}{12} + \frac{9}{4} + \frac{10}{3} - \frac{3}{2} \Rightarrow \frac{-13+27+40-18}{12} \Rightarrow$$

$$\Rightarrow \frac{-31+67}{12} \Rightarrow$$