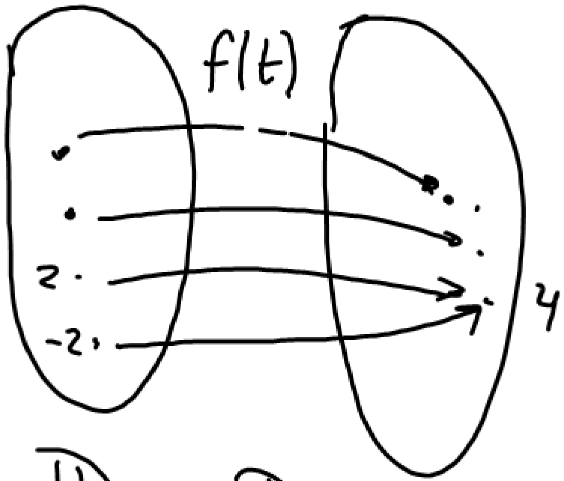
Capítulo 3.

- Transformada de Laplace
- Sistemas de EDO's.

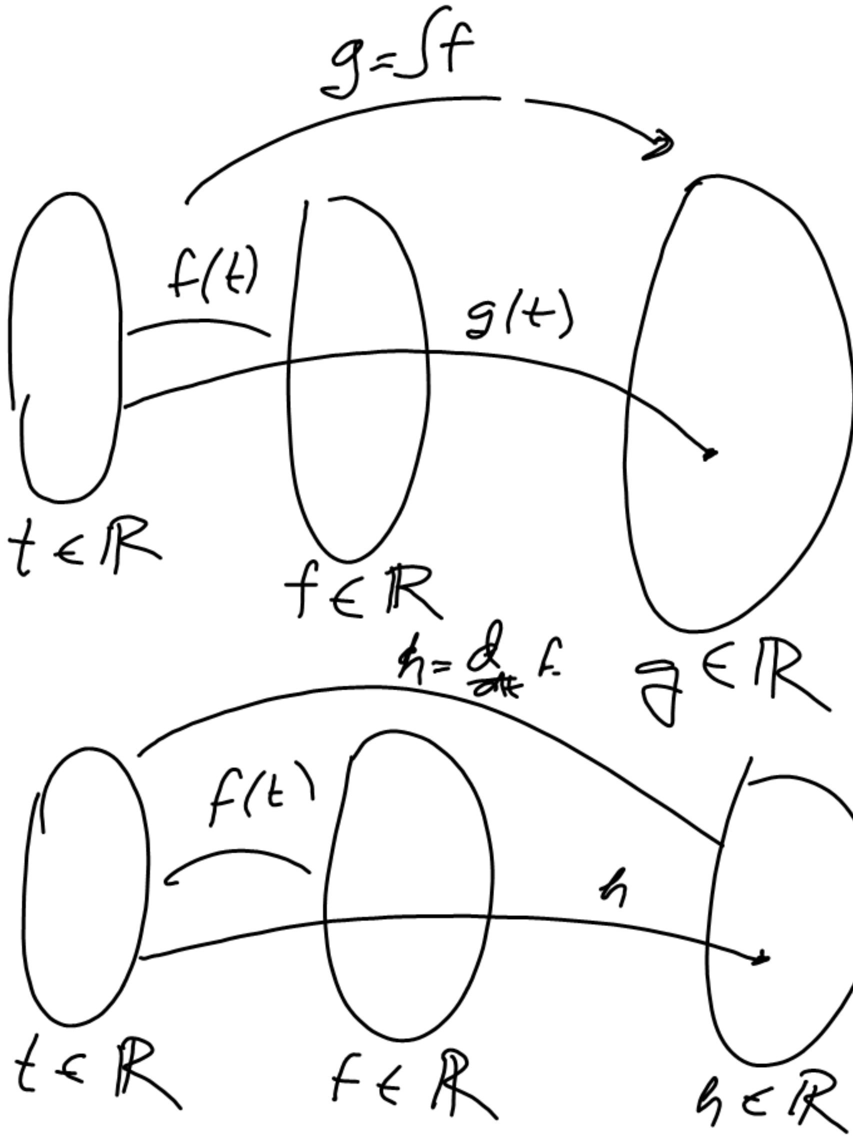
T. L.



$f = t^2$ ✓

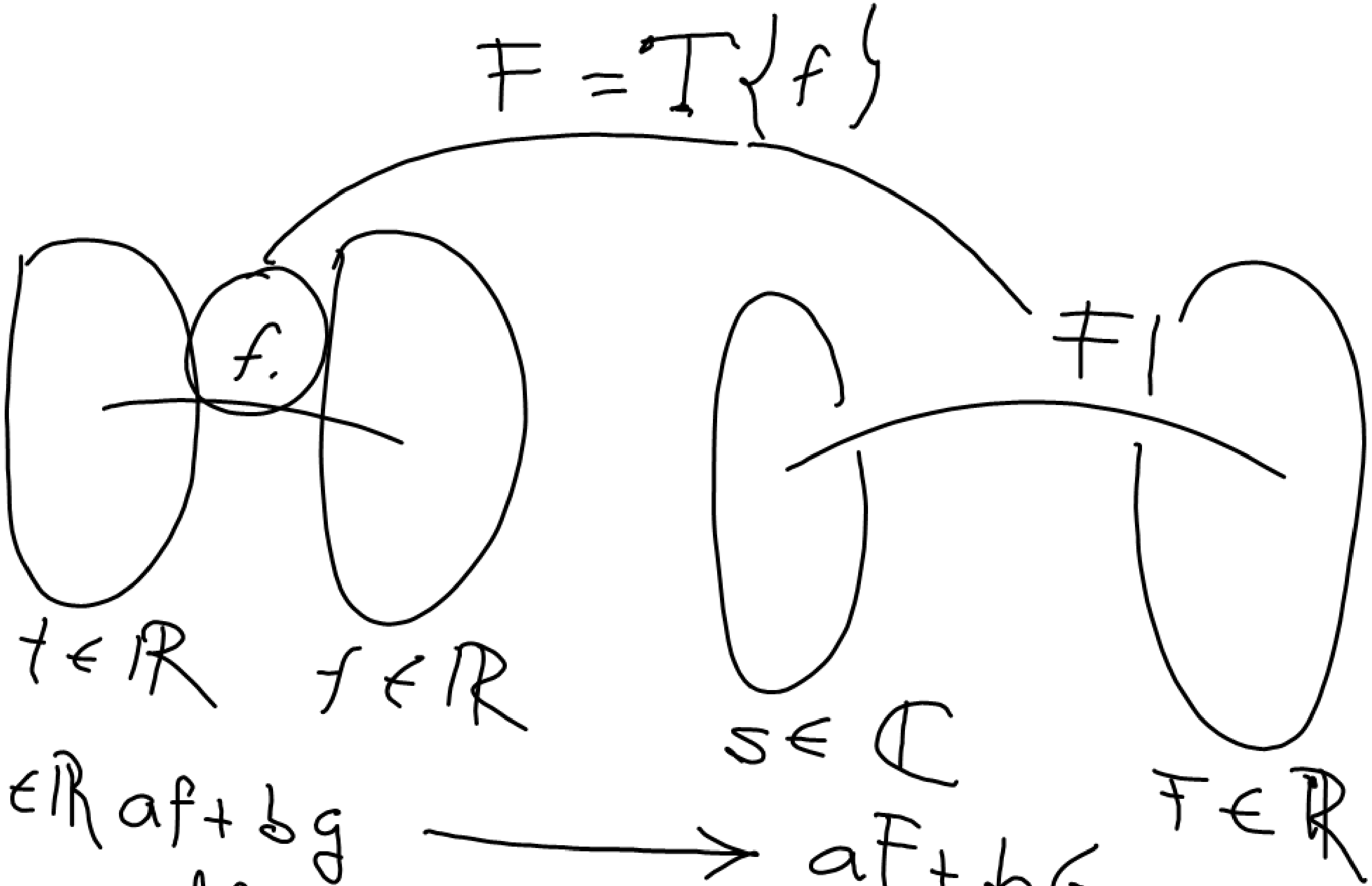
$f = \pm\sqrt{t}$ ✗

$t \in D \in \mathbb{R}$ $f \in C \in \mathbb{R}$



$$\int t\,dt = \frac{t^2}{2}$$

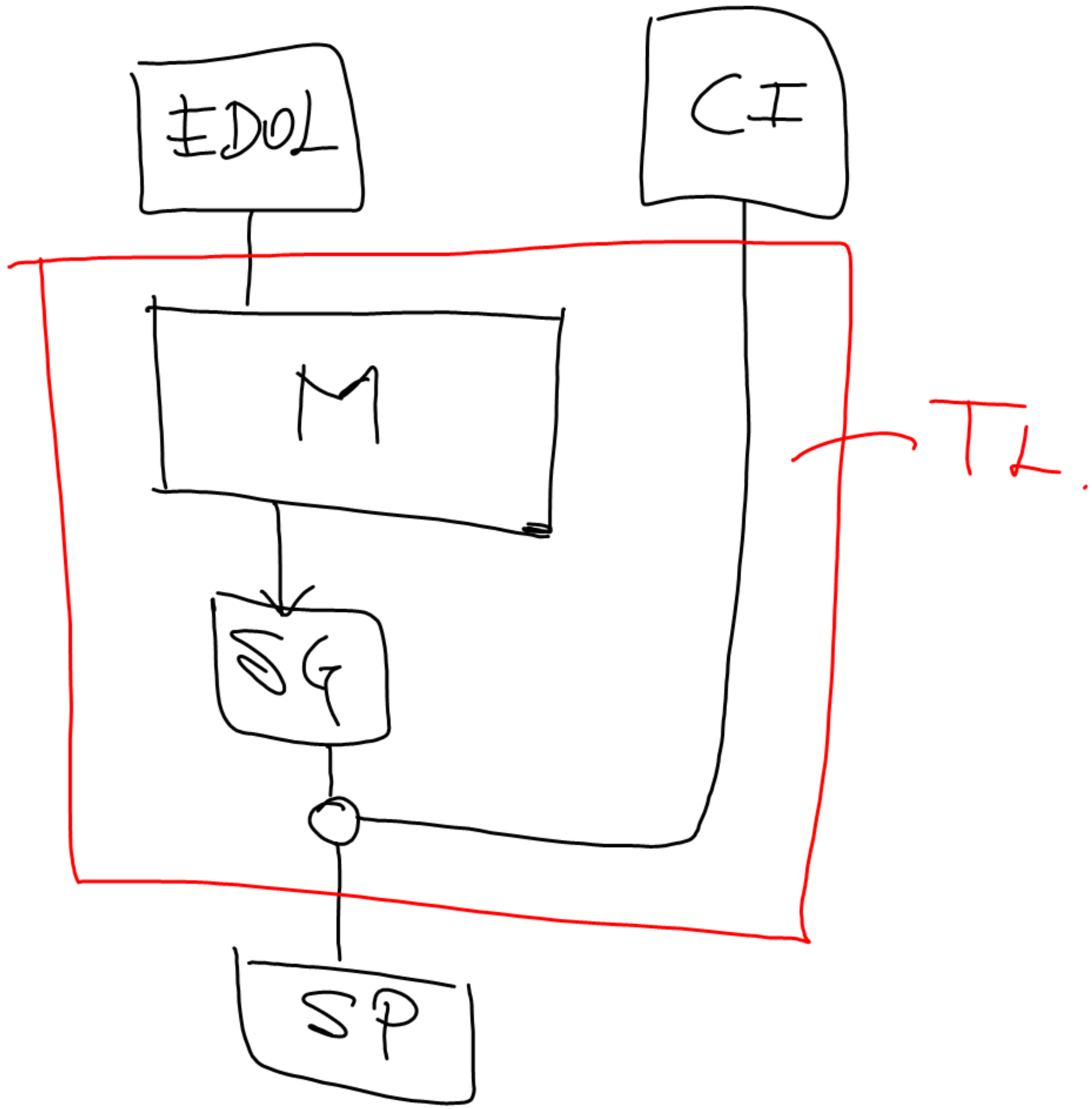
$$\frac{d}{dt} t^3 = 3t^2$$



$a, b \in \mathbb{R}$ $\quad af + bg \longrightarrow aF + bG$

$$\frac{df}{dt} \longrightarrow sF$$

$$\int f\,dt \longleftarrow \frac{F}{s}$$


EDOL
CI
M
SG
SP
TL.

$$T\{f(t)\} = F(s) \quad s \in \mathbb{R}.$$ $\left\{\begin{array}{l}\text{existe} \\ \text{y es única } F(s)\end{array}\right.$

$$T^{-1}\{F(s)\} = f(t)$$ existe ? y no es única

$$T\{f(t)\} = F(s)$$

$$T\{f(t)\} = \int_{-\infty}^{\infty} N(s,t) f(t)\, dt.$$

Laplace

$$N(s,t) = \begin{cases} 0 \ ; \ t < 0 \\ e^{-st} \ ; \ t \geq 0 \end{cases}$$

$$\boxed{L\{f(t)\} = \int_0^{\infty} e^{-st} f(t)\, dt}$$

$$f(t) = 1$$

$$\mathcal{L}\{1\} = \int_0^{\infty} e^{-st} \cdot (1) \cdot dt$$

$$= \left[\int e^{-st} dt \right]_0^{\infty}$$

$$= \left[\frac{e^{-st}}{-s} \right]_0^{\infty}$$

$$= \left(\frac{e^{-s(\infty)}}{-s} - \frac{e^{-s(0)}}{-s} \right)$$

$$\mathcal{L}\{1\} = \left(\frac{e^{-\infty s}}{-s} + \frac{1}{s} \right) \Rightarrow \frac{1}{s}$$

$$\lim_{b \to \infty} e^{-sb} = \lim_{b \to \infty} \frac{1}{e^{sb}} \Rightarrow \lim_{k \to \infty} \frac{1}{k} \Rightarrow 0$$

$$\lim_{b \to \infty} e^{sb} \to \infty$$

$$\mathcal{L}\{5\} = \int_0^\infty e^{-st}(5)\,dt = 5\int_0^\infty e^{-st}\,dt$$

$$= 5\left(\frac{1}{s}\right)$$

$$\mathcal{L}\{5\} = \frac{5}{s}$$

$$\mathcal{L}\{5\} = 5\mathcal{L}\{1\}$$

$$= 5\left(\frac{1}{s}\right).$$

$$\mathcal{L}\{t\} = \int_0^\infty e^{-st}\, t\, dt$$

$$= \left[\int e^{-st}\, t\, dt\right]_0^\infty$$

$$\left.\begin{array}{ll} u = t & du = dt \\ dV = e^{-st}dt & \\ & V = \dfrac{e^{-st}}{-s} \end{array}\right\} = \left[\frac{te^{-st}}{-s} - \int -\frac{e^{-st}}{s}\, dt\right]_0^\infty$$

$$= \left[\frac{te^{-st}}{-s} + \frac{1}{s}\left(\frac{e^{-st}}{-s}\right)\right]_0^\infty$$

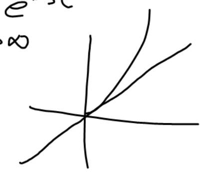
$$= \left[\frac{te^{-st}}{-s} - \frac{1}{s^2}e^{-st}\right]_0^\infty$$

$$= \left[\frac{te^{-st}}{-s}\right]_0^\infty - \frac{1}{s^2}\left[e^{-st}\right]_0^\infty$$

$$= \qquad -\frac{1}{s^2}\left[0 - 1\right]$$

$$+\frac{1}{s^2}$$

$$\boxed{\lim_{t\to\infty} te^{-st}} = \lim_{t\to\infty} \frac{t}{e^{+st}} = 0$$



$$\lim_{t\to\infty} t \cdot \lim_{t\to\infty} \frac{1}{e^{st}} =$$

$$(\infty)\cdot(0) = 0.$$

$$\mathcal{L}\{t\} = \frac{1}{s^2}$$

$$\boxed{\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}}$$

$$\mathcal{L}\{1\} = \frac{1}{s}$$

$$\mathcal{L}\{t\} = \frac{1}{s^2}$$

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$$

$f(t)$	$F(s)$
t	$\frac{1}{s^2}$
e^{at}	$\frac{1}{s-a}$
$e^{at} f(t)$	$F(s-a)$
te^{at}	$\frac{1}{(s-a)^2}$
$\cos(bt)$	$\frac{s}{s^2+b^2}$
$\operatorname{sen}(bt)$	$\frac{b}{s^2+b^2}$
$e^{at}\cos(bt)$	$\frac{(s-a)}{(s-a)^2+b^2}$

$$\mathcal{L}^{-1}\{F(s)\} = f(t)$$

$$f(t) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{st}F(s)\,ds$$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-8)^2}\right\} = te^{8t}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} = t$$

$$\mathcal{L}\{af(t)+bg(t)\} = aF(s)+bG(s) \quad a,b\in\mathbb{R}$$

$$\mathcal{L}\{f(at)\} = \frac{1}{a}F\left(\frac{s}{a}\right)$$

$$\mathcal{L}\{f'(t)\} = sF(s)-f(0)$$

$$\mathcal{L}\{f^{(n)}(t)\} = s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)- \cdots\cdots -sf^{(n-2)}(0)-f^{(n-1)}(0).$$

$$\mathcal{L}\{f''(t)\} = s^2F(s)-sf(0)-f'(0)$$

$$\mathcal{L}\{f'''(t)\} = s^3F(s)-s^2f(0)-sf'(0)-f''(0)$$

$$\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y = 4e^{3t} \qquad y(0) = 2, \quad y'(0) = 5.$$

$$\mathcal{L}\left\{\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y\right\} = \mathcal{L}\{4e^{3t}\}.$$

$$\mathcal{L}\left\{\frac{d^2y}{dt^2}\right\} - 5\mathcal{L}\left\{\frac{dy}{dt}\right\} + 6\mathcal{L}\{y(t)\} = 4\mathcal{L}\{e^{3t}\}.$$

$$\left(s^2\cancel{\mathcal{L}}\{y\} - s(2) - (5)\right) - 5\left(s\cancel{\mathcal{L}}\{y\} - (2)\right) + 6\cancel{\mathcal{L}}\{y\} = \frac{4}{s-3}$$

$$(s^2 - 5s + 6)\mathcal{L}\{y\} - 2s - 5 + 10 = \frac{4}{s-3}$$

$$\Rightarrow (s^2 - 5s + 6)\mathcal{L}\{y\} - 2s + 5 = \frac{4}{s-3}$$

$$(s^2 - 5s + 6)\mathcal{L}\{y\} = \frac{4}{s-3} + 2s - 5$$

$$= \frac{4 + (2s-5)(s-3)}{(s-3)}$$

$$(s^2 - 5s + 6)\mathcal{L}\{y\} = \frac{2s^2 - 11s + 19}{(s-3)}$$

$$\boxed{\mathcal{L}\{y\} = \frac{2s^2 - 11s + 19}{(s-3)(s^2 - 5s + 6)}}$$

$$\mathcal{L}\{y\} = \frac{2s^2 - 11s + 19}{(s-3)(s-3)(s-2)}$$

$$= \frac{2s^2 - 11s + 19}{(s-3)^2 \cdot (s-2)}$$

$$\frac{2s^2 - 11s + 19}{(s-3)^2(s-2)} = \frac{A}{(s-3)^2} + \frac{B}{s-3} + \frac{C}{s-2}$$

$$2s^2 - 11s + 19 = A(s-2) + B(s-2)(s-3) + C(s-3)^2$$

$$= As - 2A + Bs^2 - 5Bs + 6B + Cs^2 - 6Cs + 9C$$

$$= (B+C)s^2 + (A - 5B - 6C)s + (-2A + 6B + 9C)$$

$$B + C = 2$$
$$A - 5B - 6C = -11$$
$$-2A + 6B + 9C = 19$$

$$\mathcal{L}\{y\} = \frac{A}{(s-3)^2} + \frac{B}{s-3} + \frac{C}{(s-2)}$$

$$y = A\mathcal{L}^{-1}\left\{\frac{1}{(s-3)^2}\right\} + B\mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\} + C\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\}$$

$$y_p = Ate^{3t} + Be^{3t} + Ce^{2t}$$