TEORÍA: EDOL (2) H.

$$\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = 0 \qquad H: \; y_p = e^{mx}$$

$$m^2 + a_1 m + a_2 = 0$$ ECUACION CARACTERÍSTICA

Método del Operador Diferencial.

$$\frac{dy}{dx} \Rightarrow Dy \qquad D(Dy) \Rightarrow D^2y \Leftarrow \frac{d^2y}{dx^2}$$

$$D(D^n y) \Rightarrow D^{n+1}y.$$

D^{-1} derivada inversa

$$\boxed{D(D^{-1}y) = y}$$

$$(D + a_1)y \Rightarrow Dy + a_1 y$$

$$D[a_1 f_1 + a_2 f_2] \Rightarrow a_1 Df_1 + a_2 Df_2$$

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0 \qquad H: y_p = e^{mx}$$

$$m^2 - 5m + 6 = 0$$

$$(m-2)(m-3) = 0 \qquad m_1 = 2 \quad m_2 = 3$$

$$y_1 = e^{2x} \qquad y_2 = e^{3x}$$

$$y_g = c_1 e^{2x} + c_2 e^{3x}$$

$$D^2 y - 5Dy + 6y = 0$$

$$(D^2 - 5D + 6)y = 0$$

$$(D-2)(D-3)y = 0$$

$$(D-2)(D-3)\left[c_1 e^{2x} + c_2 e^{3x}\right] = 0$$

$$(D-2)\left[2c_1 e^{2x} + 3\cancel{c_2} e^{3x} - 3c_1 e^{2x} - 3\cancel{c_2} e^{3x}\right] = 0$$

$$(D-2)\left[-c_1 e^{2x}\right] = 0$$

$$-2\cancel{c_1} e^{2x} + 2\cancel{c_1} e^{2x} = 0$$

$$0 \equiv 0$$

$$\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = 0$$

$$(D^2 + a_1 D + a_2)y = 0 \qquad m^2 + a_1 m + a_2 = 0$$

$$(D - m_1)(D - m_2)y = 0 \qquad m_1 \quad m_2$$

$$y_g = c_1 e^{m_1 x} + c_2 e^{m_2 x}. \qquad e^{m_1 x} \quad e^{m_2 x}$$

$$y = C_1 e^{m_1 x} + C_2 e^{m_2 x}$$

$$|W| \neq 0 \quad \begin{vmatrix} e^{m_1 x} & e^{m_2 x} \\ m_1 e^{m_1 x} & m_2 e^{m_2 x} \end{vmatrix} \neq 0$$

$$m_2 e^{m_1 x} e^{m_2 x} - m_1 e^{m_1 x} e^{m_2 x} \neq 0$$

$$(m_2 - m_1) e^{m_1 x} e^{m_2 x} \neq 0$$

$$m_2 \neq m_1$$

$$m^2 + a_1 m + a_2 = 0 \begin{cases} m_1, m_2 \in \mathbb{R} & m_1 \neq m_2 \quad \text{I} \\ m_1, m_2 \in \mathbb{R} & m_1 = m_2 \quad \text{II} \\ m_1, m_2 \in \mathbb{C} & m_1 \neq m_2 \quad \text{III} \end{cases}$$

$$m_{1,2} = a \pm bi$$

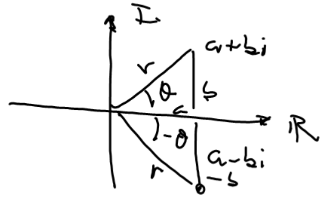
$$y(x) \quad x \in \mathbb{R} \quad y = C_1 e^{(a+bi)x} + C_2 e^{(a-bi)x}$$

$$y \in \mathbb{R}$$

¿Qué pasa cuando m_1 m_2 son un par complejo?

Euler: $e^{\pi i} = -1$

$$e^{\theta i} = \cos(\theta) + i\ \text{sen}(\theta)$$
$$e^{-\theta i} = \cos(\theta) - i\ \text{sen}(\theta).$$



$$b_1 = r\,\text{sen}\,\theta$$
$$a_1 = r\cos\theta$$
$$re^{\theta} = r\cos\theta \qquad e^{\theta} = \cos\theta$$
$$re^{\theta i} = ir\,\text{sen}\,\theta \qquad e^{\theta i} = (\text{sen}\,\theta)\,i$$

$$e^{(a+bi)x} = e^{ax}e^{(bx)i} = e^{ax}\big(\cos(bx) + i\ \text{sen}(bx)\big)$$
$$e^{(a-bi)x} = e^{ax}e^{(-bx)i} = e^{ax}\big(\cos(bx) - i\ \text{sen}(bx)\big)$$
$$y_g = c_1 e^{(a+bi)x} + c_2 e^{(a-bi)x}$$
$$y_g = c_1\ e^{ax}\big(\cos(bx) + i\ \text{sen}(bx)\big) + c_2 e^{ax}\big(\cos(bx) - i\ \text{sen}(bx)\big)$$
$$y_g = (c_1 + c_2)e^{ax}\cos(bx) + (c_1 i - c_2 i)e^{ax}\,\text{sen}(bx) \qquad x \in \mathbb{R},\ y \in \mathbb{R}$$

$$\boxed{y_g = C_{10}\,e^{ax}\cos(bx) + C_{20}\,e^{ax}\,\text{sen}(bx).}$$

$m_1 = a + bi$

$m_2 = a - bi$

CASO II .- Las raíces de la EC. serán reales e iguales.

$\Rightarrow \quad m^2 + a_1 m + a_2 = 0 \qquad m_1 = m_2 \quad y = e^{m_1 x}$

$$y_g = c_1 e^{m_1 x} + c_2 (\quad)$$

Si las raíces de una ecuación de segundo grado son reales y distintas nunca van a satisfacen la primera derivada de la e

$m^2 + a_1 m + a_2 = 0 \qquad m_1 \neq m_2$

$\frac{d}{dx}$

$(m - m_1)(m - m_2) = 0$

$2m + a_1 = 0$

$\frac{d}{dx}$

$(m - m_1) + (m - m_2) = 0$

Si son iguales sí satisfacen derivada.

$m^2 + a_1 m + a_2 = 0 \qquad m_1 = m_2$

$\frac{d}{dm}$

$(m - m_1)^2 = 0$

$2m + a_1 = 0$

$\frac{d}{dm}$

$2(m - m_1) = 0$

$$D^2 y + a_1 D y + a_2 y = 0$$

$m^2 + a_1 m + a_2 = 0 \qquad m_1 = m_2$

$\frac{d}{dm}$

$e^{mx} \xrightarrow{m = m_1} e^{m_1 x}$

$x e^{mx} \xrightarrow{m = m_1} x e^{m_1 x}$

$$\boxed{y_g = c_1 e^{m_1 x} + c_2 x e^{m_1 x}}$$

$m_{1,2} = 2 \pm 3i \qquad m_{3,4} = 2 \pm 3i \qquad m_1 = m_3 = 2 + 3i$

$m_2 = m_4 = 2 - 3i$

$$y_g = c_1 e^{2x} \cos(3x) + c_2 e^{2x} \operatorname{sen}(3x) + c_3 x e^{2x} \cos(3x) + c_4 x e^{2x} \operatorname{sen}(3x)$$