TRANSFORMADA DE LAPLACE.

$$F(s)=\mathcal{L}\{f(t)\}=\int_0^{\infty} e^{-st} f(t)\,dt$$

operador

nucleo

argumento.

$F \in \mathbb{R}$
$s \in \mathbb{C}$

$t, f \in \mathbb{R}$

$t\in\mathbb{R}$ $\quad f\in\mathbb{R}$ $\qquad s\in\mathbb{C}$ $\quad F\in\mathbb{R}$

$$af+bg \rightarrow aF+bG.$$

$$\mathcal{L}\left\{\frac{d}{dt}f\right\} \longrightarrow sF-f(0)$$

$$\mathcal{L}\left\{\int f\,dt\right\} \longleftarrow \frac{F}{s}$$

$$\mathcal{L}\{f*g\} \longleftarrow F\cdot G$$

$$f*g=\int_0^t f(\tau)\cdot g(t-\tau)\,d\tau$$

$$f(t)=\mathcal{L}^{-1}\{F(s)\}=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{st}F(s)\,ds$$

(1) $\mathcal{L}\{af(t)+bg(t)\} = aF(s) + bG(s)$

(2) $\mathcal{L}\{f(at)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$

$$\mathcal{L}\{\cos(t)\} = \frac{s}{s^2+1^2}$$

$$\mathcal{L}\{\cos(2t)\} = \frac{1}{2}\left(\frac{\frac{s}{2}}{\left(\frac{s}{2}\right)^2+1^2}\right)$$

$$= \frac{\frac{s}{4}}{\left(\frac{s^2}{4}\right)+1^2}$$

$$\mathcal{L}\{\cos(2t)\} = \frac{\frac{s}{4}}{\frac{s^2+4}{4}} \Rightarrow \frac{s}{s^2+4}$$

$$\mathcal{L}\{\cos(bt)\} = \frac{s}{s^2+b^2}$$

$$\mathcal{L}\{e^t\} = \frac{1}{s-1}$$

$$\mathcal{L}\{e^{3t}\} = \frac{1}{3}\left(\frac{1}{\frac{s}{3}-1}\right)$$

$$\mathcal{L}\{e^{3t}\} = \frac{\frac{1}{3}}{\frac{s-3}{3}} = \frac{1}{s-3}$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$$

3

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

$$\mathcal{L}\{f^{(n)}(t)\} = s^nF(s) - \Big(\underbrace{s^{n-1}f(0) + s^{n-2}f'(0) + \ldots + f^{(n-1)}(0)}_{\text{"n" términos}}\Big)$$

$$\mathcal{L}\{f'''(t)\} = s^3F(s) - \underbrace{s^2f(0) - sf'(0) - f''(0)}_{3\text{ términos}}$$

4

$$\mathcal{L}^{-1}\{F'(s)\} = -t\,f(t)$$

$$\mathcal{L}^{-1}\{F^{(n)}(s)\} = (-1)^n t^n f(t)$$

$$\boxed{\begin{aligned} F(s) &= \frac{1}{s-4} \\ \mathcal{L}^{-1}\{F(s)\} &= e^{4t} \end{aligned}}$$

$$\frac{d}{ds}F = \frac{d}{ds}\Big((s-4)^{-1}\Big)$$

$$= -(s-4)^{-2}$$

$$= -\frac{1}{(s-4)^2}$$

$$\mathcal{L}^{-1}\left\{-\frac{1}{(s-4)^2}\right\} = -(-1)^1\,te^{4t}$$

$$= te^{4t}$$

5 $\mathcal{L}\left\{\int_0^t f(\tau)\,d\tau\right\} = \dfrac{F(s)}{s}$

6 $\mathcal{L}^{-1}\left\{\int_s^\infty F(\sigma)\,d\sigma\right\} = \dfrac{f(t)}{t}$

7 $\mathcal{L}\left\{f(t)e^{at}\right\} = F(s-a)$

8 $\mathcal{L}^{-1}\left\{e^{-sa}F(s)\right\} = f(t-a)\cdot u(t-a)$

(9) $\mathcal{L}^{-1}\{F(s)\cdot G(s)\} = f(t) * g(t)$

operador convolución

$$f(t) * g(t) = \int_0^t f(\tau)\cdot g(t-\tau)\,d\tau.$$

Teorema de existencia de la Transformada de Laplace.

Para que una función $f(t)$ tenga una Transformada $F(s)$ debe ser de clase "A".

Para que una $f(t)$ sea de clase "A"

a) ser de orden exponencial

$$|f(t)| \le M e^{At}$$

e^{t^2}

b) sea seccionalmente continua

Heaviside

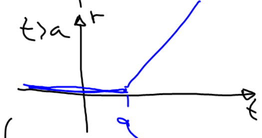
$$u(t-a) = \begin{cases} 0 & t<a \\ 1 & t>a \end{cases}$$

$u(t-a)$

función escalón unitario

$$r(t-a) = \begin{cases} 0 & t<a \\ t & t>a \end{cases}$$

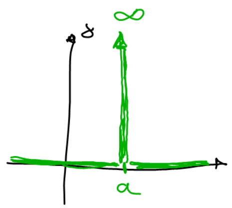
t * Heaviside.



$r(t-a)$

rampa unitaria

$$\delta(t-a) \begin{cases} 0 & t\neq a \\ \int_{-\infty}^{\infty} \delta\, dt = 1. \end{cases}$$



$$\delta(t-a) = \frac{d}{dt}\left(u(t-a)\right)$$

Dirac

$$\mathcal{L}\{r(t-5)\} = \frac{e^{-5s}}{s^2}$$

$$\mathcal{L}\{u(t-5)\} = \frac{e^{-5s}}{s}$$

$$\mathcal{L}\{u'(t-5)\} = s\mathcal{L}\{u(t-5)\} - u|_0$$

$$\mathcal{L}\{u'(t-5)\} = s\left[\frac{e^{-5s}}{s}\right] - (0)$$

$$\mathcal{L}\{u'(t-5)\} = e^{-5s}$$

$$\mathcal{L}\{u'(t-5)\} = \mathcal{L}\{\delta(t-5)\}$$

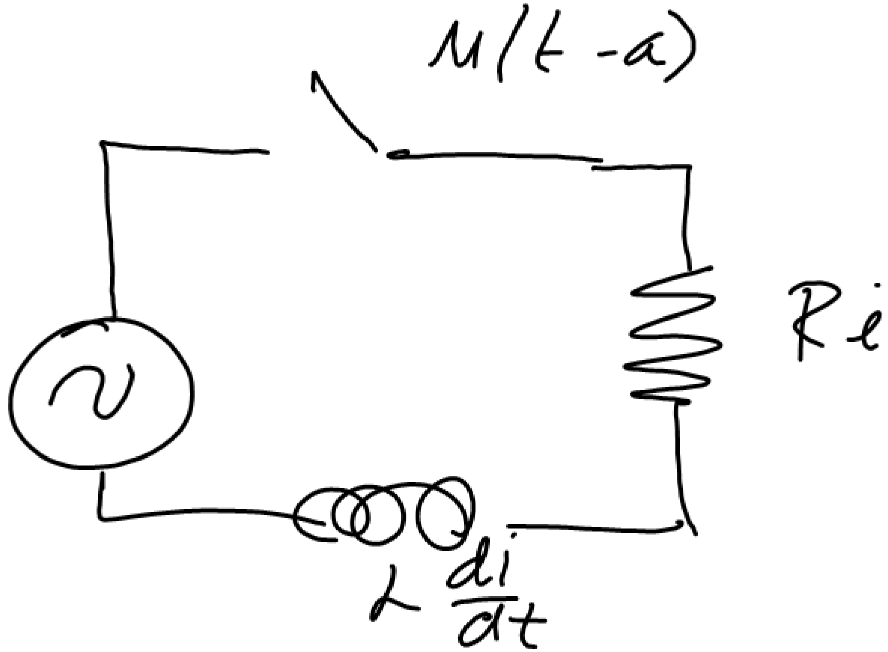
$$u'(t-5) = \delta(t-5)$$

$$\mathcal{L}\{r'(t-5)\} = s\left[\frac{e^{-5s}}{s^2}\right] - (0)$$

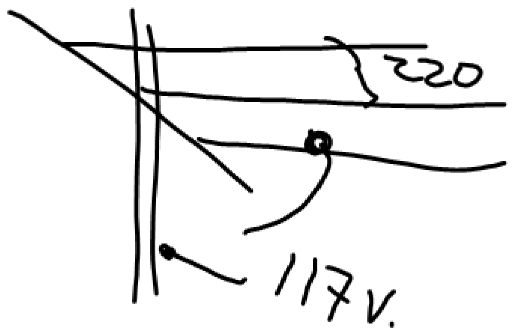
$$\mathcal{L}\{r'(t-5)\} = \frac{e^{-5s}}{s}$$

$$\mathcal{L}\{r'(t-5)\} = \mathcal{L}\{u(t-5)\}$$

$$r'(t-5) = u(t-5)$$



$$V = 117 \operatorname{sen}(60t) \qquad Ri + L\frac{di}{dt} = u(t-a)\,117 \operatorname{sen}(60t)$$



$$i(0) = 0$$