

PROBLEMA DE LA BALA.

$t = ?$ $200 \frac{m}{s}$ $80 \frac{m}{s}$ Material
 $\frac{dv}{dt} = -k v^2$
 $U_0 = 200; t_0 = 0$ 0.10 m $\frac{dv}{v^2} = -k dt$
 $V = \frac{1}{kt - C}$ $X_0 = 0$ $\int \frac{dv}{v^2} = -k \int dt + C$
 $200 = \frac{1}{-C}$ $\int \frac{1}{v^2} dv = -k t + C$
 $C = -\frac{1}{200}$ $\frac{v^{-1}}{-1} = -k t + C$
 $V = \frac{1}{kt + \frac{1}{200}}$ (SP) $\frac{1}{v} = kt - C$
 $V_f = 80 \frac{m}{s}; t_f = ?$ $V = \frac{1}{kt - C}$ (Sg)

$\frac{dx}{dt} = \frac{1}{kt + \frac{1}{200}}$ $\int dx = \int \frac{dt}{kt + \frac{1}{200}}$
 $\frac{dx}{dt} = \frac{1}{kt + \frac{1}{200}}$ $\int dx = \int \frac{dt}{kt + \frac{1}{200}}$
 $t_f = \frac{e^{\frac{1}{200k}} - 1}{\frac{1}{200k}}$ $X = \frac{1}{k} \ln \left(\frac{kt + \frac{1}{200}}{\frac{1}{200}} \right) + C_2$
 $V = \frac{1}{kt + \frac{1}{200}}$ $X = \frac{1}{k} \ln \left(\frac{kt + \frac{1}{200}}{\frac{1}{200}} \right) + C_2$
 $kt + \frac{1}{200} = \frac{1}{V}$ $0 = \frac{1}{k} \ln \left(\frac{\frac{1}{200}}{\frac{1}{200}} \right) + C_2$
 $kt = \frac{1}{V} - \frac{1}{200}$ $C_2 = -\frac{1}{k} \ln \left(\frac{1}{200} \right)$
 $k = \frac{\frac{1}{V} - \frac{1}{200}}{t}$ $X = \frac{1}{k} \ln \left(\frac{kt + \frac{1}{200}}{\frac{1}{200}} \right) - \frac{1}{k} \ln \left(\frac{1}{200} \right)$
 $t_f = \frac{e^{\left(\frac{1}{80} - \frac{1}{200} \right)} - 1}{\frac{1}{200} \left(\frac{1}{80} - \frac{1}{200} \right)}$ $kx = \ln \left(\frac{kt + \frac{1}{200}}{\frac{1}{200}} \right)$
 $t_f \left(200 \left(\frac{1}{80} - \frac{1}{200} \right) \right) = \frac{kt + \frac{1}{200}}{\frac{1}{200}} - 1$ $\frac{kt + \frac{1}{200}}{\frac{1}{200}} = e^{kx}$
 $e^{\left(\frac{1}{80} - \frac{1}{200} \right) t_f} - 1 = \frac{kt + \frac{1}{200}}{\frac{1}{200}} - 1$ $kt + \frac{1}{200} = \frac{e^{kx}}{200}$
 $200 \left(\frac{1}{80} - \frac{1}{200} \right) + 1 = e^{\left(\frac{1}{80} - \frac{1}{200} \right) t_f}$ $200kt = e^{kx} - 1$
 $\frac{5}{2} = e^{\left(\frac{1}{80} - \frac{1}{200} \right) t_f}$ $t = \frac{e^{kx} - 1}{200k}$
 $\frac{1}{80} - \frac{1}{200} = \ln \left(\frac{5}{2} \right)$ $a = e^b$
 $\frac{1}{10} t_f = \frac{\ln \left(\frac{5}{2} \right)}{\frac{1}{80} - \frac{1}{200}}$ $b = \ln(a)$
 $t_f = \frac{\ln \left(\frac{5}{2} \right)}{\frac{1}{80} - \frac{1}{200}}$ $t = 0.0008852 \text{ [s]}$
 $t_f = 8.852 \times 10^{-4}$

$$k = \frac{\frac{1}{80} - \frac{1}{200}}{t_f} \Rightarrow k = 9.16$$

$$V = \frac{1}{9.16t + \frac{1}{200}} \quad X = \frac{1}{9.16} \ln \left(\frac{9.16t + \frac{1}{200}}{\frac{1}{200}} \right)$$

$$t_f = \frac{e^{\frac{9.16}{200}} - 1}{200(9.16)} = 2.86 \times 10^{-3}$$

$$V = 32 \frac{m}{s}$$

CAPÍTULO 2. EDO(n) LINEAL

$$\frac{dy}{dx} + p(x)y = q(x) \quad \text{EDO(1) L. C. v. NH}$$

$$y(x) = C e^{-\int p(x) dx} + e^{-\int p(x) dx} \int e^{\int p(x) dx} q(x) dx$$

$$y_{g/NH} = y_{g/H} + y_{p/q}$$

EDO(2) LCCH.

$$\frac{d^2 y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0$$

$$H_0: y = e^{mx} \quad \frac{dy}{dx} = m e^{mx} \quad \frac{d^2 y}{dx^2} = m^2 e^{mx}$$

$$[m^2 e^{mx}] + a_1 [m e^{mx}] + a_2 [e^{mx}] = 0$$

$$(m^2 + a_1 m + a_2) e^{mx} = 0$$

$$H_1: y = 0 \text{ trivial}$$

$$m_1, m_2 \left\{ \begin{array}{l} e^{mx} = 0 \\ m^2 + a_1 m + a_2 = 0 \end{array} \right. \quad \swarrow$$

Ecuación CARACTERÍSTICA

$$\frac{d^2 y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0 \quad \text{EDO(2) LCCH.}$$

$$y = c_1 y_1 + c_2 y_2$$

$$y = e^{mx}$$

y_1, y_2 SOLUCIONES PARTICULARES
FUNDAMENTALES.

$$y_1 = e^{m_1 x}$$

$$y_2 = e^{m_2 x}$$

$$\frac{d^2 y}{dx^2} - 5 \frac{dy}{dx} + 6y = 0$$

$$H_0 \quad y = e^{mx} \rightarrow m^2 - 5m + 6 = 0$$

$$y_g = c_1 e^{2x} + c_2 e^{3x} \quad (m-3)(m-2) = 0$$

$$\left. \begin{array}{l} m_1 = 2 \\ m_2 = 3 \end{array} \right\} m_1 \neq m_2 \in \mathbb{R}$$

$$\checkmark y_1 = e^{2x} \quad \frac{dy}{dx} = 2e^{2x} \quad \frac{d^2 y}{dx^2} = 4e^{2x}$$

$$[4e^{2x}] - 5[2e^{2x}] + 6[e^{2x}] = 0$$

$$(10e^{2x} - 10e^{2x}) = 0$$

$$0 \equiv 0$$

$$\checkmark y_2 = e^{3x} \quad \frac{dy}{dx} = 3e^{3x} \quad \frac{d^2 y}{dx^2} = 9e^{3x}$$

$$[9e^{3x}] - 5[3e^{3x}] + 6[e^{3x}] = 0$$

$$[15e^{3x} - 15e^{3x}] = 0$$

$$0 \equiv 0$$

Sol gral $y = C_1 e^x + C_2 e^{-x}$ EDO(2) L: H

$$m_1 = 1 \quad m_2 = -1$$

$$(m-1)(m+1) = 0$$

$$m^2 - 1 = 0 \quad \text{Ecuación Característica.}$$

$$\frac{d^2 y}{dx^2} - y = 0$$

