ECUACIONES EN DERIVADAS PARCIALES

MÉTODO DE SEPARACIÓN DE VARIABLES

ZILL, DENNIS G, CULLEN, MICHAEL R.
"ECUACIONES DIFERENCIALES con problemas de valores en la frontera" QUINTA EDICIÓN, pp. 521, ED. THOMSON LEARNING, 2002.

1.- Tipo prueba y error
2.- AL MENOS UNA DE LAS SOLS. GRALES.
3.- SE PUEDEN OBTENER SOL. PARTS. APROX.

EJEMPLO MÁS SENCILLO

$$\frac{\partial^2 z}{\partial y^2} - 4\frac{\partial z}{\partial x} = 0 \qquad z(x,y)$$

EDenDP(2) L. H.

H_0: $z(x,y) = F(x)\cdot G(y)$

$$\frac{\partial z}{\partial x} = F'(x)\cdot G(y) + F(x)\cdot(0) \Rightarrow F'(x)\cdot G(y)$$

$$\frac{\partial z}{\partial y} = F(x)\cdot G'(y)$$

$$\frac{\partial^2 z}{\partial y^2} = F(x)\cdot G''(y)$$

Sust. EenDP

$$\left[F(x)\cdot G''(y)\right] - 4\left[F'(x)\cdot G(y)\right] = 0$$

$$G''(y)\cdot F(x) = 4F'(x)G(y)$$

$$\boxed{\frac{G''(y)}{4G(y)} = \frac{F'(x)}{F(x)}}$$

Si logramos separar las variables.

H_1: $z(x,y) = \frac{F(x)}{G(y)}$ $\qquad$ H_2: $z(x,y) = F(x) + G(y)$

H_3: $z(x,y) = F(x)^y$ $\qquad$ H_4: $z(x,y) = G(y)^x$

H_5: $z(x,y) = F(x)\,\mathrm{Log}(y)$

$$\frac{G''(y)}{4G(y)} = \frac{F'(x)}{F(x)} = \alpha$$

$$\frac{G''(y)}{4G(y)} = \alpha \qquad \frac{F'(x)}{F(x)} = \alpha \qquad \alpha = \begin{cases} \alpha = 0 \\ \alpha > 0 \\ \alpha < 0 \end{cases}$$

$$G''(y) = 4\alpha G(y) \qquad F'(x) = \alpha F(x).$$

Para $\alpha = 0$

$$G''(y) = 0 \qquad F'(x) = 0$$

$$G'(y) = C_1 \qquad F(x) = k_1$$

$$G(y) = C_1 y + C_2$$

Sol GRAL

$$\hookrightarrow z(x,y) = k_1 (C_1 y + C_2)$$

$$z_g^{①}(x,y) = C_{10} y + C_{20} \qquad \begin{cases} \frac{\partial z}{\partial y} = C_{10} \\ \frac{\partial^2 z}{\partial y^2} = 0 \\ \frac{\partial z}{\partial x} = 0 \end{cases}$$

$$\frac{\partial^2 z}{\partial y^2} - 4\frac{\partial z}{\partial x} = 0$$

$$(0) - 4(0) = 0$$

$$0 = 0.$$

$$\frac{G''(y)}{4G(y)} = \frac{F'(x)}{F(x)} = \alpha$$

para $\alpha > 0 \longrightarrow \alpha = \beta^2 \quad \forall \beta \neq 0$

$$G''(y) = 4\beta^2 G(y) \qquad F'(x) = \beta^2 F(x)$$

$$G''(y) - 4\beta^2 G(y) = 0 \qquad F'(x) - \beta^2 F(x) = 0$$

$$(D^2 - 4\beta^2) G(y) = 0 \qquad (D - \beta^2) F(x) = 0$$

EDOL(2) cc H.

$$F(x) = k_1 e^{\beta^2 x}$$

$$(D - 2\beta)(D + 2\beta) G(y) = 0$$

$$G(y) = C_1 e^{2\beta y} + C_2 e^{-2\beta y}$$

SOLGRAL$_2$

$$Z_g^{②}(x,y) = k_1 e^{\beta^2 x}\left(C_1 e^{2\beta y} + C_2 e^{-2\beta y}\right)$$

$$\boxed{Z_g^{②}(x,y) = e^{\beta^2 x}\left(C_{10} e^{2\beta y} + C_{20} e^{-2\beta y}\right)}$$

$$\frac{\partial z}{\partial y} = e^{\beta^2 x}\left(2\beta C_{10} e^{2\beta y} - 2\beta C_{20} e^{-2\beta y}\right)$$

$$\frac{\partial^2 z}{\partial y^2} = e^{\beta^2 x}\left(4\beta^2 C_{10} e^{2\beta y} + 4\beta^2 C_{20} e^{-2\beta y}\right)$$

$$\frac{\partial z}{\partial x} = \beta^2 e^{\beta^2 x}\left(C_{10} e^{2\beta y} + C_{20} e^{-2\beta y}\right)$$

$$4\beta^2 e^{\beta^2 x}\left(C_{10} e^{2\beta y} + C_{20} e^{-2\beta y}\right) - 4\left(\beta^2 e^{\beta^2 x}\left(C_{10} e^{2\beta y} + C_{20} e^{-2\beta y}\right)\right) = 0$$

$$(0)\, e^{\beta^2 x}\left(C_{10} e^{2\beta y} + C_{20} e^{-2\beta y}\right) = 0$$

$$0 = 0$$

para $\alpha < 0$ $\quad \alpha = -\beta^2 \quad \forall \beta \neq 0$

$$F'(x) = -\beta^2 F(x)$$

$$F'(x) + \beta^2 F(x) = 0$$

$$F(x) = k_1 e^{-\beta^2 x}$$

$$G''(y) = -4\beta^2 G(y)$$

$$G''(y) + 4\beta^2 G(y) = 0$$

$$(D^2 + 4\beta^2) G(y) = 0$$

$$m^2 + 4\beta^2 = 0 \qquad m_1 = 2\beta i \qquad m_2 = -2\beta i$$

$$G(y) = C_1 \cos(2\beta y) + C_2 \operatorname{sen}(2\beta y).$$

Sol. GPDL$_3$

$$Z(x,y) = k_1 e^{-\beta^2 x}\left(C_1 \cos(2\beta y) + C_2 \operatorname{sen}(2\beta y)\right).$$

$$\boxed{Z_g^{(3)}(x,y) = e^{-\beta^2 x}\left(C_{10} \cos(2\beta y) + C_{20} \operatorname{sen}(2\beta y)\right).}$$

$$\frac{\partial^2 z}{\partial x^2} - 6\frac{\partial^2 z}{\partial x \partial y} + 8\frac{\partial z}{\partial y} = 0$$

H_0: $z = FG$

$$\frac{\partial z}{\partial x} = F'G \qquad \frac{\partial z}{\partial y} = FG'$$

$$\frac{\partial^2 z}{\partial x^2} = F''G \qquad \frac{\partial^2 z}{\partial x \partial y} = F'G'$$

$$F''G - 6F'G' + 8FG' = 0$$

H_1: $z = F + G$.

$$\frac{\partial z}{\partial x} = F' \qquad \frac{\partial z}{\partial y} = G'$$

$$\frac{\partial^2 z}{\partial x^2} = F'' \qquad \frac{\partial^2 z}{\partial x \partial y} = 0$$

$$F'' - 6(0) + 8G' = 0$$

SERIE TRIGONOMÉTRICA DE FOURIER

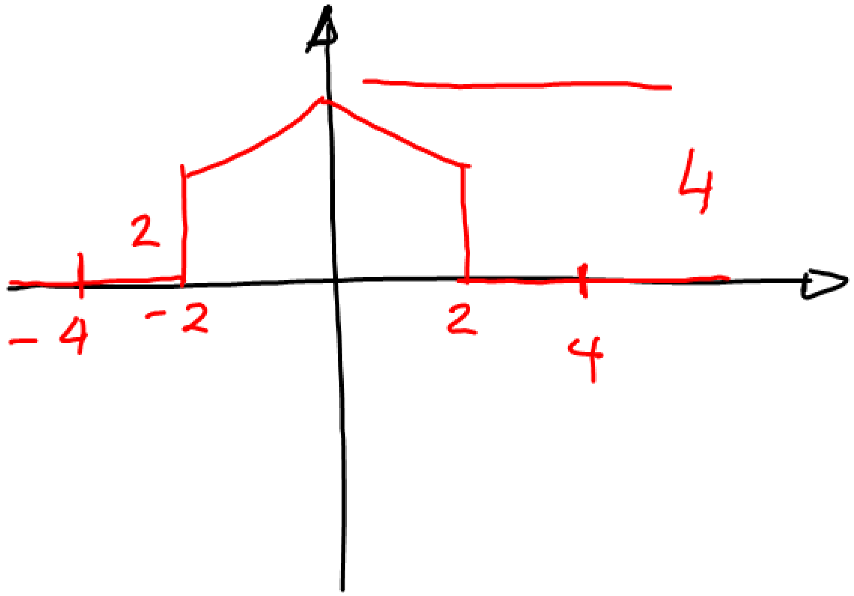
$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{L} + b_n \operatorname{sen} \frac{n\pi x}{L} \right)$$

$$x = (-L, L)$$

$$a_0 = \frac{1}{L} \int_{-L}^{L} f(x)\, dx$$

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos\left(\frac{n\pi}{L} x\right) dx$$

$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \operatorname{sen}\left(\frac{n\pi}{L} x\right) dx$$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