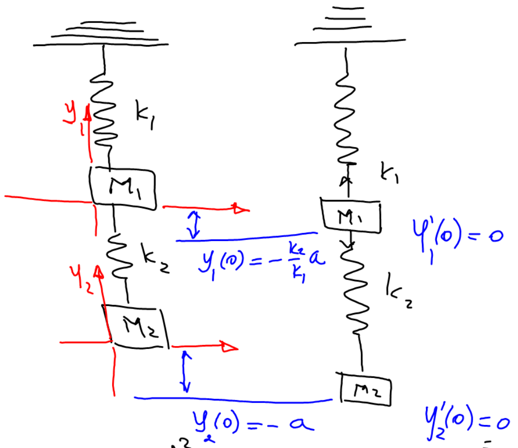
EJEMPLO 1.- SISTEMAS.



$$M_1 \frac{d^2y_1}{dt^2} = -k_1 y_1 + k_2(y_2 - y_1)$$

$$M_2 \frac{d^2y_2}{dt^2} = -k_2(y_2 - y_1)$$

$$\frac{dy_3}{dt} = \frac{(-k_1 - k_2)y_1}{M_1} + \frac{k_2}{M_1}y_2$$

$$\frac{dy_4}{dt} = \frac{k_2}{M_2}y_1 - \frac{k_2}{M_2}y_2$$

$$\frac{dy_1}{dt} = y_3$$

$$\frac{dy_2}{dt} = y_4$$

$$\frac{d}{dt}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{(k_1+k_2)}{M_1} & \frac{k_2}{M_1} & 0 & 0 \\ \frac{k_2}{M_2} & -\frac{k_2}{M_2} & 0 & 0 \end{bmatrix}}_{A} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

$$\frac{d}{dt}\bar{y} = A\bar{y}$$

$$\bar{y}_p = \left[e^{At}\right]\bar{y}(0)$$

$$\bar{y}(0) = \begin{bmatrix} y_1(0) \\ y_2(0) \\ y_3(0) \\ y_4(0) \end{bmatrix}$$

$$\frac{d}{dt}\left[e^{At}\right] = A\left[e^{At}\right]$$

$$\left[e^{At}\right]_{t=0} = \underline{I}$$

$$\left[e^{At}\right]\left[e^{At}\right]^{-1} = \underline{I}.$$

$$\left[e^{At}\right]\left[e^{A(-t)}\right] = I.$$

$$M_1 \frac{d^2y_1}{dt^2} = -k_1 y_1 + k_2(y_2 - y_1)$$
$$M_2 \frac{d^2y_2}{dt^2} = -k_2(y_2 - y_1)$$

$$\mathcal{L}\left\{M_1 \frac{d^2y_1}{dt^2}\right\} = \mathcal{L}\left\{-k_1 y_1 + k_2(y_2 - y_1)\right\} \quad y_1(0) = \frac{k_2}{k_1} a \quad y_1'(0) = 0$$
$$\mathcal{L}\left\{M_2 \frac{d^2y_2}{dt^2}\right\} = \mathcal{L}\left\{-k_2(y_2 - y_1)\right\} \quad y_2(0) = a \quad y_2'(0) = 0$$

$$\left(s^2 \mathcal{L}\{y_1\} - s\left(-\tfrac{1}{5}\right) - (0)\right) = -\mathcal{L}\{y_1\} + 2\left(\mathcal{L}\{y_2\} - \mathcal{L}\{y_1\}\right)$$
$$\left(s^2 \mathcal{L}\{y_2\} - s\left(-\tfrac{1}{10}\right) - (0)\right) = -2\left(\mathcal{L}\{y_2\} - \mathcal{L}\{y_1\}\right)$$

$$s^2 \mathcal{L}\{y_1\} + \mathcal{L}\{y_1\} - 2\mathcal{L}\{y_2\} + 2\mathcal{L}\{y_1\} = -\frac{s}{5}$$
$$s^2 \mathcal{L}\{y_2\} + 2\mathcal{L}\{y_2\} - 2\mathcal{L}\{y_1\} = -\frac{s}{10}$$

$$s^2 \mathcal{L}\{y_1\} + 3\mathcal{L}\{y_1\} - 2\mathcal{L}\{y_2\} = -\frac{s}{5}$$
$$s^2 \mathcal{L}\{y_2\} + 2\mathcal{L}\{y_2\} - 2\mathcal{L}\{y_1\} = -\frac{s}{10}$$

$$\mathcal{L}\{y_1\} = \frac{1}{2}\left(s^2 \mathcal{L}\{y_2\} + 2\mathcal{L}\{y_2\} + \frac{s}{10}\right)$$

$$s^2\left(\frac{1}{2}\left(s^2 \mathcal{L}\{y_2\} + 2\mathcal{L}\{y_2\} + \frac{s}{10}\right)\right) + 3\left(\frac{1}{2}\left(s^2 \mathcal{L}\{y_2\} + 2\mathcal{L}\{y_2\} + \frac{s}{10}\right)\right) - 2\mathcal{L}\{y_2\} = -\frac{s}{5}$$

$$\frac{s^4 \cancel{\mathcal{L}\{y_2\}}}{2} + \cancel{s^2 \mathcal{L}\{y_2\}} + \frac{s^3}{20} + \frac{s^2\, \cancel{3\mathcal{L}\{y_2\}}}{2} + 3\mathcal{L}\{y_2\} + \frac{3s}{20} - 2\mathcal{L}\{y_2\} = -\frac{s}{5}$$

$$\left(\frac{s^4}{2} + \frac{5}{2}s^2 + 1\right)\mathcal{L}\{y_2\} = -\frac{s^3}{20} - \frac{3s}{20} - \frac{s}{5}$$

$$\mathcal{L}\{y_2\} = \frac{-\left(\frac{s^3}{20} + \frac{7s}{20}\right)}{\frac{s^4}{2} + \frac{5}{2}s^2 + 1} = \frac{-\frac{1}{10}s^3 - \frac{7}{10}s}{s^4 + 5s^2 + 2}$$

$$= \frac{Es + F}{s^2 + a^2} + \frac{Gs + H}{s^2 + b^2}$$