

> restart :

3) DADO EL SIGUIENTE PROBLEMA DE CONDICIONES INICIALES & UTILIZANDO EXCLUSIVAMENTE EL MÉTODO DE VARIACIÓN DE PARÁMETROS (sin utilizar dsolve)

a) OBTENER SU SOLUCIÓN PARTICULAR

b) GRAFICAR EL RESULTADO DEL INCISO a) EN UN INTERVALO  $0 < t < 1$

> restart

> EDO :=  $\frac{d^4}{dt^4} y(t) + 5 \left( \frac{d^2}{dt^2} y(t) \right) - 4 y(t) = 5 e^{-3t} \cos(2t)$

$$EDO := \frac{d^4}{dt^4} y(t) + 5 \left( \frac{d^2}{dt^2} y(t) \right) - 4 y(t) = 5 e^{-3t} \cos(2t) \quad (1)$$

> ConUno :=  $y(0) = -2$ ; ConDos :=  $D(y)(0) = 0$ ; ConTres :=  $D^{(2)}(y)(0) = 7$ ; ConCuatro :=  $D^{(3)}(y)(0) = -5$

$$ConUno := y(0) = -2$$

$$ConDos := D(y)(0) = 0$$

$$ConTres := D^{(2)}(y)(0) = 7$$

$$ConCuatro := D^{(3)}(y)(0) = -5 \quad (2)$$

> EDOH :=  $lhs(EDO) = 0$

$$EDOH := \frac{d^4}{dt^4} y(t) + 5 \left( \frac{d^2}{dt^2} y(t) \right) - 4 y(t) = 0 \quad (3)$$

> EcuCarac :=  $m \cdot 4 + 5 \cdot m \cdot 2 - 4 = 0$

$$EcuCarac := m^4 + 5 m^2 - 4 = 0 \quad (4)$$

> Raiz :=  $solve(EcuCarac)$

$$Raiz := \frac{1}{2} I \sqrt{2 \sqrt{41} + 10}, -\frac{1}{2} I \sqrt{2 \sqrt{41} + 10}, \frac{1}{2} \sqrt{-10 + 2 \sqrt{41}}, -\frac{1}{2} \sqrt{-10 + 2 \sqrt{41}} \quad (5)$$

> evalf(%o, 3)

$$2.38 I, -2.38 I, 0.835, -0.835 \quad (6)$$

> PartUno :=  $y(t) = \exp(Raiz[3] \cdot t) : evalf(\%, 3)$ ; PartDos :=  $y(t) = \exp(Raiz[4] \cdot t) : evalf(\%, 3)$ ; PartTres :=  $y(t) = \cos(\text{Im}(Raiz[1]) \cdot t) : evalf(\%, 3)$ ; PartCuatro :=  $y(t) = \sin(\text{Im}(Raiz[1]) \cdot t) : evalf(\%, 3)$

$$y(t) = e^{0.835 t}$$

$$y(t) = e^{-0.835 t}$$

$$y(t) = \cos(2.38 t)$$

$$y(t) = \sin(2.38 t) \quad (7)$$

> SGH :=  $y(t) = C1 \cdot rhs(PartUno) + C2 \cdot rhs(PartDos) + C3 \cdot rhs(PartTres) + C4 \cdot rhs(PartCuatro) : evalf(\%, 3)$

$$y(t) = C1 e^{0.835 t} + C2 e^{-0.835 t} + C3 \cos(2.38 t) + C4 \sin(2.38 t) \quad (8)$$

> SGNH :=  $y(t) = A(t) \cdot rhs(PartUno) + B(t) \cdot rhs(PartDos) + D(t) \cdot rhs(PartTres) + E(t) \cdot rhs(PartCuatro) : evalf(\%, 2)$

$$y(t) = A(t) e^{0.85 t} + B(t) e^{-0.85 t} + D(t) \cos(2.4 t) + E(t) \sin(2.4 t) \quad (9)$$

$$\begin{aligned} &> \text{with(linalg)} : \\ &> \text{PNH} := Q(t) = \text{rhs}(\text{EDO}) \\ &\quad \text{PNH} := Q(t) = 5 e^{-3t} \cos(2t) \end{aligned} \quad (10)$$

$$\begin{aligned} &> \text{MM} := \text{wronskian}([ \text{rhs}(\text{PartUno}), \text{rhs}(\text{PartDos}), \text{rhs}(\text{PartTres}), \text{rhs}(\text{PartCuatro}) ], t) : \\ &\quad \text{evalf}(\text{MM}[3, 4], 3) \\ &\quad -5.70 \sin(2.38 t) \end{aligned} \quad (11)$$

$$\begin{aligned} &> \text{BB} := \text{array}([0, 0, 0, \text{rhs}(\text{PNH})]) \\ &\quad \text{BB} := \begin{bmatrix} 0 & 0 & 0 & 5 e^{-3t} \cos(2t) \end{bmatrix} \end{aligned} \quad (12)$$

$$\begin{aligned} &> \text{ParDer} := \text{linsolve}(\text{MM}, \text{BB}) : \text{evalf}(\text{ParDer}[1]); \\ &\quad \frac{0.4661385439 e^{-3 \cdot t} \cos(2 \cdot t)}{e^{0.8375930500 t}} \end{aligned} \quad (13)$$

$$\begin{aligned} &> \text{Aprima} := \text{ParDer}[1] : \text{evalf}(\%, 3); \text{Bprima} := \text{ParDer}[2] : \text{evalf}(\%, 3); \text{Dprima} \\ &\quad := \text{ParDer}[3] : \text{evalf}(\%, 3); \text{Eprima} := \text{ParDer}[4] : \text{evalf}(\%, 3); \\ &\quad \frac{0.467 e^{-3 \cdot t} \cos(2 \cdot t)}{e^{0.835 t}} \\ &\quad - \frac{0.467 e^{-3 \cdot t} \cos(2 \cdot t)}{e^{-0.835 t}} \\ &\quad \frac{0.327 \sin(2.38 t) e^{-3 \cdot t} \cos(2 \cdot t)}{\cos(2.38 t)^2 + \sin(2.38 t)^2} \\ &\quad - \frac{0.327 \cos(2.38 t) e^{-3 \cdot t} \cos(2 \cdot t)}{\cos(2.38 t)^2 + \sin(2.38 t)^2} \end{aligned} \quad (14)$$

$$\begin{aligned} &> \text{PVuno} := A(t) = \text{simplify}(\text{int}(\text{Aprima}, t) + C1) : \\ &> \text{PVdos} := B(t) = \text{simplify}(\text{int}(\text{Bprima}, t) + C2) : \\ &> \text{PVtres} := D(t) = \text{simplify}(\text{int}(\text{Dprima}, t) + C3) : \\ &> \text{PVcuatro} := E(t) = \text{simplify}(\text{int}(\text{Eprima}, t) + C4) : \\ &> \text{SolucionGeneral} := \text{simplify}(\text{subs}(A(t) = \text{rhs}(\text{PVuno}), B(t) = \text{rhs}(\text{PVdos}), D(t) = \text{rhs}(\text{PVtres}), \\ &\quad E(t) = \text{rhs}(\text{PVcuatro}), \text{SGNH})) : \text{evalf}(\%, 3) \\ &\quad y(t) = -0.00630 e^{-3 \cdot t} \sin(0.385 t) \sin(2.38 t) - 0.0260 e^{-3 \cdot t} \cos(4.38 t) \cos(2.38 t) \end{aligned} \quad (15)$$

$$\begin{aligned} &\quad + 0.0180 e^{-3 \cdot t} \cos(4.38 t) \sin(2.38 t) - 0.0178 e^{-3 \cdot t} \sin(4.38 t) \cos(2.38 t) \\ &\quad - 0.0260 e^{-3 \cdot t} \sin(4.38 t) \sin(2.38 t) - 0.00644 e^{-3 \cdot t} \cos(0.385 t) \cos(2.38 t) \\ &\quad + 0.0536 e^{-3 \cdot t} \cos(0.385 t) \sin(2.38 t) - 0.0541 e^{-3 \cdot t} \sin(0.385 t) \cos(2.38 t) \\ &\quad - 0.0578 e^{-3 \cdot t} \sin(2 \cdot t) + 1.00 C2 e^{-0.835 t} + 1.00 C3 \cos(2.38 t) + 1.00 C4 \sin(2.38 t) \\ &\quad + 0.0206 e^{-3 \cdot t} \cos(2 \cdot t) + 1.00 C1 e^{0.835 t} \end{aligned}$$

$$\begin{aligned} &> \text{PARAuno} := \text{eval}(\text{subs}(t=0, \text{rhs}(\text{SolucionGeneral}) = \text{rhs}(\text{ConUno}))) : \text{evalf}(\%, 3) \\ &\quad -0.0116 + 1.00 C1 + 1.00 C3 + 1.00 C2 = -2. \end{aligned} \quad (16)$$

$$\begin{aligned} &> \text{PARAdos} := \text{eval}(\text{subs}(t=0, \text{rhs}(\text{diff}(\text{SolucionGeneral}, t)) = \text{rhs}(\text{ConDos}))) : \text{evalf}(\%, 3) \\ &\quad -0.0116 - 0.836 C2 + 0.836 C1 + 2.39 C4 = 0. \end{aligned} \quad (17)$$

$$\begin{aligned} &> \text{PARAtres} := \text{eval}(\text{subs}(t=0, \text{rhs}(\text{diff}(\text{SolucionGeneral}, t\$2)) = \text{rhs}(\text{ConTres}))) : \text{evalf}(\%, 3) \\ &\quad 0.152 - 5.73 C3 + 0.706 C2 + 0.706 C1 = 7. \end{aligned} \quad (18)$$

$$\begin{aligned} &> \text{PARAcuatro} := \text{eval}(\text{subs}(t=0, \text{rhs}(\text{diff}(\text{SolucionGeneral}, t\$3)) = \text{rhs}(\text{ConCuatro}))) : \\ &\quad \text{evalf}(\%, 3) \end{aligned}$$

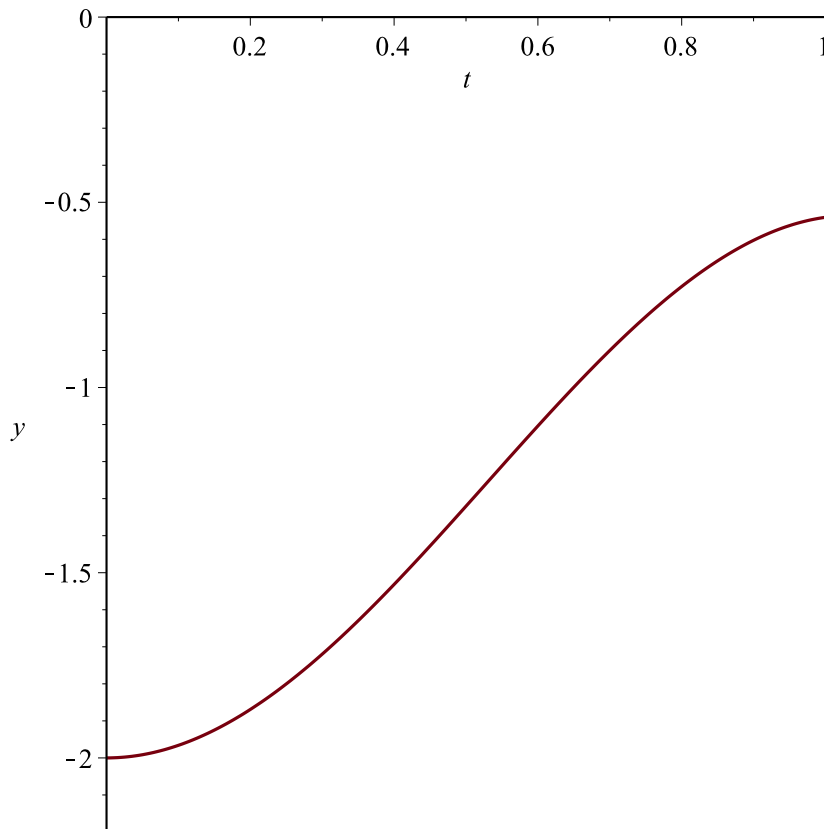
$$-1.16 - 13.6 C4 - 0.581 C2 + 0.581 C1 = -5. \quad (19)$$

> *PARA* := solve( {*PARA*uno, *PARA*dos, *PARA*tres, *PARA*cuatro}, {*C1*, *C2*, *C3*, *C4*}):evalf(%, 3)  
 $\{C1 = -0.721, C2 = 0.00777, C3 = -1.28, C4 = 0.258\}$  (20)

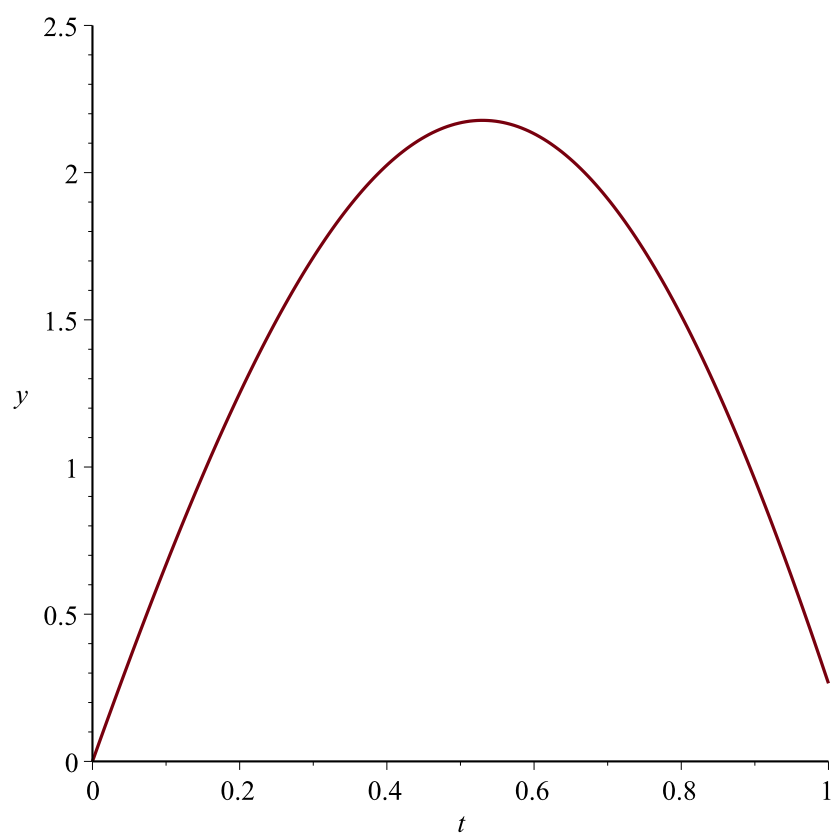
> *SolucionParticular* := subs(*C1* = rhs(*PARA*[1]), *C2* = rhs(*PARA*[2]), *C3* = rhs(*PARA*[3]), *C4* = rhs(*PARA*[4]), *SolucionGeneral*):evalf(%, 3)

$$\begin{aligned} y(t) = & -0.725 e^{0.835 t} + 0.00772 e^{-0.835 t} - 0.00630 e^{-3 \cdot t} \sin(0.385 t) \sin(2.38 t) \\ & - 0.0260 e^{-3 \cdot t} \cos(4.38 t) \cos(2.38 t) + 0.0178 e^{-3 \cdot t} \cos(4.38 t) \sin(2.38 t) \\ & - 0.0175 e^{-3 \cdot t} \sin(4.38 t) \cos(2.38 t) - 0.0260 e^{-3 \cdot t} \sin(4.38 t) \sin(2.38 t) \\ & - 0.00644 e^{-3 \cdot t} \cos(0.385 t) \cos(2.38 t) + 0.0540 e^{-3 \cdot t} \cos(0.385 t) \sin(2.38 t) \\ & - 0.0541 e^{-3 \cdot t} \sin(0.385 t) \cos(2.38 t) - 1.28 \cos(2.38 t) + 0.258 \sin(2.38 t) \\ & - 0.0578 e^{-3 \cdot t} \sin(2 \cdot t) + 0.0206 e^{-3 \cdot t} \cos(2 \cdot t) \end{aligned} \quad (21)$$

> plot(rhs(*SolucionParticular*), t = 0 .. 1, y = -2.2 .. 0)

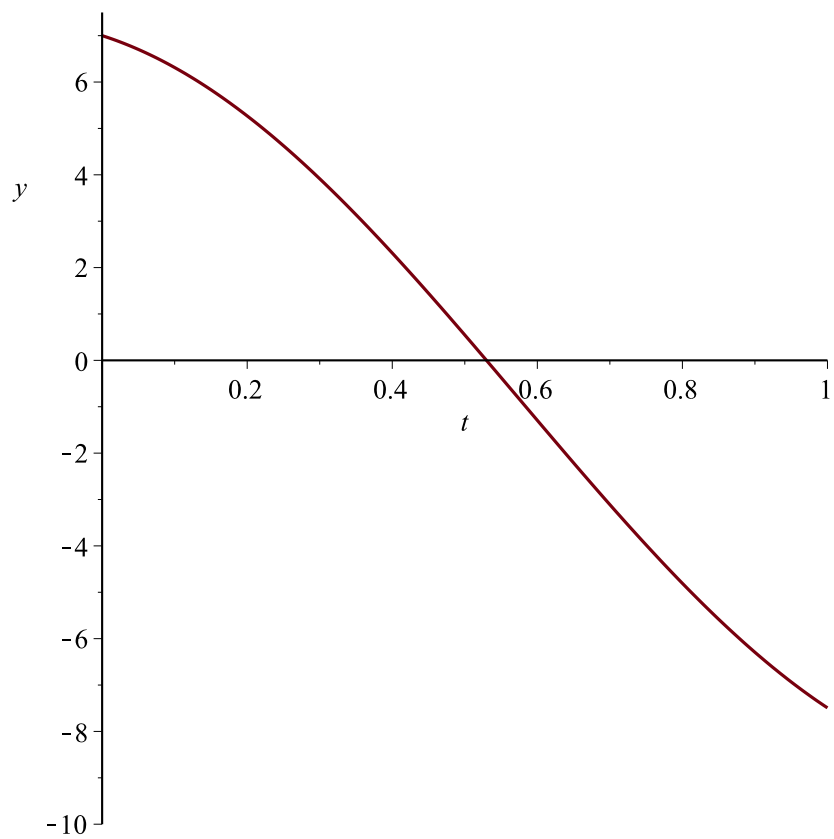


> plot(rhs(diff(*SolucionParticular*, t)), t = 0 .. 1, y = 0 .. 2.5)



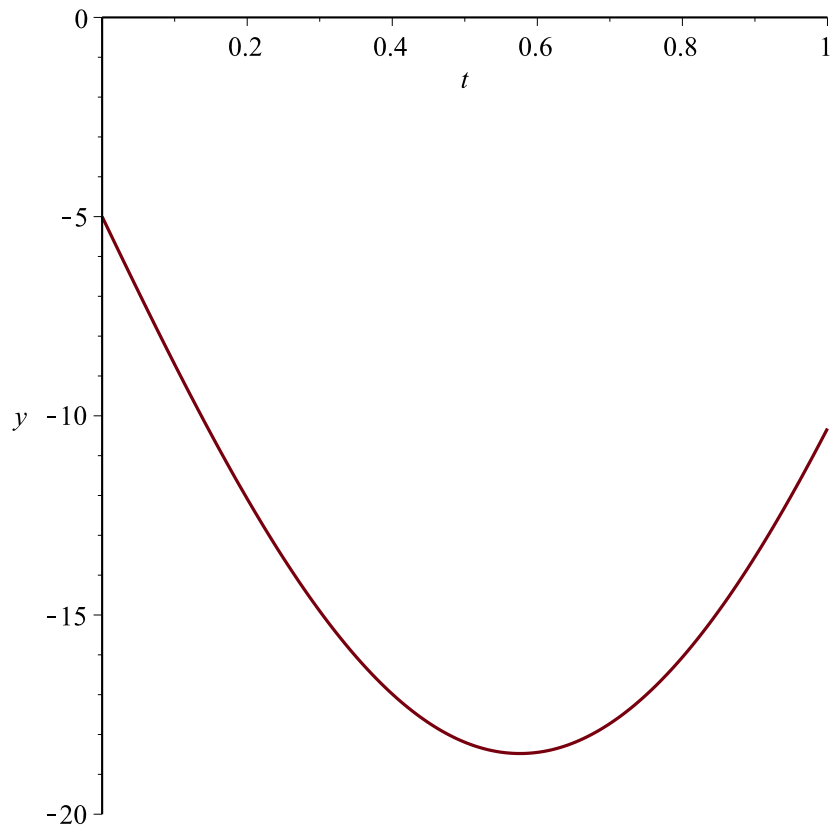

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> `plot(rhs(diff(SolucionParticular,t$2)),t=0..1,y=-10..7.5)`




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**>** `plot(rhs(diff(SolucionParticular,t$3)),t=0..1,y=-20..0)`



**> restart**

3) OBTENGA Y GRAFIQUE { EN EL INTERVALO - 1..1 } LA SOLUCIÓN PARTICULAR DE LOS SIGUIENTES PROBLEMAS:

a) CON CONDICIONES EN LA FRONTERA

$$\frac{d^3}{dx^3} y(x) + \frac{d^2}{dx^2} y(x) + \frac{d}{dx} y(x) + y(x) = 0$$

$$y(0) = 0$$

$$y\left(\frac{1}{2} \pi\right) = 10$$

$$y\left(\frac{3}{2} \pi\right) = 5$$

**(22)**

**> restart**

**> EDO :=**  $\frac{d^3}{dx^3} y(x) + \frac{d^2}{dx^2} y(x) + \frac{d}{dx} y(x) + y(x) = 0$

$$EDO := \frac{d^3}{dx^3} y(x) + \frac{d^2}{dx^2} y(x) + \frac{d}{dx} y(x) + y(x) = 0$$

**(23)**

$$\begin{aligned} &> \text{Cond} := y(0) = 0, y\left(\frac{1}{2} \pi\right) = 10, y\left(\frac{3}{2} \pi\right) = 5 \\ &\quad \text{Cond} := y(0) = 0, y\left(\frac{1}{2} \pi\right) = 10, y\left(\frac{3}{2} \pi\right) = 5 \end{aligned} \quad (24)$$

$$\begin{aligned} &> \text{EcuaCarac} := m \cdot 3 + m \cdot 2 + m + 1 = 0 \\ &\quad \text{EcuaCarac} := m^3 + m^2 + m + 1 = 0 \end{aligned} \quad (25)$$

$$\begin{aligned} &> \text{Raiz} := \text{solve}(\text{EcuaCarac}) \\ &\quad \text{Raiz} := -1, I, -I \end{aligned} \quad (26)$$

$$\begin{aligned} &> \text{SolUno} := y(x) = \exp(\text{Raiz}[1] \cdot x) \\ &\quad \text{SolUno} := y(x) = e^{-x} \end{aligned} \quad (27)$$

$$\begin{aligned} &> \text{SolDos} := y(x) = \cos(\text{Im}(\text{Raiz}[2]) \cdot x) \\ &\quad \text{SolDos} := y(x) = \cos(x) \end{aligned} \quad (28)$$

$$\begin{aligned} &> \text{SolTres} := y(x) = \sin(\text{Im}(\text{Raiz}[2]) \cdot x) \\ &\quad \text{SolTres} := y(x) = \sin(x) \end{aligned} \quad (29)$$

$$\begin{aligned} &> \text{SolucionGeneral} := y(x) = C1 \cdot \text{rhs}(\text{SolUno}) + C2 \cdot \text{rhs}(\text{SolDos}) + C3 \cdot \text{rhs}(\text{SolTres}) \\ &\quad \text{SolucionGeneral} := y(x) = C1 e^{-x} + C2 \cos(x) + C3 \sin(x) \end{aligned} \quad (30)$$

$$\begin{aligned} &> \text{PARAuno} := \text{eval}(\text{subs}(x=0, \text{rhs}(\text{SolucionGeneral}) = \text{rhs}(\text{Cond}[1]))) \\ &\quad \text{PARAuno} := C1 + C2 = 0 \end{aligned} \quad (31)$$

$$\begin{aligned} &> \text{PARAdos} := \text{eval}\left(\text{subs}\left(x = \frac{\text{Pi}}{2}, \text{rhs}(\text{SolucionGeneral}) = \text{rhs}(\text{Cond}[2])\right)\right) \\ &\quad \text{PARAdos} := C1 e^{-\frac{1}{2} \pi} + C3 = 10 \end{aligned} \quad (32)$$

$$\begin{aligned} &> \text{PARAtres} := \text{eval}\left(\text{subs}\left(x = \frac{3 \cdot \text{Pi}}{2}, \text{rhs}(\text{SolucionGeneral}) = \text{rhs}(\text{Cond}[3])\right)\right) \\ &\quad \text{PARAtres} := C1 e^{-\frac{3}{2} \pi} - C3 = 5 \end{aligned} \quad (33)$$

$$\begin{aligned} &> \text{PARA} := \text{solve}(\{\text{PARAuno}, \text{PARAdos}, \text{PARAtres}\}, \{C1, C2, C3\}) : \text{evalf}(\%, 2) \\ &\quad \{C1 = 72., C2 = -72., C3 = -4.3\} \end{aligned} \quad (34)$$

$$\begin{aligned} &> \text{SolucionParticular} := \text{simplify}(\text{subs}(C1 = \text{rhs}(\text{PARA}[1]), C2 = \text{rhs}(\text{PARA}[2]), C3 \\ &\quad = \text{rhs}(\text{PARA}[3]), \text{SolucionGeneral})) \\ &\quad \text{SolucionParticular} := y(x) = \frac{5 \left( 3 e^{-x} - 3 \cos(x) + 2 \sin(x) e^{-\frac{3}{2} \pi} - \sin(x) e^{-\frac{1}{2} \pi} \right) e^{\frac{1}{2} \pi}}{1 + e^{-\pi}} \end{aligned} \quad (35)$$

$$> \text{plot}\left(\text{rhs}(\text{SolucionParticular}), x=0 .. \frac{3 \cdot \text{Pi}}{2}\right)$$

