```
> restart
>
```

RESOLUCIÓN ALGEBRAICA DEL
PROBLEMA DE DOS RESORTES CON LOS PARÁMETROS INICIALES

```
> EcuaUno := M[1]·diff(y[1](t), t$2) = -Hooke[1]·y[1](t) + Hooke[2]·(y[2](t) - y[1](t));
```

$$EcuaUno := M_1 \left(\frac{d^2}{dt^2} y_1(t) \right) = -Hooke_1 y_1(t) + Hooke_2 \left(y_2(t) - y_1(t) \right) \quad (1)$$

```
> EcuaDos := M[2]·diff(y[2](t), t$2) = -Hooke[2]·(y[2](t) - y[1](t))
```

$$EcuaDos := M_2 \left(\frac{d^2}{dt^2} y_2(t) \right) = -Hooke_2 \left(y_2(t) - y_1(t) \right) \quad (2)$$

```
> Condiciones := y[1](0) = -1/5, y[2](0) = -1/10, D(y[1])(0) = 0, D(y[2])(0) = 0
```

$$Condiciones := y_1(0) = -\frac{1}{5}, y_2(0) = -\frac{1}{10}, D\left(y_1\right)(0) = 0, D\left(y_2\right)(0) = 0 \quad (3)$$

```
> M[1] := 1; M[2] := 1; Hooke[1] := 1; Hooke[2] := 2
```

$$M_1 := 1$$
$$M_2 := 1$$
$$Hooke_1 := 1$$
$$Hooke_2 := 2 \quad (4)$$

```
> EcuaUno
```

$$\frac{d^2}{dt^2} y_1(t) = -3 y_1(t) + 2 y_2(t) \quad (5)$$

```
> EcuaDos
```

$$\frac{d^2}{dt^2} y_2(t) = -2 y_2(t) + 2 y_1(t) \quad (6)$$

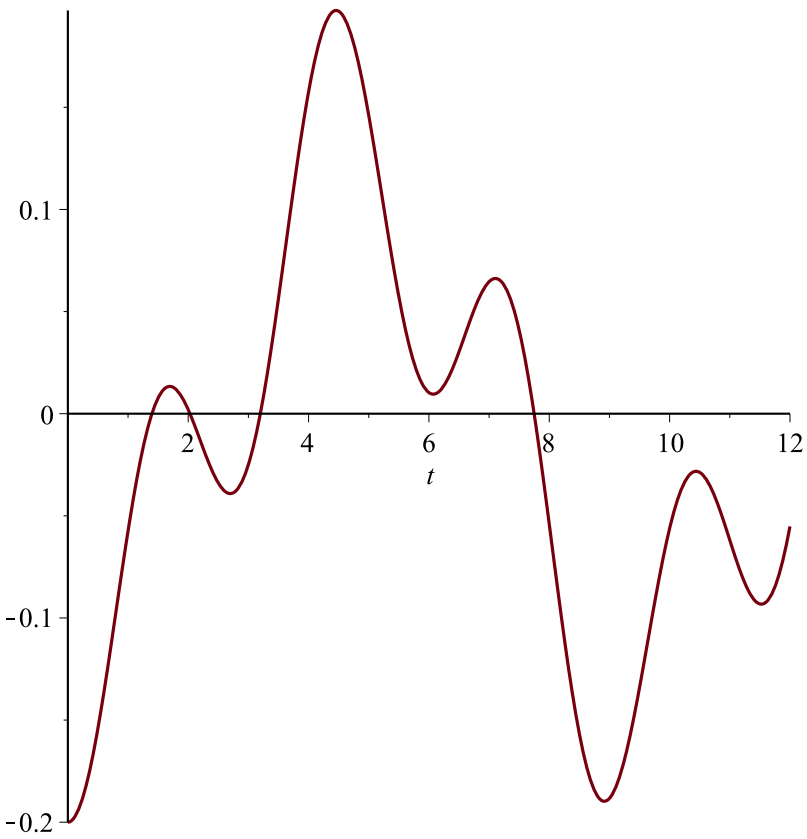
```
> SolPart := (dsolve({EcuaUno, EcuaDos, Condiciones})) :
> evalf(SolPart[1], 3)
```

$$y_1(t) = -0.0759 \cos(2.14\, t) - 0.124 \cos(0.665\, t) \quad (7)$$

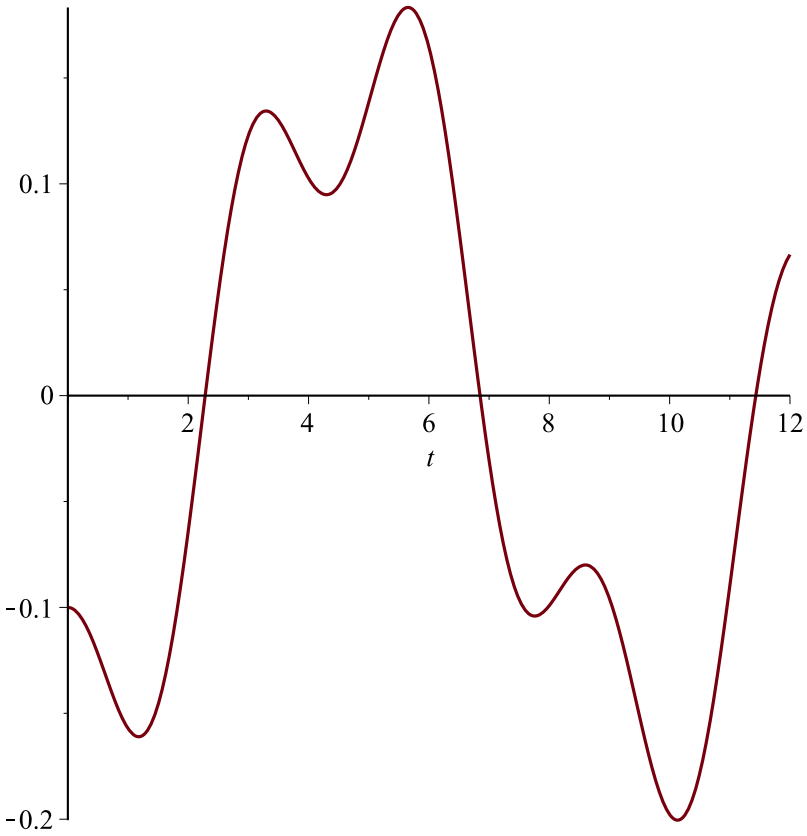
```
> evalf(SolPart[2], 3)
```

$$y_2(t) = 0.0590 \cos(2.14\, t) - 0.159 \cos(0.665\, t) \quad (8)$$

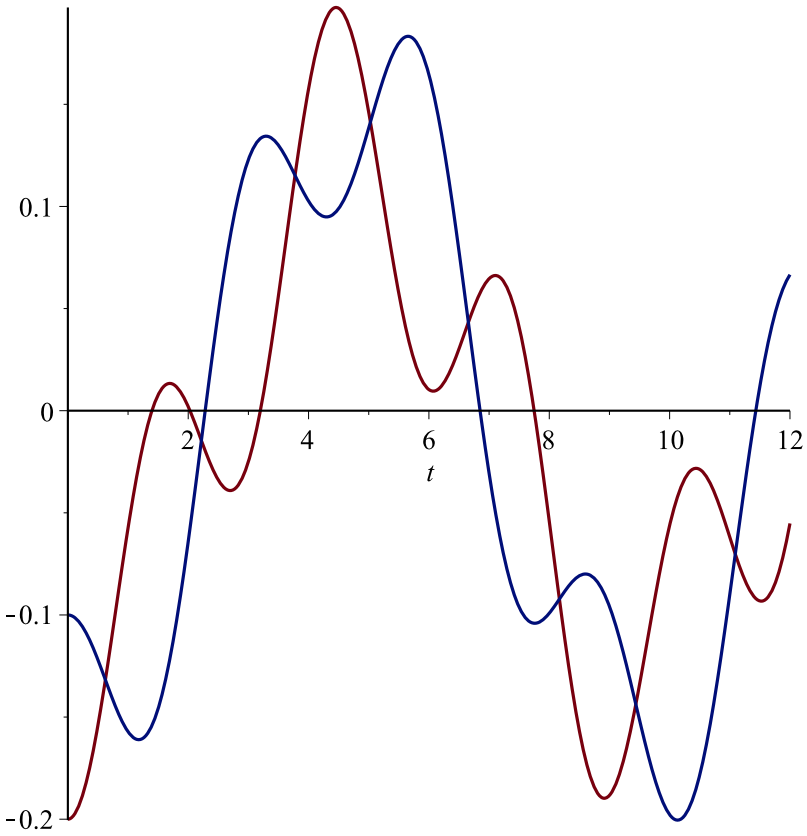
```
> plot(rhs(SolPart[1]), t = 0..12)
```



> $plot(rhs(SolPart[2]), t = 0..12)$



> $plot([rhs(SolPart[1]), rhs(SolPart[2])], t = 0..12)$




```
> with(inttrans) :
```

RESOLUCIÓN CON TRANSFORMADA DE LAPLACE DEL
PROBLEMA DE DOS RESORTES CON LOS PARÁMETROS INICIALES

```
>
> EcuaTransUno := subs(Condiciones, laplace(EcuaUno, t, s))
```

$$EcuaTransUno := s^2\, laplace\left(y_1(t), t, s\right) + \frac{1}{5}\, s = -3\, laplace\left(y_1(t), t, s\right) + 2\, laplace\left(y_2(t), t, s\right) \quad (9)$$

```
> EcuaTransDos := subs(Condiciones, laplace(EcuaDos, t, s))
```

$$EcuaTransDos := s^2\, laplace\left(y_2(t), t, s\right) + \frac{1}{10}\, s = -2\, laplace\left(y_2(t), t, s\right) + 2\, laplace\left(y_1(t), t, s\right) \quad (10)$$

```
> SolDosTrans := isolate(EcuaTransUno, laplace(y[2](t), t, s))
```

$$SolDosTrans := laplace\left(y_2(t), t, s\right) = \frac{1}{2}\, s^2\, laplace\left(y_1(t), t, s\right) + \frac{1}{10}\, s + \frac{3}{2}\, laplace\left(y_1(t), t, s\right) \quad (11)$$

```
> SolUnoTrans := simplify(isolate(subs(laplace(y[2](t), t, s) = rhs(SolDosTrans),
      EcuaTransDos), laplace(y[1](t), t, s)))
```

$$SolUnoTrans := laplace(y_1(t), t, s) = -\frac{1}{5}\ \frac{s\,(s^2+3)}{s^4+5\,s^2+2} \tag{12}$$

>

> $SolTrans := solve(\{EcuaTransUno, EcuaTransDos\}, \{laplace(y[1](t), t, s), laplace(y[2](t), t, s)\})$

$$SolTrans := \left\{laplace(y_1(t), t, s) = -\frac{1}{5}\ \frac{s\,(s^2+3)}{s^4+5\,s^2+2}, laplace(y_2(t), t, s) = -\frac{1}{10}\ \frac{s\,(s^2+7)}{s^4+5\,s^2+2}\right\} \tag{13}$$

>

> $Raiz := evalf(solve(2+s^4+5\,s^2), 3)$

$$Raiz := 2.14\ I,\ -2.14\ I,\ 0.665\ I,\ -0.665\ I \tag{14}$$

> $PoliUno := eval(expand((s-Raiz[1])\cdot(s-Raiz[2])))$

$$PoliUno := s^2 + 4.5796 + 0.\ I \tag{15}$$

> $PoliDos := eval(expand((s-Raiz[3])\cdot(s-Raiz[4])))$

$$PoliDos := s^2 + 0.442225 + 0.\ I \tag{16}$$

> $expand\left(-\frac{1}{5}\ s\,(s^2+3)\right) = factor(expand((A\cdot s+B)\cdot PoliUno + (C\cdot s+D)\cdot PoliDos))$

$$-\frac{1}{5}\ s^3 - \frac{3}{5}\ s = s^3\,A + s^2\,B + 4.5796\,A\,s + 4.5796\,B + s^3\,C + s^2\,D + 0.442225\,C\,s + 0.442225\,D \tag{17}$$

> $Sistema := A + C = -\frac{1}{5}, B + D = 0, 4.561552813\,A + 0.4384471867\,C = -\frac{3}{5}, 4.561552813\,B + 0.4384471867\,D = 0:$

> $Sistema[1]; Sistema[2]; Sistema[3]; Sistema[4]$

$$A + C = -\frac{1}{5}$$
$$B + D = 0$$
$$4.561552813\,A + 0.4384471867\,C = -\frac{3}{5}$$
$$4.561552813\,B + 0.4384471867\,D = 0 \tag{18}$$

> $ParaUno := solve(\{Sistema\}, \{A, B, C, D\}):$

> $ParaUno[1]; ParaUno[2]; ParaUno[3]; ParaUno[4];$

$$A = -0.1242535625$$
$$B = 0.$$
$$C = -0.07574643749$$
$$D = 0. \tag{19}$$

> $SolPartTransUno := \frac{rhs(ParaUno[1])\cdot s + rhs(ParaUno[2])}{PoliDos} + \frac{rhs(ParaUno[3])\cdot s + rhs(ParaUno[4])}{PoliUno}$

$$SolPartTransUno := -\frac{0.1242535625\, s}{s^2 + 0.442225 + 0.\,\mathrm{I}} - \frac{0.07574643749\, s}{s^2 + 4.5796 + 0.\,\mathrm{I}} \quad (20)$$

> $SolPartUno := y[1](t) = invlaplace(SolPartTransUno, s, t)$:

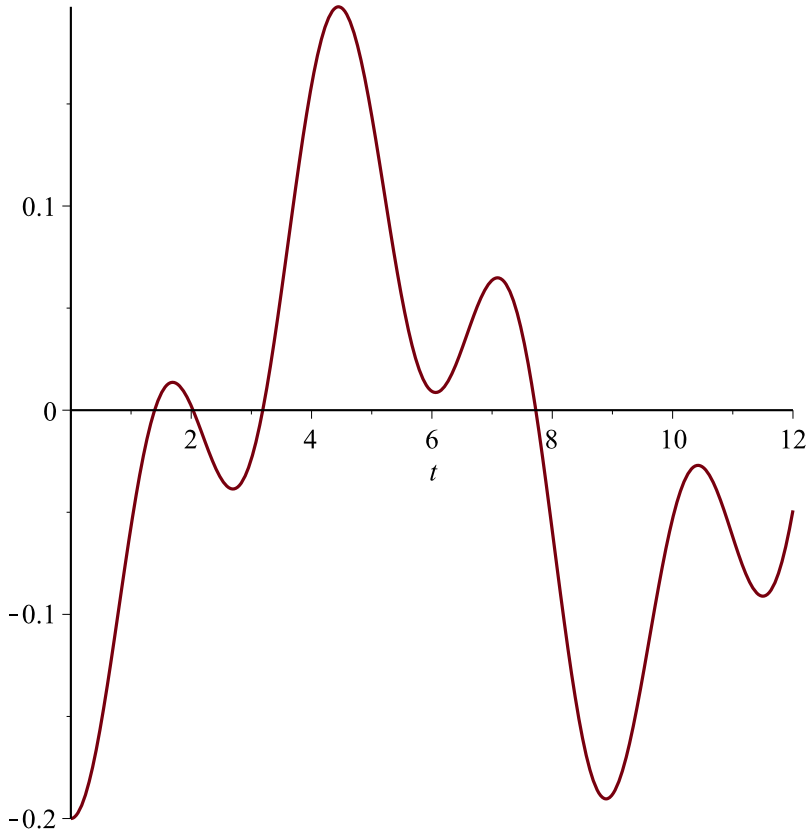
> $evalf(\%, 3)$

$$y_1(t) = -0.124 \cos(0.665\, t) - 0.0757 \cos(2.14\, t) \quad (21)$$

> $evalf(subs(t = 0, SolPartUno), 3)$

$$y_1(0) = -0.200 \quad (22)$$

> $plot(rhs(SolPartUno), t = 0..12)$



>

> $expand\left(-\frac{1}{10}\, s\left(s^2 + 7\right)\right) = factor(expand((E \cdot s + F) \cdot PoliUno + (G \cdot s + H) \cdot PoliDos))$

$$-\frac{1}{10}\, s^3 - \frac{7}{10}\, s = s^3 E + s^2 F + 4.5796\, E\, s + 4.5796\, F + s^3 G + s^2 H + 0.442225\, G\, s + 0.442225\, H \quad (23)$$

> $SistemaDos := E + G = -\frac{1}{10}, F + H = 0, 4.561552813\, E + 0.4384471867\, G = -\frac{7}{10}, 4.561552813\, F + 0.4384471867\, H = 0$:

> $SistemaDos[1]; SistemaDos[2]; SistemaDos[3]; SistemaDos[4]$

$$E + G = -\frac{1}{10}$$

$$F + H = 0$$

$$4.561552813\, E + 0.4384471867\, G = -\frac{7}{10}$$

$$4.561552813\, F + 0.4384471867\, H = 0 \quad \textbf{(24)}$$

> $ParaDos := solve(\{SistemaDos\}, \{E, F, G, H\})$

$$ParaDos := \{E = -0.1591410313, F = 0., G = 0.05914103125, H = 0.\} \quad \textbf{(25)}$$

> $SolPartTransDos := \frac{rhs(ParaDos[1]) \cdot s + rhs(ParaDos[2])}{PoliDos} + \frac{rhs(ParaDos[3]) \cdot s + rhs(ParaDos[4])}{PoliUno}$

$$SolPartTransDos := -\frac{0.1591410313\, s}{s^2 + 0.442225 + 0.\,\mathrm{I}} + \frac{0.05914103125\, s}{s^2 + 4.5796 + 0.\,\mathrm{I}} \quad \textbf{(26)}$$

> $SolPartDos := y[2](t) = invlaplace(SolPartTransDos, s, t) :$

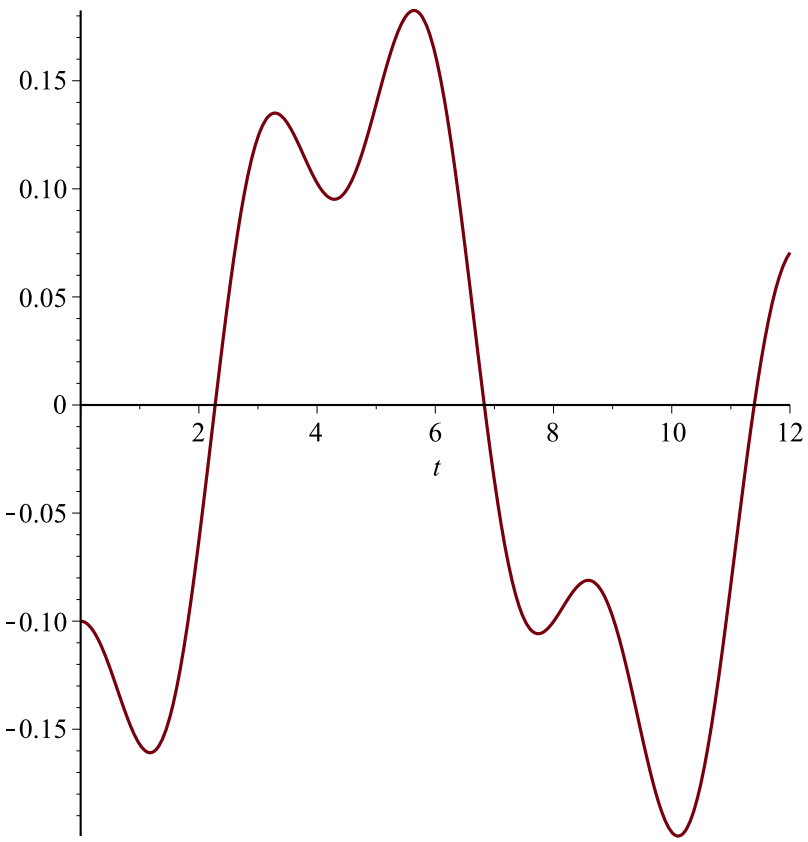
> $evalf(\%, 3)$

$$y_2(t) = -0.159 \cos(0.665\, t) + 0.0591 \cos(2.14\, t) \quad \textbf{(27)}$$

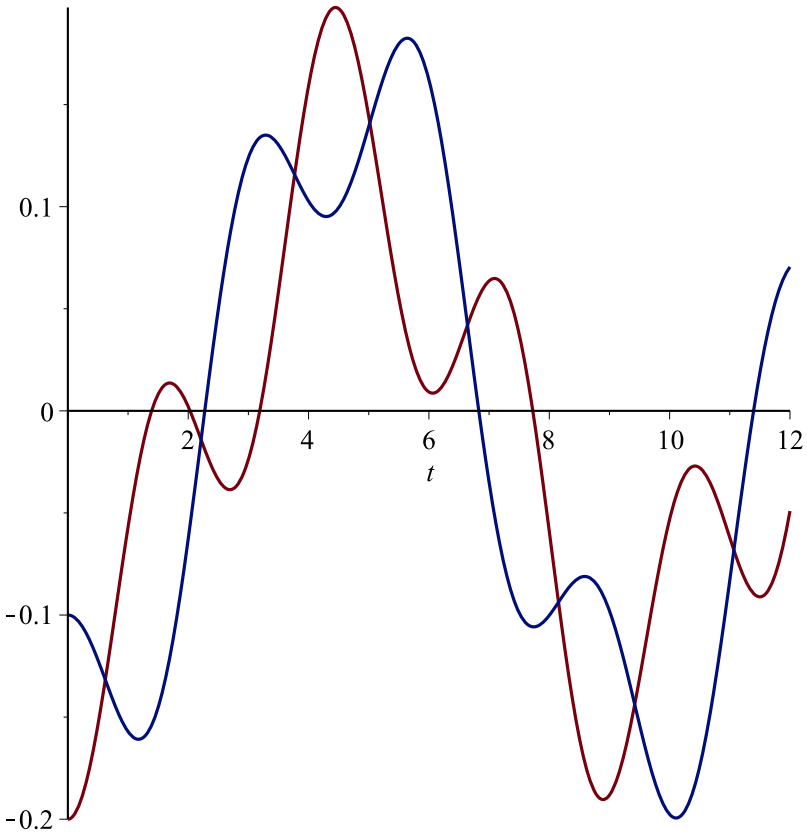
> $evalf(subs(t = 0, SolPartDos), 3)$

$$y_2(0) = -0.0999 \quad \textbf{(28)}$$

> $plot(rhs(SolPartDos), t = 0..12)$



> $plot([rhs(SolPartUno), rhs(SolPartDos)], t = 0..12)$



>

> *restart*

RESOLUCIÓN CON MATRIZ EXPONENCIAL DEL
PROBLEMA DE DOS RESORTES CON LOS PARÁMETROS INICIALES

>

> $ParaUno := M[1] = 1;\ ParaDos := M[2] = 1;\ ParaTres := Hooke[1] = 1;\ ParaCuatro := Hooke[2] = 2;$

$$ParaUno := M_1 = 1$$

$$ParaDos := M_2 = 1$$

$$ParaTres := Hooke_1 = 1$$

$$ParaCuatro := Hooke_2 = 2 \quad \textbf{(29)}$$

> $a := -\frac{1}{10}$

$$a := -\frac{1}{10} \quad \textbf{(30)}$$

> $CondicionesIniciales := Ycero = array\left(\left[\frac{rhs(ParaCuatro)}{rhs(ParaTres)} \cdot a,\ a,\ 0,\ 0\right]\right)$

$$CondicionesIniciales := Ycero = \begin{bmatrix} -\frac{1}{5} & -\frac{1}{10} & 0 & 0 \end{bmatrix} \quad (31)$$

```
> A := array([[0, 0, 1, 0], [0, 0, 0, 1], [-((rhs(ParaTres) + rhs(ParaCuatro))/rhs(ParaUno)),
      rhs(ParaCuatro)/rhs(ParaUno), 0, 0], [rhs(ParaCuatro)/rhs(ParaDos), -rhs(ParaCuatro)/rhs(ParaDos), 0, 0]]) :
```

$$A := \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & 2 & 0 & 0 \\ 2 & -2 & 0 & 0 \end{bmatrix} \quad (32)$$

```
> with(linalg) :
> MatExp := exponential(A, t) :
> evalf(MatExp[1, 1], 3)
```

$$0.383 \cos(0.665\, t) + 0.618 \cos(2.14\, t) \quad (33)$$

```
> SolPart := evalm(MatExp &* rhs(CondicionesIniciales)) :
> evalf(SolPart[1], 3)
```

$$-0.125 \cos(0.665\, t) - 0.0753 \cos(2.14\, t) \quad (34)$$

```
> evalf(SolPart[2], 3)
```

$$-0.159 \cos(0.665\, t) + 0.0598 \cos(2.14\, t) \quad (35)$$

```
> evalf(subs(t = 0, SolPart[1]), 3)
```

$$-0.200 + 0.\,I \quad (36)$$

```
> evalf(subs(t = 0, SolPart[2]), 3)
```

$$-0.100 + 0.\,I \quad (37)$$

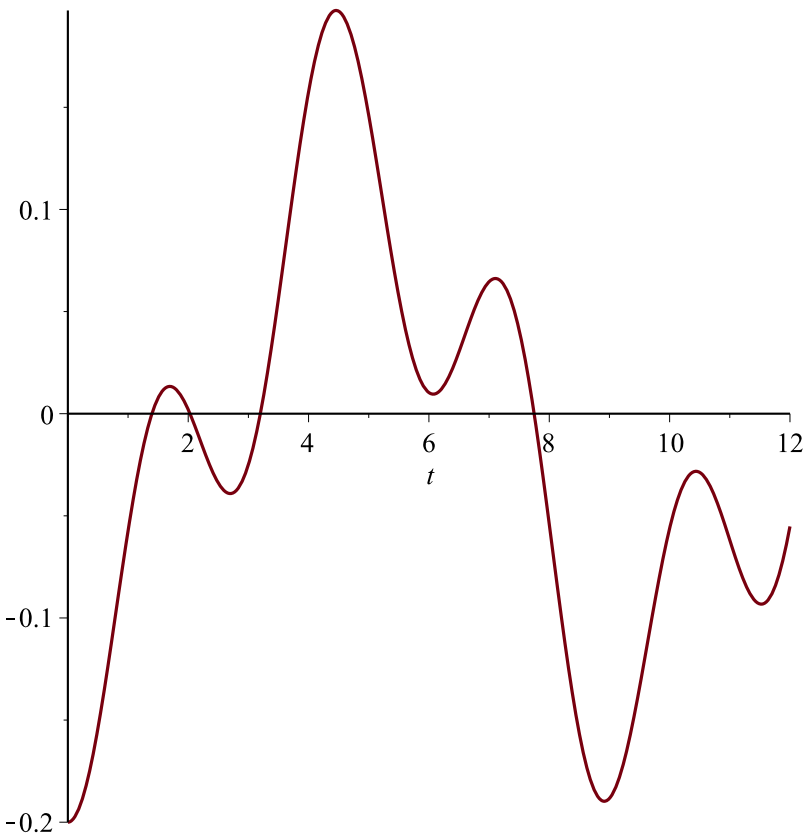
```
> eval(subs(t = 0, SolPart[3]))
```

$$0 \quad (38)$$

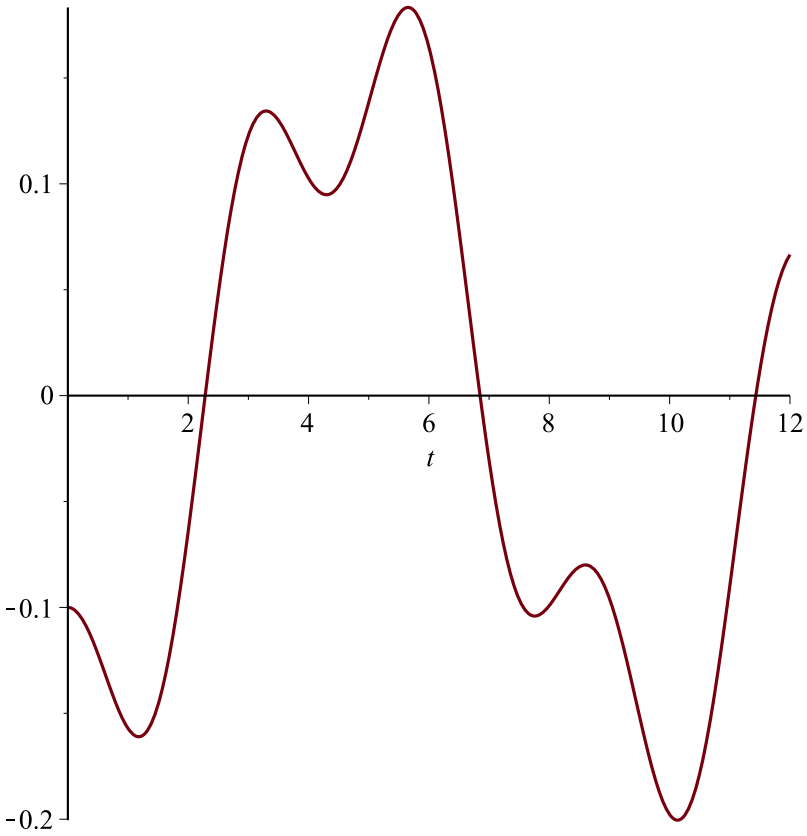
```
> eval(subs(t = 0, SolPart[4]))
```

$$0 \quad (39)$$

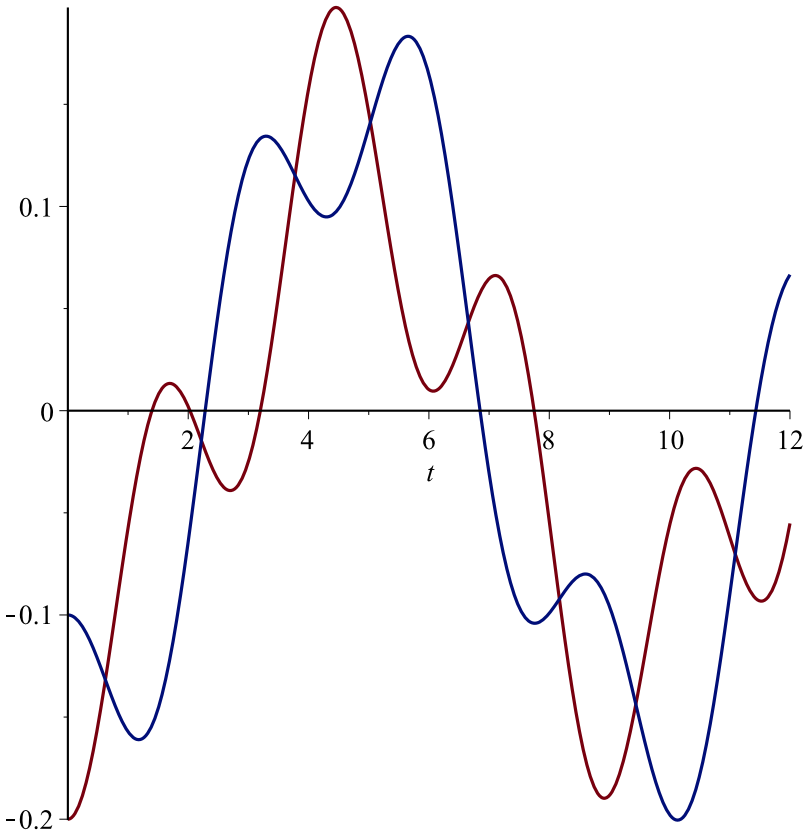
```
> plot(SolPart[1], t = 0..12)
```



> $plot(SolPart[2], t = 0..12)$



> $plot([SolPart[1], SolPart[2]], t = 0..12)$



>

>

> *restart*

PROBLEMA DE DOS RESORTES CON OTROS PARÁMETROS CAMBIADOS

> $ParaUno := M[1] = 1; ParaDos := M[2] = 1; ParaTres := Hooke[1] = 2; ParaCuatro := Hooke[2] = 1;$

$$ParaUno := M_1 = 1$$

$$ParaDos := M_2 = 1$$

$$ParaTres := Hooke_1 = 2$$

$$ParaCuatro := Hooke_2 = 1 \quad \textbf{(40)}$$

> $a := -\frac{1}{10}$

$$a := -\frac{1}{10} \quad \textbf{(41)}$$

> $CondicionesIniciales := Ycero = array\left(\left[\frac{rhs(ParaCuatro)}{rhs(ParaTres)} \cdot a, a, 0, 0\right]\right)$

$$CondicionesIniciales := Ycero = \begin{bmatrix} -\frac{1}{20} & -\frac{1}{10} & 0 & 0 \end{bmatrix} \quad (42)$$

```
> A := array([[0, 0, 1, 0], [0, 0, 0, 1], [-(rhs(ParaTres) + rhs(ParaCuatro))/rhs(ParaUno), rhs(ParaCuatro)/rhs(ParaUno), 0, 0], [rhs(ParaCuatro)/rhs(ParaDos), -rhs(ParaCuatro)/rhs(ParaDos), 0, 0]]) :
```

$$A := \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \quad (43)$$

```
> with(linalg) :
> MatExp := exponential(A, t) :
> evalf(MatExp[1, 1], 3)
```

$$0.146 \cos(0.768\, t) + 0.850 \cos(1.85\, t) \quad (44)$$

```
> SolPart := evalm(MatExp &* rhs(CondicionesIniciales)) :
> evalf(SolPart[1], 3)
```

$$-0.0427 \cos(0.768\, t) - 0.0071 \cos(1.85\, t) \quad (45)$$

```
> evalf(SolPart[2], 3)
```

$$-0.103 \cos(0.768\, t) + 0.0034 \cos(1.85\, t) \quad (46)$$

```
> evalf(subs(t = 0, SolPart[1]), 3)
```

$$-0.0496 + 0.\,I \quad (47)$$

```
> evalf(subs(t = 0, SolPart[2]), 3)
```

$$-0.0993 + 0.\,I \quad (48)$$

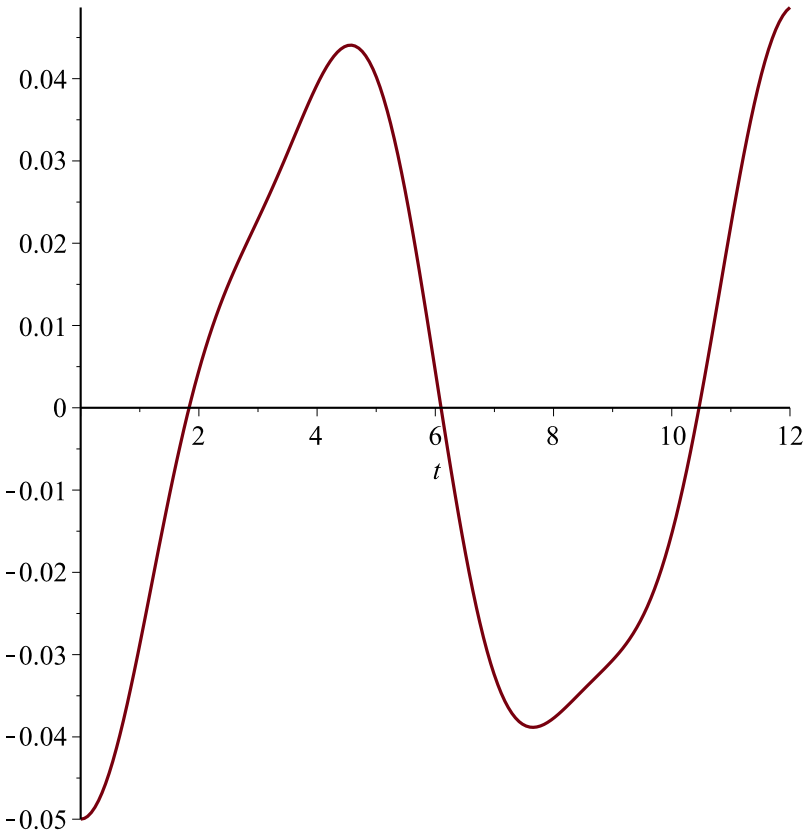
```
> eval(subs(t = 0, SolPart[3]))
```

$$0 \quad (49)$$

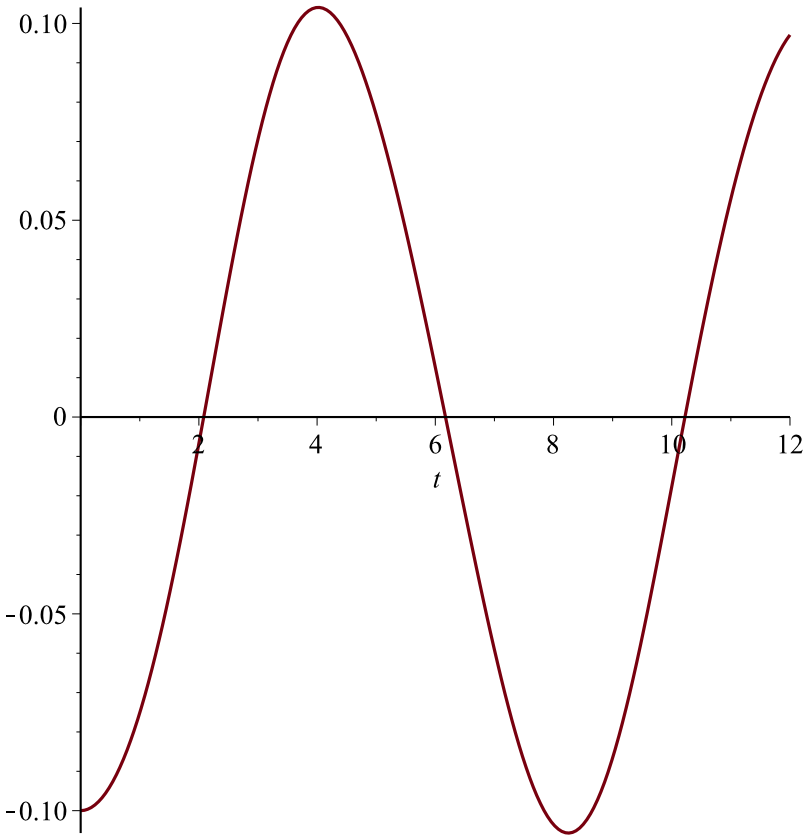
```
> eval(subs(t = 0, SolPart[4]))
```

$$0 \quad (50)$$

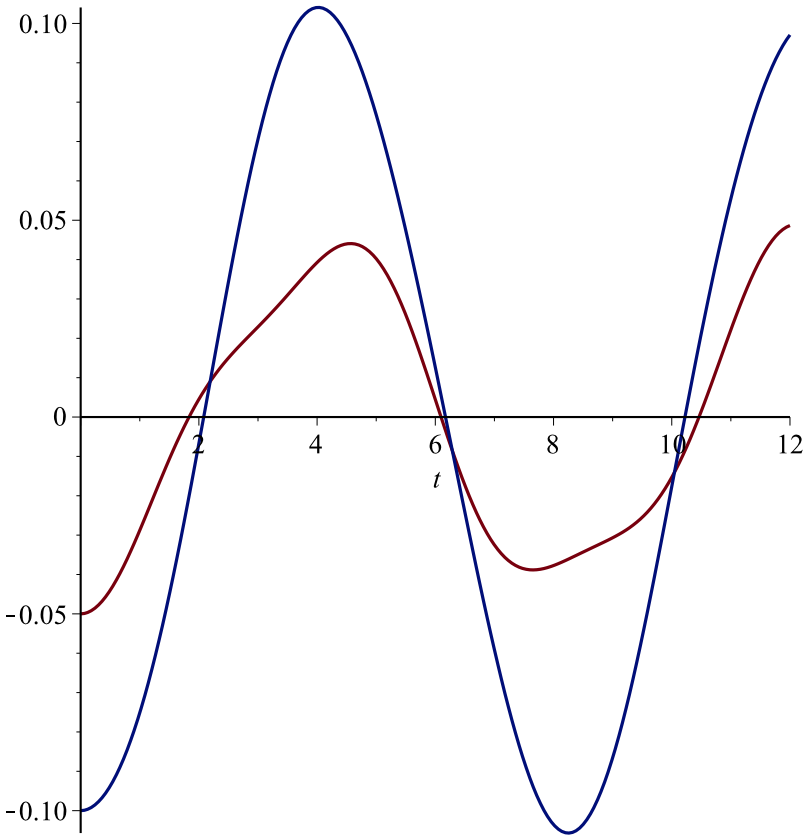
```
> plot(SolPart[1], t = 0..12)
```




```
> plot(SolPart[2], t = 0 ..12)
```




```
> plot( [SolPart[1], SolPart[2]], t = 0 ..12 )
```



> *restart*

> *ParaUno* := *M*[1] = 1; *ParaDos* := *M*[2] = 2; *ParaTres* := *Hooke*[1] = 1; *ParaCuatro* := *Hooke*[2] = 1;

$$ParaUno := M_1 = 1$$

$$ParaDos := M_2 = 2$$

$$ParaTres := Hooke_1 = 1$$

$$ParaCuatro := Hooke_2 = 1 \quad \textbf{(51)}$$

> $a := -\frac{1}{10}$

$$a := -\frac{1}{10} \quad \textbf{(52)}$$

> $CondicionesIniciales := Ycero = array\left(\left[\frac{rhs(ParaCuatro)}{rhs(ParaTres)} \cdot a, a, 0, 0\right]\right)$

$$CondicionesIniciales := Ycero = \left[\begin{array}{cccc} -\frac{1}{10} & -\frac{1}{10} & 0 & 0 \end{array}\right] \quad \textbf{(53)}$$

```
> A := array([[0, 0, 1, 0], [0, 0, 0, 1], [-(rhs(ParaTres) + rhs(ParaCuatro))/rhs(ParaUno),
      rhs(ParaCuatro)/rhs(ParaUno), 0, 0], [rhs(ParaCuatro)/rhs(ParaDos), -rhs(ParaCuatro)/rhs(ParaDos), 0, 0]]) :
```

$$A := \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & 1 & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 & 0 \end{bmatrix} \tag{54}$$

```
> with(linalg) :
> MatExp := exponential(A, t) :
> evalf(MatExp[1, 1], 3)
```

$$0.134 \cos(0.469\, t) + 0.863 \cos(1.51\, t) \tag{55}$$

```
> SolPart := evalm(MatExp &* rhs(CondicionesIniciales)) :
> evalf(SolPart[1], 3)
```

$$-0.0622 \cos(0.469\, t) - 0.0375 \cos(1.51\, t) \tag{56}$$

```
> evalf(SolPart[2], 3)
```

$$-0.110 \cos(0.469\, t) + 0.0106 \cos(1.51\, t) \tag{57}$$

```
> evalf(subs(t = 0, SolPart[1]), 3)
```

$$-0.100 + 0.\,I \tag{58}$$

```
> evalf(subs(t = 0, SolPart[2]), 3)
```

$$-0.100 + 0.\,I \tag{59}$$

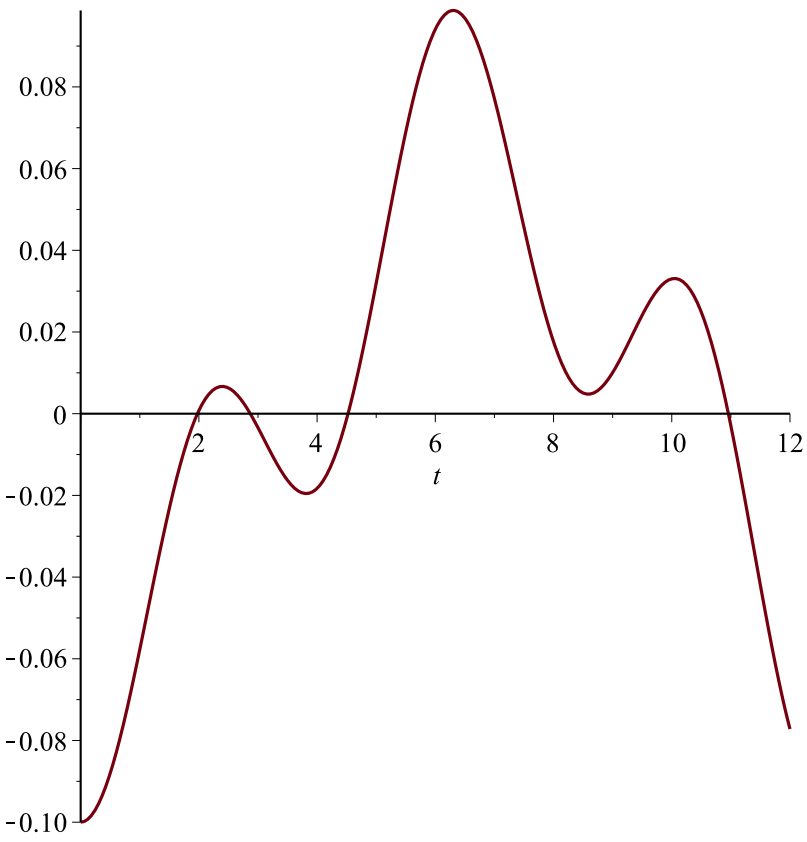
```
> eval(subs(t = 0, SolPart[3]))
```

$$0 \tag{60}$$

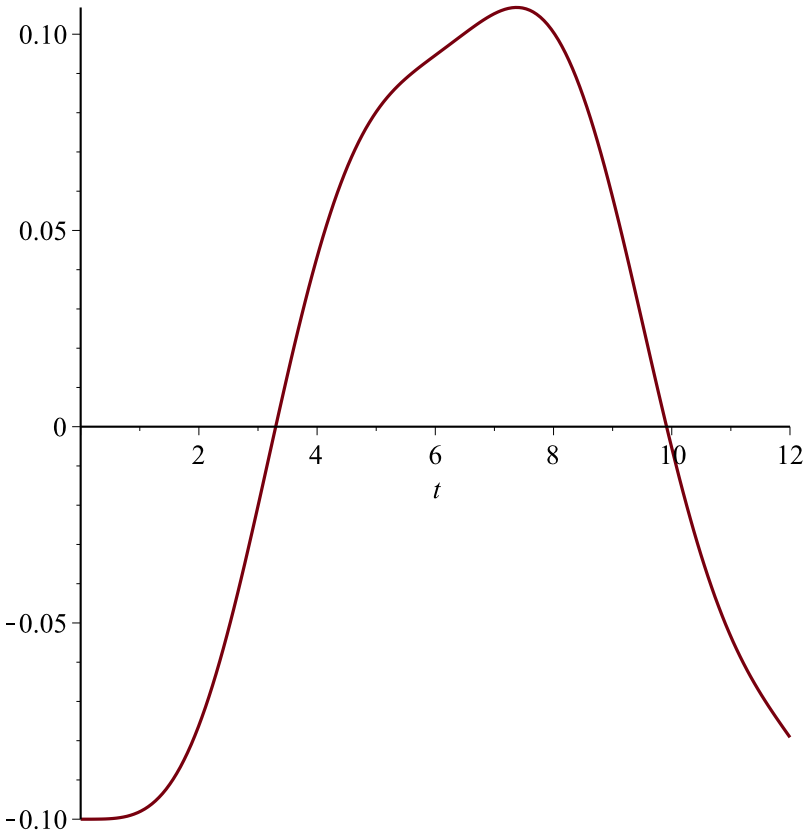
```
> eval(subs(t = 0, SolPart[4]))
```

$$0 \tag{61}$$

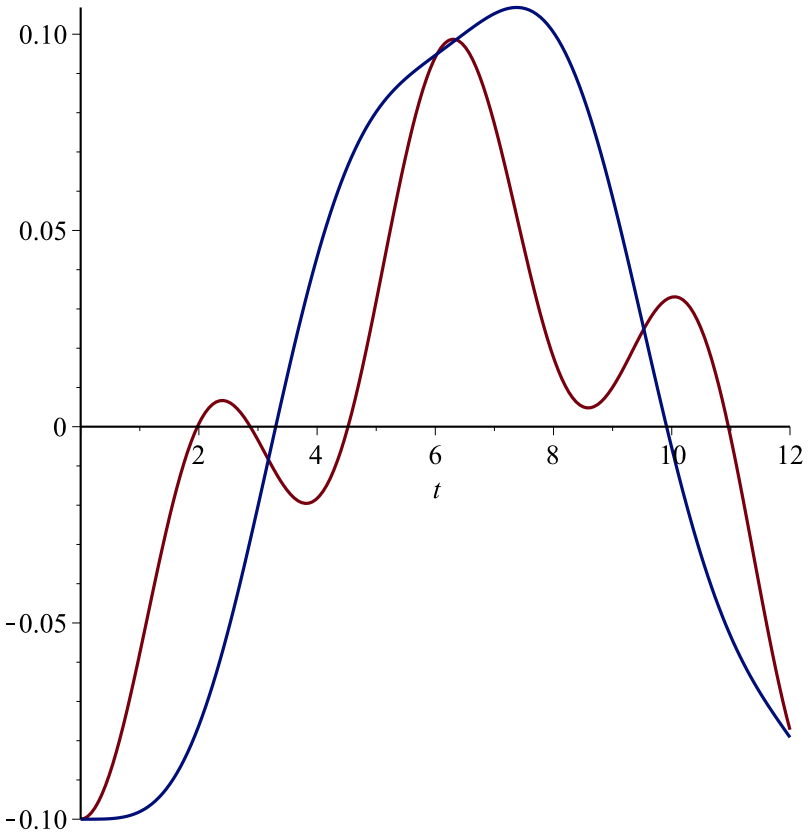
```
> plot(SolPart[1], t = 0..12)
```



> $plot(SolPart[2], t = 0..12)$




```
> plot( [SolPart[1], SolPart[2]], t = 0 ..12 )
```



> *restart*

> *ParaUno* := *M*[1] = 1; *ParaDos* := *M*[2] = 2; *ParaTres* := *Hooke*[1] = 1; *ParaCuatro* := *Hooke*[2] = 2;

$$ParaUno := M_1 = 1$$

$$ParaDos := M_2 = 2$$

$$ParaTres := Hooke_1 = 1$$

$$ParaCuatro := Hooke_2 = 2 \quad \textbf{(62)}$$

> $a := -\frac{1}{10}$

$$a := -\frac{1}{10} \quad \textbf{(63)}$$

> $CondicionesIniciales := Ycero = array\left(\left[\frac{rhs(ParaCuatro)}{rhs(ParaTres)} \cdot a, a, 0, 0\right]\right)$

$$CondicionesIniciales := Ycero = \left[\begin{array}{cccc} -\frac{1}{5} & -\frac{1}{10} & 0 & 0 \end{array}\right] \quad \textbf{(64)}$$

```
> A := array([[0, 0, 1, 0], [0, 0, 0, 1], [-((rhs(ParaTres) + rhs(ParaCuatro))/rhs(ParaUno)),
      rhs(ParaCuatro)/rhs(ParaUno), 0, 0], [rhs(ParaCuatro)/rhs(ParaDos), -rhs(ParaCuatro)/rhs(ParaDos), 0, 0]]) :
```

$$A := \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & 2 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \tag{65}$$

```
> with(linalg) :
> MatExp := exponential(A, t) :
> evalf(MatExp[1, 1], 3)
```

$$0.212 \cos(-0.515\, t) + 0.784 \cos(1.92\, t) \tag{66}$$

```
> SolPart := evalm(MatExp &* rhs(CondicionesIniciales)) :
> evalf(SolPart[1], 3)
```

$$-0.0997 \cos(-0.515\, t) - 0.0997 \cos(1.92\, t) \tag{67}$$

```
> evalf(SolPart[2], 3)
```

$$-0.136 \cos(-0.515\, t) + 0.0361 \cos(1.92\, t) \tag{68}$$

```
> evalf(subs(t = 0, SolPart[1]), 3)
```

$$-0.199 \tag{69}$$

```
> evalf(subs(t = 0, SolPart[2]), 3)
```

$$-0.0996 \tag{70}$$

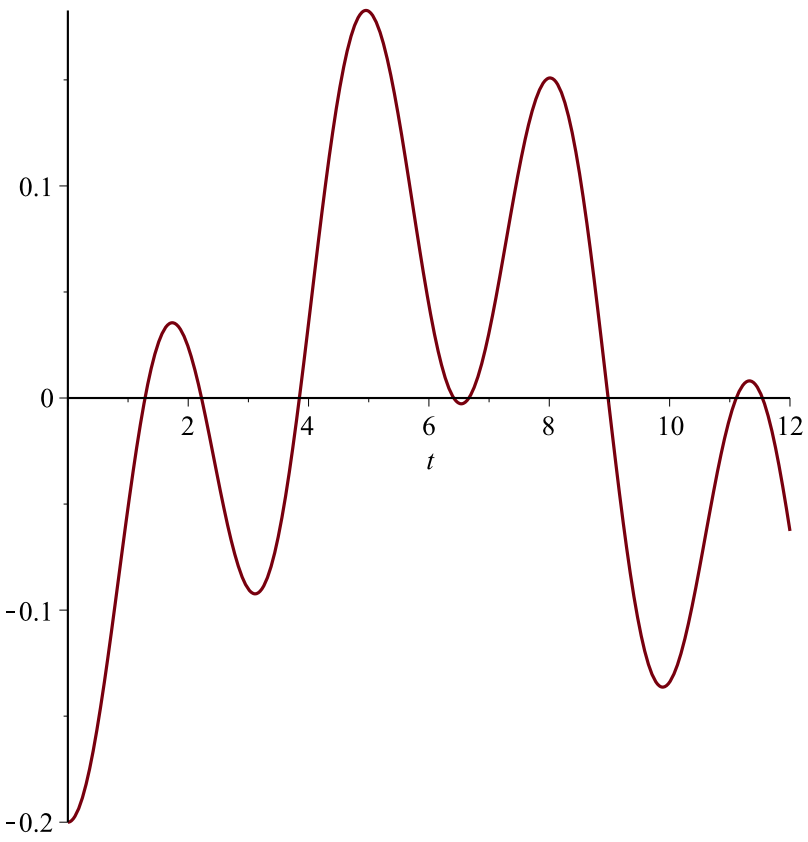
```
> evalf(subs(t = 0, SolPart[3]))
```

$$0. \tag{71}$$

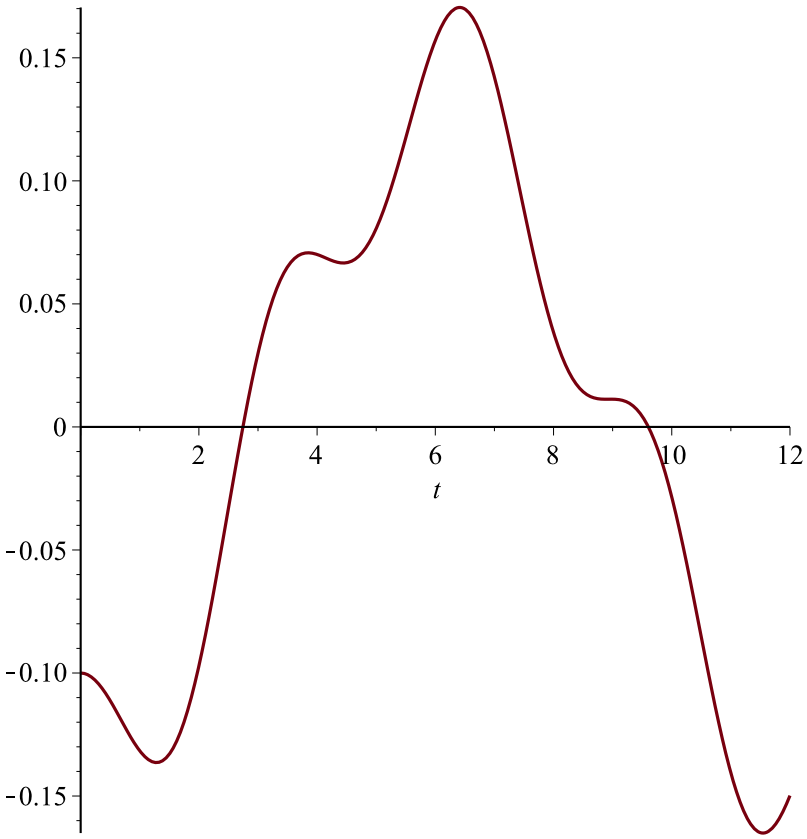
```
> evalf(subs(t = 0, SolPart[4]))
```

$$-0. \tag{72}$$

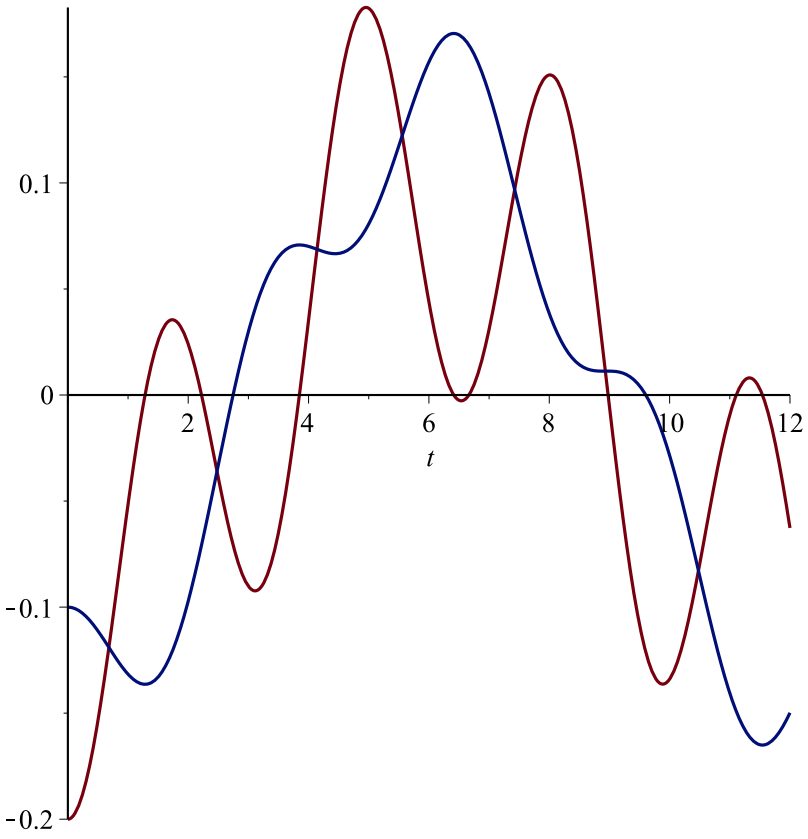
```
> plot(SolPart[1], t = 0..12)
```



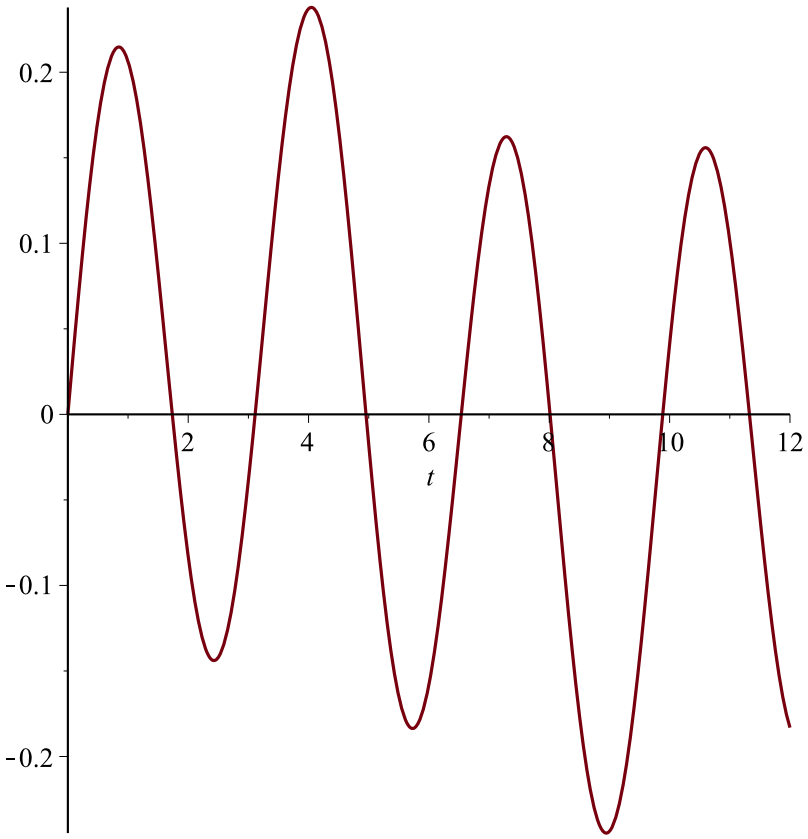
> $plot(SolPart[2], t = 0..12)$



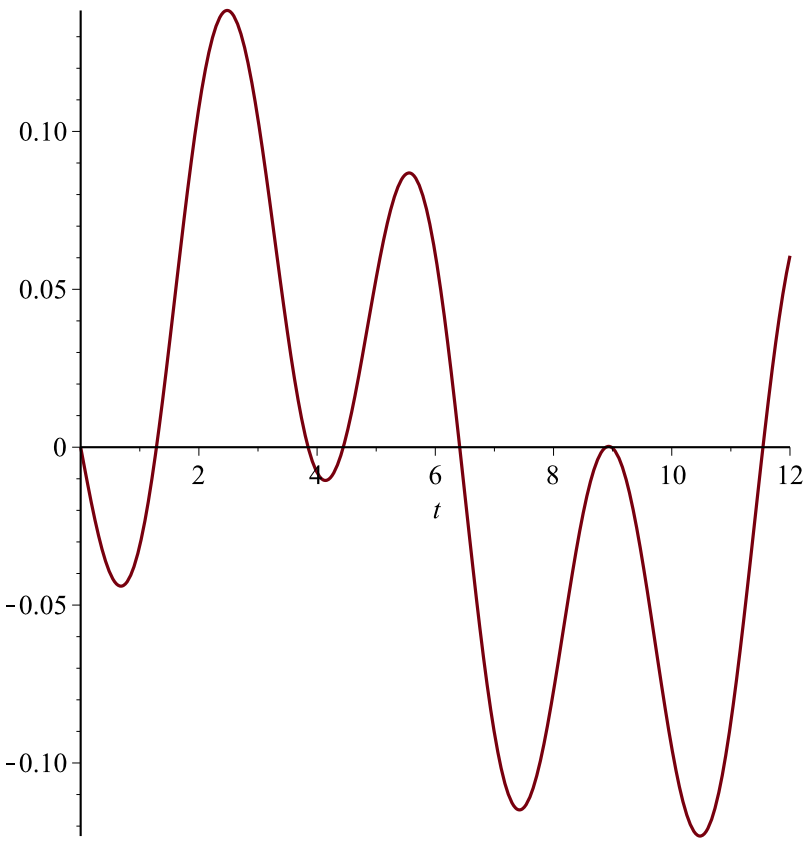
> $plot([SolPart[1], SolPart[2]], t = 0..12)$



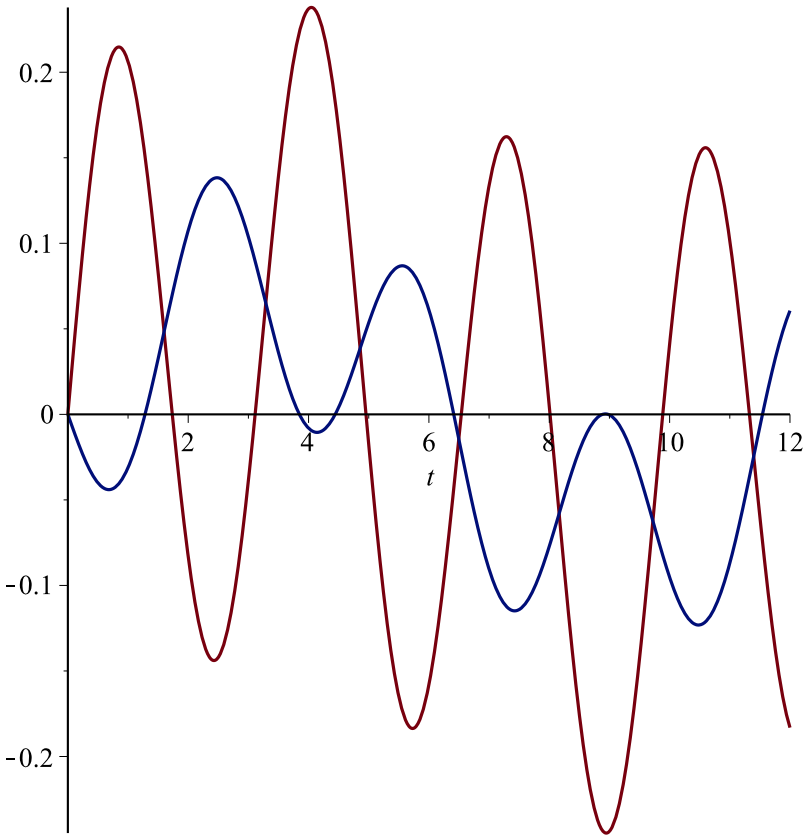
> $plot(SolPart[3], t = 0..12)$



> $plot(SolPart[4], t = 0..12)$



> $plot([SolPart[3], SolPart[4]], t = 0..12)$




```
> restart
>
```