FACULTAD DE INGENIERÍA
ECUACIONES DIFERENCIALES
SEMESTRE 2019-2
SERIE 1 (CAPÍTULO I)

2019-02-28

> restart

5) OBTENER LA SOLUCIÓN GENERAL DE LA SIGUIENTE ECUACIÓN DIFERENCIAL NO LINEAL (sin usar dsolve o relativos)

$$y(x)\,\left(y(x)^2+2\,x^2\right)-2\,x\,\left(x^2+y(x)^2\right)\left(\frac{d}{dx}\,y(x)\right)=0 \tag{1}$$

> Ecua := y(x) (y(x)^2 + 2 x^2) − 2 x (x^2 + y(x)^2) (d/dx y(x)) = 0

$$Ecua := y(x)\,\left(y(x)^2+2\,x^2\right)-2\,x\,\left(x^2+y(x)^2\right)\left(\frac{d}{dx}\,y(x)\right)=0 \tag{2}$$

> with(DEtools)

[AreSimilar, Closure, DEnormal, DEplot, DEplot3d, DEplot_polygon, DFactor, DFactorLCLM, DFactorsols, Dchangevar, Desingularize, FunctionDecomposition, GCRD, Gosper, Heunsols, Homomorphisms, IVPsol, IsHyperexponential, LCLM, MeijerGsols, MultiplicativeDecomposition, ODEInvariants, PDEchangecoords, PolynomialNormalForm, RationalCanonicalForm, ReduceHyperexp, RiemannPsols, Xchange, Xcommutator, Xgauge, Zeilberger, abelsol, adjoint, autonomous, bernoullisol, buildsol, buildsym, canoni, caseplot, casesplit, checkrank, chinisol, clairautsol, constcoeffsols, convertAlg, convertsys, dalembertsol, dcoeffs, de2diffop, dfieldplot, diff_table, diffop2de, dperiodic_sols, dpolyform, dsubs, eigenring, endomorphism_charpoly, equinv, eta_k, eulersols, exactsol, expsols, exterior_power, firint, firtest, formal_sol, gen_exp, generate_ic, genhomosol, gensys, hamilton_eqs, hypergeomsols, hyperode, indicialeq, infgen, initialdata, integrate_sols, intfactor, invariants, kovacicsols, leftdivision, liesol, line_int, linearsol, matrixDE, matrix_riccati, maxdimsystems, moser_reduce, muchange, mult, mutest, newton_polygon, normalG2, ode_int_y, ode_y1, odeadvisor, odepde, parametricsol, particularsol, phaseportrait, poincare, polysols, power_equivalent, rational_equivalent, ratsols, redode, reduceOrder, reduce_order, regular_parts, regularsp, remove_RootOf, riccati_system, riccatisol, rifread, rifsimp, rightdivision, rtaylor, separablesol, singularities, solve_group, super_reduce, symgen, symmetric_power, symmetric_product, symtest, transinv, translate, untranslate, varparam, zoom] **(3)**

> odeadvisor(Ecua)

[[_homogeneous, class A], _rational, _dAlembert] **(4)**

> EcuaDos := isolate(simplify(eval(subs(y(x) = v(x)·x, Ecua))), diff(v(x), x))

$$EcuaDos := \frac{d}{dx}\,v(x) = -\frac{v(x)^3}{2\,v(x)^2\,x+2\,x} \tag{5}$$

> EcuaTres := lhs(EcuaDos) − rhs(EcuaDos) = 0

$$EcuaTres := \frac{d}{dx}\,v(x) + \frac{v(x)^3}{2\,v(x)^2\,x+2\,x} = 0 \tag{6}$$

> $MM := v^{\cdot\cdot}3$

$$MM := v^3 \quad (7)$$

> $NN := factor(2\,v^2\,x + 2\,x)$

$$NN := 2\,x\,(v^2 + 1) \quad (8)$$

> $P := 1; Q := v^{\cdot\cdot}3; R := 2\,x; S := v^{\cdot\cdot}2 + 1$

$$P := 1$$
$$Q := v^3$$
$$R := 2\,x$$
$$S := v^2 + 1 \quad (9)$$

> $SolGral := int\left(\frac{P}{R}, x\right) + int\left(\frac{S}{Q}, v\right) = C$

$$SolGral := \frac{1}{2}\ln(x) + \ln(v) - \frac{1}{2\,v^2} = C \quad (10)$$

> $SolGralDos := subs\left(v = \frac{y(x)}{x}, SolGral\right)$

$$SolGralDos := \frac{1}{2}\ln(x) + \ln\left(\frac{y(x)}{x}\right) - \frac{1}{2}\,\frac{x^2}{y(x)^2} = C \quad (11)$$

> $SolGralTres := genhomosol(Ecua)$

$$SolGralTres := \left\{y(x) = \sqrt{\frac{1}{\mathrm{LambertW}(\_C1\,x)}}\,x\right\} \quad (12)$$

> $SolGralCuatro := isolate(SolGralDos, y(x))$

$$SolGralCuatro := y(x) = \sqrt{\frac{1}{\mathrm{LambertW}(\mathrm{e}^{-2\,C}\,x)}}\,x \quad (13)$$

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ECUACION DIFERENCIAL LINEAL DE SEGUNDO ORDEN

> $Ecua := diff(y(x), x\$2) + 3\cdot diff(y(x), x) - 4\cdot y(x) = 0$

$$Ecua := \frac{d^2}{dx^2}\,y(x) + 3\left(\frac{d}{dx}\,y(x)\right) - 4\,y(x) = 0 \quad (14)$$

> $EcuaCarac := m^{\cdot\cdot}2 + 3\cdot m - 4 = 0$

$$EcuaCarac := m^2 + 3\,m - 4 = 0 \quad (15)$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := 1, -4 \quad (16)$$

> $SolGral := y(x) = C[1]\cdot\exp(Raiz[1]\cdot x) + C[2]\cdot\exp(Raiz[2]\cdot x)$

$$SolGral := y(x) = C_1\,\mathrm{e}^{x} + C_2\,\mathrm{e}^{-4\,x} \quad (17)$$

> $SolGralDos := dsolve(Ecua)$

$$SolGralDos := y(x) = \_C1\,\mathrm{e}^{-4\,x} + \_C2\,\mathrm{e}^{x} \quad (18)$$

> $ComprobacionUno := eval(subs(y(x) = rhs(SolGral), Ecua))$

(19)

$$ComprobacionUno := 0 = 0 \quad (19)$$

> $ComprobacionDos := eval(subs(y(x) = rhs(SolGralDos), Ecua))$

$$ComprobacionDos := 0 = 0 \quad (20)$$

> $restart$

> $Ecua := diff(y(t), t\$2) - y(t) = 0$

$$Ecua := \frac{d^2}{dt^2}\, y(t) - y(t) = 0 \quad (21)$$

> $Sol := dsolve(Ecua)$

$$Sol := y(t) = \_C1\, e^{-t} + \_C2\, e^{t} \quad (22)$$

> $m[1] := 1;\, m[2] := -1$

$$m_1 := 1$$

$$m_2 := -1 \quad (23)$$

> $EcuaCarac := expand((m - m[1]) \cdot (m - m[2])) = 0$

$$EcuaCarac := m^2 - 1 = 0 \quad (24)$$

> $DerSol := diff(Sol, t)$

$$DerSol := \frac{d}{dt}\, y(t) = -\_C1\, e^{-t} + \_C2\, e^{t} \quad (25)$$

> $DerSolDos := diff(Sol, t\$2)$

$$DerSolDos := \frac{d^2}{dt^2}\, y(t) = \_C1\, e^{-t} + \_C2\, e^{t} \quad (26)$$

> $Parametros := solve(\{DerSol, DerSolDos\}, \{\_C1, \_C2\})$

$$Parametros := \left\{ \_C1 = -\frac{1}{2}\, \frac{-\left(\frac{d^2}{dt^2}\, y(t)\right) + \frac{d}{dt}\, y(t)}{e^{-t}},\ \_C2 = \frac{1}{2}\, \frac{\frac{d^2}{dt^2}\, y(t) + \frac{d}{dt}\, y(t)}{e^{t}} \right\} \quad (27)$$

> $EcuaTres := subs(\_C1 = rhs(Parametros[1]), \_C2 = rhs(Parametros[2]), Sol)$

$$EcuaTres := y(t) = \frac{d^2}{dt^2}\, y(t) \quad (28)$$

> $EcuaFinal := rhs(EcuaTres) - lhs(EcuaTres) = 0$

$$EcuaFinal := \frac{d^2}{dt^2}\, y(t) - y(t) = 0 \quad (29)$$

> $SolFinal := dsolve(EcuaFinal)$

$$SolFinal := y(t) = \_C1\, e^{t} + \_C2\, e^{-t} \quad (30)$$

> $restart$

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PROBLEMA DE LA BALA

> $Ecua := diff(v(t), t) = -K \cdot v(t) \cdot \cdot 2$

$$Ecua := \frac{d}{dt}\, v(t) = -K\, v(t)^2 \quad (31)$$

> $SolGral := dsolve(Ecua)$

$$SolGral := v(t) = \frac{1}{K\,t + \_C1} \quad \textbf{(32)}$$

> $SolPart := dsolve(\{Ecua, v(0) = 200\})$

$$SolPart := v(t) = \frac{200}{200\,K\,t + 1} \quad \textbf{(33)}$$

> $EcuaDos := diff(x(t), t) = rhs(SolPart)$

$$EcuaDos := \frac{d}{dt}\,x(t) = \frac{200}{200\,K\,t + 1} \quad \textbf{(34)}$$

> $SolGralDos := dsolve(EcuaDos)$

$$SolGralDos := x(t) = \frac{\ln(200\,K\,t + 1)}{K} + \_C1 \quad \textbf{(35)}$$

> $SolPartDos := dsolve(\{EcuaDos, x(0) = 0\})$

$$SolPartDos := x(t) = \frac{\ln(200\,K\,t + 1)}{K} \quad \textbf{(36)}$$

> $Tiempo := t = solve\left(rhs(SolPartDos) = \frac{1}{10}, t\right)$

$$Tiempo := t = \frac{1}{200}\,\frac{e^{\frac{1}{10}\,K} - 1}{K} \quad \textbf{(37)}$$

> $Parametro := K = solve(rhs(SolPart) = 80, K)$

$$Parametro := K = \frac{3}{400\,t} \quad \textbf{(38)}$$

> $TiempoFinal := isolate(subs(K = rhs(Parametro), Tiempo), t); evalf(\%, 4)$

$$TiempoFinal := t = \frac{3}{4000\,\ln\left(\frac{5}{2}\right)}$$

$$t = 0.0008182 \quad \textbf{(39)}$$

> $ParametroFinal := subs(t = rhs(TiempoFinal), Parametro); evalf(\%, 4)$

$$ParametroFinal := K = 10\,\ln\left(\frac{5}{2}\right)$$

$$K = 9.163 \quad \textbf{(40)}$$

> $SolVelFinal := subs(K = rhs(ParametroFinal), SolPart); evalf(\%, 4)$

$$SolVelFinal := v(t) = \frac{200}{2000\,\ln\left(\frac{5}{2}\right)\,t + 1}$$

$$v(t) = \frac{200.}{1833.\,t + 1.} \quad \textbf{(41)}$$

> $SolDesplFinal := subs(K = rhs(ParametroFinal), SolPartDos); evalf(\%, 4)$

$$SolDesplFinal := x(t) = \frac{1}{10}\,\frac{\ln\left(2000\,\ln\left(\frac{5}{2}\right)\,t + 1\right)}{\ln\left(\frac{5}{2}\right)}$$

$$x(t) = 0.1091 \ln(1833.\, t + 1.) \tag{42}$$

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