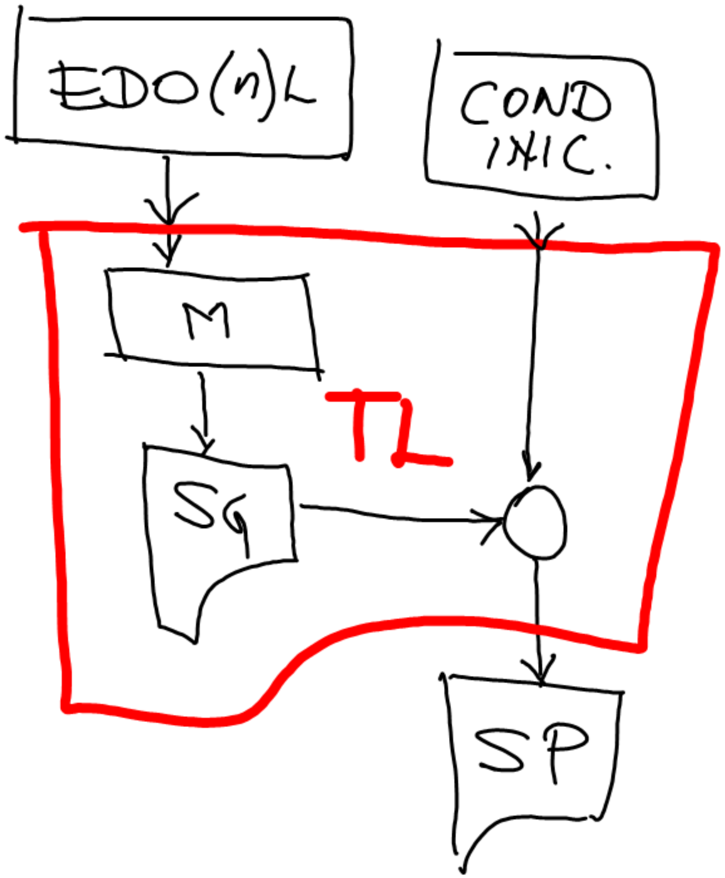
CAPÍTULO III: Transformada de Laplace y sistemas de Ecuaciones Dif.

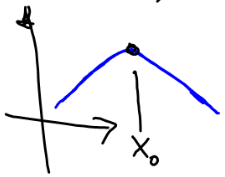


Cálculo diferencial integral clásico.

función será derivable

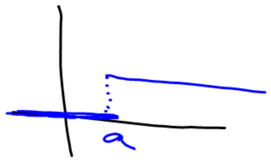
a) Si es continua desde $-\infty < x < \infty$

b) si en punto dado



$$\lim_{x \to x_0^+} f(x) = \lim_{x \to x_0^-} f(x)$$

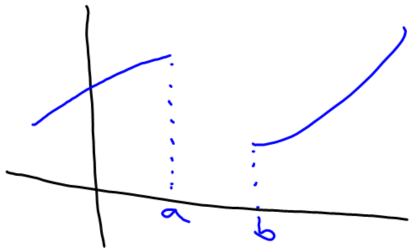
$u(t-a)$



función interruptor (switch).

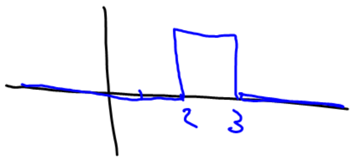
$r(t-a)$

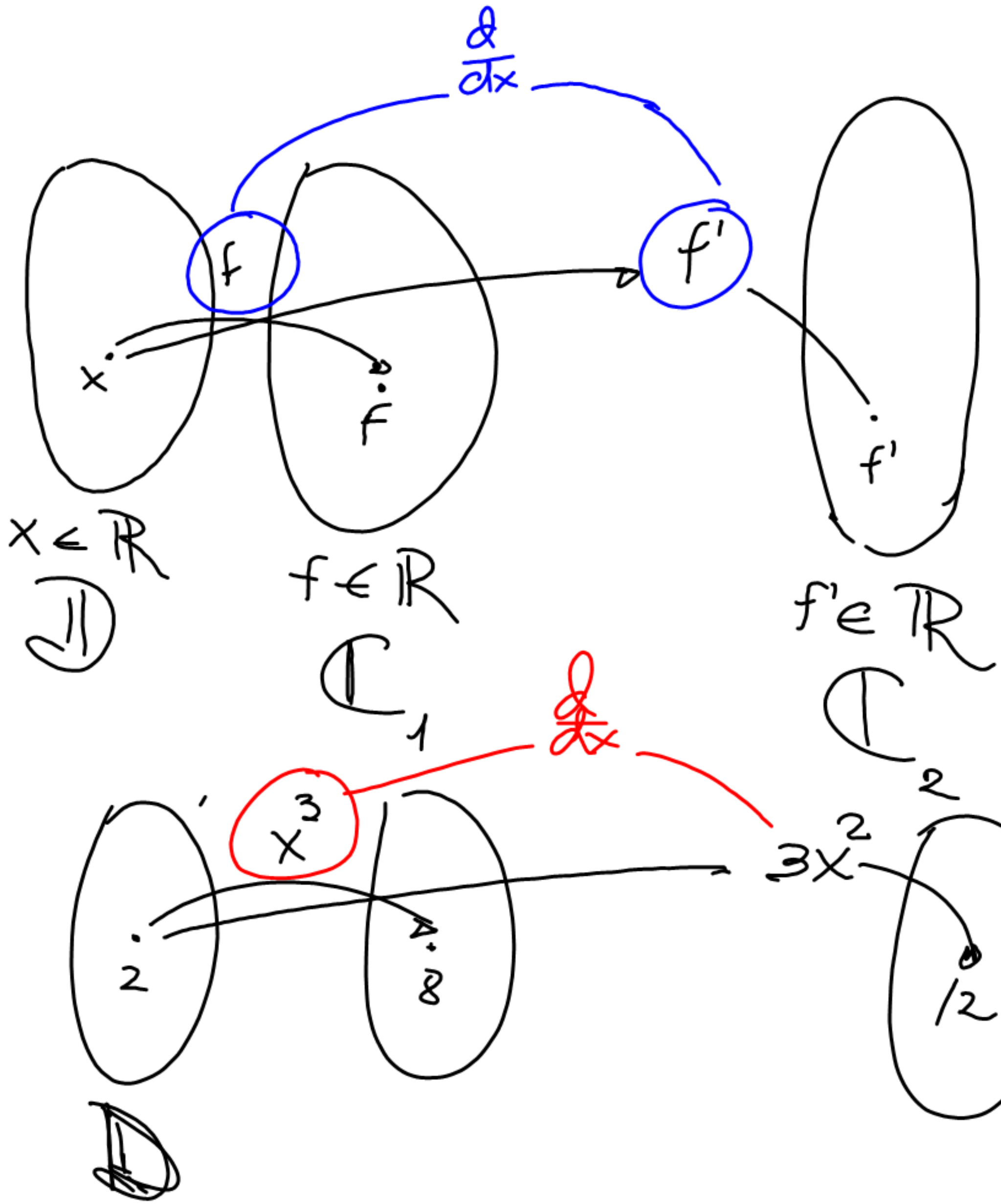
$$\frac{d}{dt} r(t-a) = u(t-a)$$



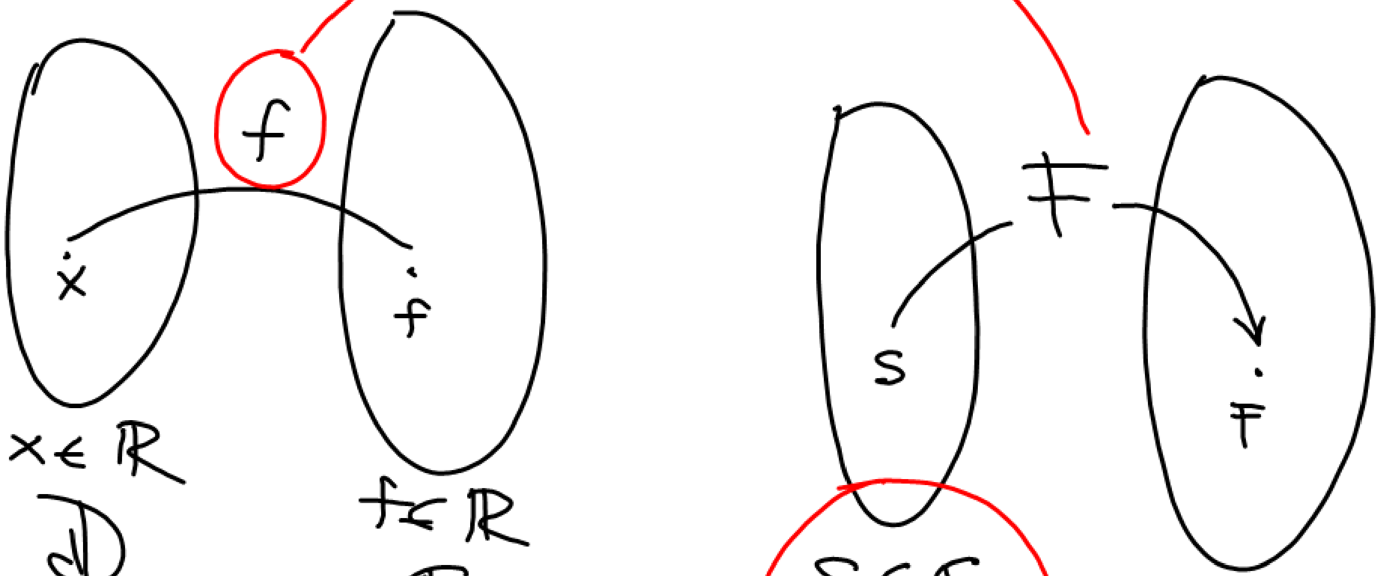
$f = u(t-2) - u(t-3)$

Heaviside.




d/dx
f
f'
x
f
f'
x ∈ ℝ
D
f ∈ ℝ
C1
f' ∈ ℝ
C2
d/dx
x3
3x2
2
8
12
D

$T\{f\} = F$



$x \in \mathbb{R}$ $\quad f \in \mathbb{R}$ $\quad s \in \mathbb{C}$ $\quad F \in \mathbb{R}$

$\mathbb{D}$ $\quad \mathbb{C}$ $\quad \mathbb{D}$ $\quad \mathbb{C}$

$T\{af + bg\} \longrightarrow aF + bG$

$a, b \in \mathbb{R}$

$f, g \in \mathbb{R}$

$t \in \mathbb{R}$

$f \in \mathbb{R}$

$$T\{f(t)\} = \int_{-\infty}^{\infty} f(t) \cdot N(s,t)\,dt.$$

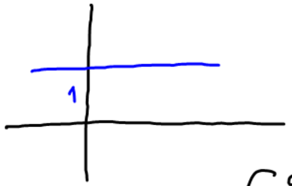
núcleo de Transformación

Laplace

$$= F(s) \qquad s \in \mathbb{C}, \quad F \in \mathbb{R}$$

$$N(s,t) = \begin{cases} 0 & ; t \leq 0 \\ e^{-st} & ; t > 0 \end{cases}$$

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t)\,dt \ni F(s)$$



$$\mathcal{L}\{1\} = \int_0^\infty e^{-st}\cdot(1)\cdot dt$$

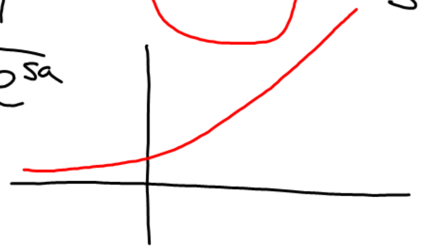
$$= \left[\int e^{-st}\,dt\right]_0^\infty$$

$$= \left[-\frac{1}{s}e^{-st}\right]_0^\infty = \lim_{a\to\infty}\frac{e^{-sa}}{-s} - \left(-\frac{1}{s}\right)$$

$$= -\frac{1}{s}\overset{0}{\lim_{a\to\infty} e^{-sa}} + \frac{1}{s}$$

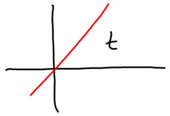
$$\lim_{a\to\infty} e^{-sa} = \lim_{a\to\infty}\frac{1}{e^{sa}}$$

$$\lim_{a\to\infty} e^{sa}\to\infty$$



$$\lim_{a\to\infty} e^{-sa} = \lim_{b\to\infty}\frac{1}{b} = 0$$

$$F(s) = \mathcal{L}\{1\} = \frac{1}{s} \quad \left\}\begin{array}{l} f\in\mathbb{R} \\ s\in\mathbb{C} \end{array}\right.$$



$$\int u\,dv = u\cdot v - \int v\,du$$

$$\mathcal{L}\{t\} = \int_0^\infty e^{-st}\cdot(t)\cdot dt$$

$$= \left[\int t e^{-st}\,dt\right]_0^\infty$$

$$\int t e^{-st}\,dt = \frac{te^{-st}}{-s} - \int \frac{e^{-st}}{-s}\,dt$$

$$\boxed{\begin{array}{ll} u = t & dv = e^{-st}dt \\ du = dt & v = \dfrac{e^{-st}}{-s} \end{array}}$$

$$\int t e^{-st}\,dt = \frac{te^{-st}}{-s} + \frac{1}{s}\int e^{-st}\,dt$$

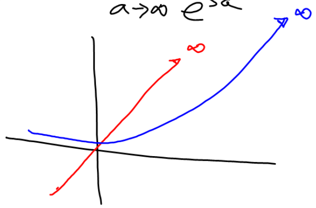
$$= -\frac{te^{-st}}{s} + \frac{1}{s}\left(\frac{e^{-st}}{-s}\right)$$

$$\int t e^{-st}\,dt = -\frac{te^{-st}}{s} - \frac{e^{-st}}{s^2}$$

$$\mathcal{L}\{t\} = \left[-\frac{te^{-st}}{s} - \frac{e^{-st}}{s^2}\right]_0^\infty$$

$$= -\frac{1}{s}\left(\cancel{\lim_{a\to\infty} a\cdot e^{-sa}} - \cancel{(0)e^{-s(0)}}\right) - \frac{1}{s^2}\left(\cancel{\lim_{a\to\infty} e^{-sa}} - (1)e^{-s(0)}\right)$$

$$\lim_{a\to\infty} a\cdot e^{-sa} = \lim_{a\to\infty} \frac{a}{e^{sa}} = 0$$



$$\mathcal{L}\{t\} = \frac{1}{s^2}$$

$$\mathcal{L}\{e^{at}\} = \int_0^\infty e^{-st} e^{at}\,dt \qquad \mathcal{L}\{1\} = \frac{1}{s} \quad \mathcal{L}\{t\} = \frac{1}{s^2}$$

$$= \int_0^\infty e^{-(s-a)t}\,dt \qquad e^{at} \quad t \in \mathbb{R} \qquad e^{at} \in \mathbb{R}$$

$$= \left[\frac{e^{-(s-a)t}}{-(s-a)} \right]_0^\infty$$

$$= \frac{1}{-(s-a)} \lim_{b \to \infty} \frac{1}{e^{(s-a)b}} \to 0 \; - \left(\frac{1}{-(s-a)} \right)$$

$$\left[\begin{array}{l} \mathcal{L}\{e^{at}\} = \dfrac{1}{(s-a)} \\ \mathcal{L}\{1\} = \dfrac{1}{s} \\ \mathcal{L}\{t\} = \dfrac{1}{s^2} \\ \mathcal{L}\{\cos(bt)\} = \dfrac{s}{s^2+b^2} \\ \mathcal{L}\{\operatorname{sen}(bt)\} = \dfrac{b}{s^2+b^2} \end{array}\right.$$

$$\mathcal{L}\{af+bg\} = aF + bG$$

$$\mathcal{L}\{f(t)e^{at}\} = F(s-a)$$

$$\mathcal{L}\{1\} = \frac{1}{s} \qquad \mathcal{L}\{1 \cdot e^{at}\} = \frac{1}{(s-a)}$$

$$\mathcal{L}\{t\} = \frac{1}{s^2} \qquad \mathcal{L}\{te^{at}\} = \frac{1}{(s-a)^2}$$

$$\mathcal{L}\{\cos(bt)\} = \frac{s}{s^2+b^2} \qquad \mathcal{L}\{e^{at}\cos(bt)\} = \frac{s-a}{(s-a)^2+b^2}$$

$$\mathcal{L}\{\text{sen}(bt)\} = \frac{b}{s^2+b^2} \qquad \mathcal{L}\{e^{at}\text{sen}(bt)\} = \frac{b}{(s-a)^2+b^2}$$

$$\mathcal{L}^{-1}\{F(s)\} = f(t)$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+2s+2}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{(s^2+2s)+2}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{s}{(s^2+2s+1)+2-1}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{s}{(s+1)^2+1}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{(s+1)-1}{(s+1)^2+1}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{(s+1)}{(s+1)^2+1}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+1}\right\}$$

$$= e^{-t}\cos(t) - e^{-t}\cdot \operatorname{sen}(t)$$

$$\mathcal{L}\left\{\frac{d}{dt}f(t)\right\} = s\mathcal{L}\{f\} - f(0)$$

$$\mathcal{L}\left\{\frac{d^2}{dt^2}f(t)\right\} = s^2\mathcal{L}\{f\} - sf(0) - f'(0)$$

$$\mathcal{L}\left\{\frac{d^n}{dt^n}f(t)\right\} = s^n\mathcal{L}\{f\} - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - f^{(n-1)}(0).$$

$$\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y = 2e^{4t} \qquad y(0) = 6 \quad y'(0) = -8$$

$$\mathcal{L}\left\{\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y\right\} = \mathcal{L}\{2e^{4t}\}.$$

$$\mathcal{L}\left\{\frac{d^2y}{dt^2}\right\} - 5\mathcal{L}\left\{\frac{dy}{dt}\right\} + 6\mathcal{L}\{y\} = 2\mathcal{L}\{e^{4t}\}$$

$$\left[s^2\mathcal{L}\{y\} - s(6) - (-8)\right] - 5\left[s\mathcal{L}\{y\} - (6)\right] + 6\mathcal{L}\{y\} = 2\left(\frac{1}{s-4}\right).$$

$$(s^2 - 5s + 6)\mathcal{L}\{y\} + (-6s + 38) = \frac{2}{(s-4)}$$

$$(s^2 - 5s + 6)\mathcal{L}\{y\} = \frac{2}{(s-4)} + 6s - 38$$

$$= \frac{2 + (6s-38)(s-4)}{(s-4)}$$

$$= \frac{6s^2 - 62s + 154}{(s-4)}$$

$$\mathcal{L}\{y\} = \frac{6s^2 - 62s + 154}{(s^2 - 5s + 6)(s-4)}$$

$$\mathcal{L}\{y\} = \frac{6s^2 - 62s + 154}{(s-3)(s-2)(s-4)}$$ fracciones racionales.

$$\mathcal{L}\{y\} = \frac{A}{s-3} + \frac{B}{s-2} + \frac{D}{s-4}.$$

$$y = A\mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\} + B\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} + D\mathcal{L}^{-1}\left\{\frac{1}{s-4}\right\}$$

$$y = Ae^{3t} + Be^{2t} + De^{4t}$$