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> restart
> with(LinearAlgebra) :
> AA := Matrix([ [1, 2], [-3, 4] ])

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$$AA := \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix} \quad (1)$$

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> MatExp := MatrixExponential(AA, t) : MatExp[1, 1]; MatExp[2, 1]; MatExp[1, 2]; MatExp[2, 2]

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$$\begin{aligned} & e^{\frac{5}{2}t} \cos\left(\frac{1}{2}t\sqrt{15}\right) - \frac{1}{5}\sqrt{15} e^{\frac{5}{2}t} \sin\left(\frac{1}{2}t\sqrt{15}\right) \\ & - \frac{2}{5}\sqrt{15} e^{\frac{5}{2}t} \sin\left(\frac{1}{2}t\sqrt{15}\right) \\ & \frac{4}{15}\sqrt{15} e^{\frac{5}{2}t} \sin\left(\frac{1}{2}t\sqrt{15}\right) \\ & e^{\frac{5}{2}t} \cos\left(\frac{1}{2}t\sqrt{15}\right) + \frac{1}{5}\sqrt{15} e^{\frac{5}{2}t} \sin\left(\frac{1}{2}t\sqrt{15}\right) \end{aligned} \quad (2)$$

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> DerMatExp := map(diff, MatExp, t) :
> Comprobacion := evalm(DerMatExp - evalm(AA &* MatExp))

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$$Comprobacion := \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (3)$$

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> Ident := map(rcurry(eval, t=0'), MatExp)

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$$Ident := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (4)$$

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> InvMatExp := map(rcurry(eval, t=-t'), MatExp) :
> IdentDos := simplify(evalm(MatExp &* InvMatExp))

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$$IdentDos := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (5)$$

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> Xcero := array([4, -2])

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$$Xcero := \begin{bmatrix} 4 & -2 \end{bmatrix} \quad (6)$$

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> BB := array([4*exp(2*t) + cos(3*t), 2*exp(2*t) + 8*sin(3*t)])

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$$BB := \begin{bmatrix} 4e^{2t} + \cos(3t) & 2e^{2t} + 8\sin(3t) \end{bmatrix} \quad (7)$$

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> SolHom := evalm(MatExp &* Xcero) : SolHom[1]; SolHom[2]

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$$\begin{aligned} & 4e^{\frac{5}{2}t} \cos\left(\frac{1}{2}t\sqrt{15}\right) - \frac{4}{3}\sqrt{15} e^{\frac{5}{2}t} \sin\left(\frac{1}{2}t\sqrt{15}\right) \\ & - 2\sqrt{15} e^{\frac{5}{2}t} \sin\left(\frac{1}{2}t\sqrt{15}\right) - 2e^{\frac{5}{2}t} \cos\left(\frac{1}{2}t\sqrt{15}\right) \end{aligned} \quad (8)$$

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> ComprobarDos := map(rcurry(eval, t=0'), SolHom)

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(9)

$$\text{ComprobarDos} := \begin{bmatrix} 4 & -2 \end{bmatrix} \quad (9)$$

> $\text{MatExpTau} := \text{map}(\text{rcurry}(\text{eval}, t = t - \text{tau}'), \text{MatExp}) :$

> $\text{BBTau} := \text{map}(\text{rcurry}(\text{eval}, t = \text{tau}'), \text{BB}) :$

> $\text{ProdTau} := \text{evalm}(\text{MatExpTau} \& \text{BBTau}) : \text{ProdTau}[1]$

$$\left(e^{\frac{5}{2}t - \frac{5}{2}\tau} \cos\left(\frac{1}{2}(t - \tau)\sqrt{15}\right) - \frac{1}{5}\sqrt{15} e^{\frac{5}{2}t - \frac{5}{2}\tau} \sin\left(\frac{1}{2}(t - \tau)\sqrt{15}\right) \right) (4 e^{2\tau} + \cos(3\tau)) + \frac{4}{15}\sqrt{15} e^{\frac{5}{2}t - \frac{5}{2}\tau} \sin\left(\frac{1}{2}(t - \tau)\sqrt{15}\right) (2 e^{2\tau} + 8 \sin(3\tau)) \quad (10)$$

>

> $\text{SolNoHom} := \text{map}(\text{int}, \text{ProdTau}, \text{tau} = 0 \dots t) : \text{SolNoHom}[1]; \text{SolNoHom}[2]$

$$\begin{aligned} & \frac{2551}{3390} \sqrt{15} e^{\frac{5}{2}t} \sin\left(\frac{1}{2}t\sqrt{15}\right) + \frac{35}{226} e^{\frac{5}{2}t} \cos\left(\frac{1}{2}t\sqrt{15}\right) + \frac{146}{113} \sin(t) \cos(t)^2 \\ & + \frac{382}{113} \cos(t)^3 - \frac{73}{226} \sin(t) - \frac{573}{226} \cos(t) - e^{2t} \\ & \frac{297}{565} \sqrt{15} e^{\frac{5}{2}t} \sin\left(\frac{1}{2}t\sqrt{15}\right) + \frac{332}{113} e^{\frac{5}{2}t} \cos\left(\frac{1}{2}t\sqrt{15}\right) - \frac{646}{113} \sin(t) \cos(t)^2 \\ & - \frac{198}{113} \cos(t)^3 - \frac{5}{2} e^{2t} + \frac{323}{226} \sin(t) + \frac{297}{226} \cos(t) \end{aligned} \quad (11)$$

> $\text{ComprobarTres} := \text{map}(\text{rcurry}(\text{eval}, t = 0'), \text{SolNoHom})$

$$\text{ComprobarTres} := \begin{bmatrix} 0 & 0 \end{bmatrix} \quad (12)$$

> $\text{SolPart} := \text{evalm}(\text{SolHom} + \text{SolNoHom}) :$

> $\text{evalf}(\text{SolPart}[1], 2); \text{evalf}(\text{SolPart}[2], 2)$

$$\begin{aligned} & 4.2 e^{2.5t} \cos(2.0t) - 2.3 e^{2.5t} \sin(2.0t) + 1.3 \sin(t) \cos(t)^2 + 3.4 \cos(t)^3 - 0.32 \sin(t) \\ & - 2.5 \cos(t) - 1. e^{2t} \\ & - 5.8 e^{2.5t} \sin(2.0t) + 0.94 e^{2.5t} \cos(2.0t) - 5.7 \sin(t) \cos(t)^2 - 1.8 \cos(t)^3 - 2.5 e^{2t} \\ & + 1.4 \sin(t) + 1.3 \cos(t) \end{aligned} \quad (13)$$

> $\text{SistUno} := \text{diff}(x[1](t), t) = x[1](t) + 2 \cdot x[2](t) + 4 \cdot \exp(2 \cdot t) + \cos(3 \cdot t);$

$$\text{SistUno} := \frac{d}{dt} x_1(t) = x_1(t) + 2 x_2(t) + 4 e^{2t} + \cos(3t) \quad (14)$$

> $\text{SistDos} := \text{diff}(x[2](t), t) = -3 \cdot x[1](t) + 4 \cdot x[2](t) + 2 \cdot \exp(2 \cdot t) + 8 \cdot \sin(3 \cdot t)$

$$\text{SistDos} := \frac{d}{dt} x_2(t) = -3 x_1(t) + 4 x_2(t) + 2 e^{2t} + 8 \sin(3t) \quad (15)$$

> $\text{XunoCero} := \text{eval}(\text{subs}(t = 0, \text{SolPart}[1]))$

$$\text{XunoCero} := 4 \quad (16)$$

> $\text{XdosCero} := \text{eval}(\text{subs}(t = 0, \text{SolPart}[2]))$

$$\text{XdosCero} := -2 \quad (17)$$

> $\text{ComprobarCuatro} := \text{simplify}(\text{eval}(\text{subs}(x[1](t) = \text{SolPart}[1], x[2](t) = \text{SolPart}[2], \text{lhs}(\text{SistUno}) - \text{rhs}(\text{SistUno}) = 0)))$

$$\text{ComprobarCuatro} := 0 = 0 \quad (18)$$

> $\text{ComprobarCinco} := \text{simplify}(\text{eval}(\text{subs}(x[1](t) = \text{SolPart}[1], x[2](t) = \text{SolPart}[2],$

$$\begin{aligned} & lhs(SistDos) - rhs(SistDos) = 0 \)) \\ & \text{ComprobarCinco} := 0 = 0 \end{aligned} \quad (19)$$

> Sistema := SistUno, SistDos

$$\begin{aligned} \text{Sistema} := \frac{d}{dt} x_1(t) = x_1(t) + 2 x_2(t) + 4 e^{2t} + \cos(3 t), \frac{d}{dt} x_2(t) = -3 x_1(t) + 4 x_2(t) + 2 e^{2t} \\ + 8 \sin(3 t) \end{aligned} \quad (20)$$

> Solucion := dsolve({Sistema}, {x[1](t), x[2](t)}) : Solucion[1]; Solucion[2]

$$\begin{aligned} x_1(t) = e^{\frac{5}{2} t} \sin\left(\frac{1}{2} t \sqrt{15}\right) _C2 + e^{\frac{5}{2} t} \cos\left(\frac{1}{2} t \sqrt{15}\right) _C1 + \frac{73}{226} \sin(3 t) + \frac{191}{226} \cos(3 t) \\ - e^{2t} \\ x_2(t) = \frac{3}{4} e^{\frac{5}{2} t} \sin\left(\frac{1}{2} t \sqrt{15}\right) _C2 + \frac{1}{4} e^{\frac{5}{2} t} \cos\left(\frac{1}{2} t \sqrt{15}\right) \sqrt{15} _C2 \\ + \frac{3}{4} e^{\frac{5}{2} t} \cos\left(\frac{1}{2} t \sqrt{15}\right) _C1 - \frac{1}{4} e^{\frac{5}{2} t} \sin\left(\frac{1}{2} t \sqrt{15}\right) \sqrt{15} _C1 - \frac{99}{226} \cos(3 t) \\ - \frac{323}{226} \sin(3 t) - \frac{5}{2} e^{2t} \end{aligned} \quad (21)$$

> Cond := x[1](0) = 4, x[2](0) = -2

$$\text{Cond} := x_1(0) = 4, x_2(0) = -2 \quad (22)$$

> SolucionParticular := dsolve({Sistema, Cond}, {x[1](t), x[2](t)}) : SolucionParticular[1]; SolucionParticular[2];

$$\begin{aligned} x_1(t) = -\frac{1969}{3390} \sqrt{15} e^{\frac{5}{2} t} \sin\left(\frac{1}{2} t \sqrt{15}\right) + \frac{939}{226} e^{\frac{5}{2} t} \cos\left(\frac{1}{2} t \sqrt{15}\right) + \frac{73}{226} \sin(3 t) \\ + \frac{191}{226} \cos(3 t) - e^{2t} \\ x_2(t) = -\frac{833}{565} \sqrt{15} e^{\frac{5}{2} t} \sin\left(\frac{1}{2} t \sqrt{15}\right) + \frac{106}{113} e^{\frac{5}{2} t} \cos\left(\frac{1}{2} t \sqrt{15}\right) - \frac{99}{226} \cos(3 t) \\ - \frac{323}{226} \sin(3 t) - \frac{5}{2} e^{2t} \end{aligned} \quad (23)$$

> restart

> Sistema := diff(x[1](t), t) = x[3](t), diff(x[2](t), t) = x[4](t), diff(x[3](t), t) =
- (H1 + H2) · x[1](t) + $\frac{H2}{M1}$ · x[2](t), diff(x[4](t), t) = $\frac{H2}{M2}$ · x[1](t) - $\frac{H2}{M2}$ · x[2](t)

$$\begin{aligned} \text{Sistema} := \frac{d}{dt} x_1(t) = x_3(t), \frac{d}{dt} x_2(t) = x_4(t), \frac{d}{dt} x_3(t) = -\frac{(H1 + H2) x_1(t)}{M1} + \frac{H2 x_2(t)}{M1}, \\ \frac{d}{dt} x_4(t) = \frac{H2 x_1(t)}{M2} - \frac{H2 x_2(t)}{M2} \end{aligned} \quad (24)$$

> Cond := x[2](0) = 5, x[1](0) = $\frac{H1}{H2}$ · 5, x[3](0) = 0, x[4](0) = 0

$$\text{Cond} := x_2(0) = 5, x_1(0) = \frac{5 H1}{H2}, x_3(0) = 0, x_4(0) = 0 \quad (25)$$

> H1 := 1; H2 := 4; M1 := 1; M2 := 2

$$\begin{aligned}
H1 &:= 1 \\
H2 &:= 4 \\
M1 &:= 1 \\
M2 &:= 2
\end{aligned} \tag{26}$$

> Sistema;

$$\frac{d}{dt} x_1(t) = x_3(t), \frac{d}{dt} x_2(t) = x_4(t), \frac{d}{dt} x_3(t) = -5 x_1(t) + 4 x_2(t), \frac{d}{dt} x_4(t) = 2 x_1(t) - 2 x_2(t) \tag{27}$$

> Cond

$$x_2(0) = 5, x_1(0) = \frac{5}{4}, x_3(0) = 0, x_4(0) = 0 \tag{28}$$

> Sol := dsolve({Sistema, Cond}, {x[1](t), x[2](t), x[3](t), x[4](t)})

$$Sol := \left\{ x_1(t) = \frac{5}{2624} (11 \sqrt{41} + 81) (2 \sqrt{41} + 14) \sqrt{41} \cos\left(\frac{1}{2} \sqrt{2 \sqrt{41} + 14} t\right) \right. \tag{29}$$

$$+ \frac{5}{2624} (-2 \sqrt{41} + 14) \sqrt{41} (-81 + 11 \sqrt{41}) \cos\left(\frac{1}{2} \sqrt{-2 \sqrt{41} + 14} t\right)$$

$$- \frac{35}{656} (11 \sqrt{41} + 81) \sqrt{41} \cos\left(\frac{1}{2} \sqrt{2 \sqrt{41} + 14} t\right) - \frac{35}{656} \sqrt{41} (-81$$

$$+ 11 \sqrt{41}) \cos\left(\frac{1}{2} \sqrt{-2 \sqrt{41} + 14} t\right), x_2(t) = -\frac{165}{2624} (11 \sqrt{41}$$

$$+ 81) \sqrt{41} \cos\left(\frac{1}{2} \sqrt{2 \sqrt{41} + 14} t\right) - \frac{165}{2624} \sqrt{41} (-81$$

$$+ 11 \sqrt{41}) \cos\left(\frac{1}{2} \sqrt{-2 \sqrt{41} + 14} t\right) + \frac{25}{10496} (11 \sqrt{41} + 81) (2 \sqrt{41}$$

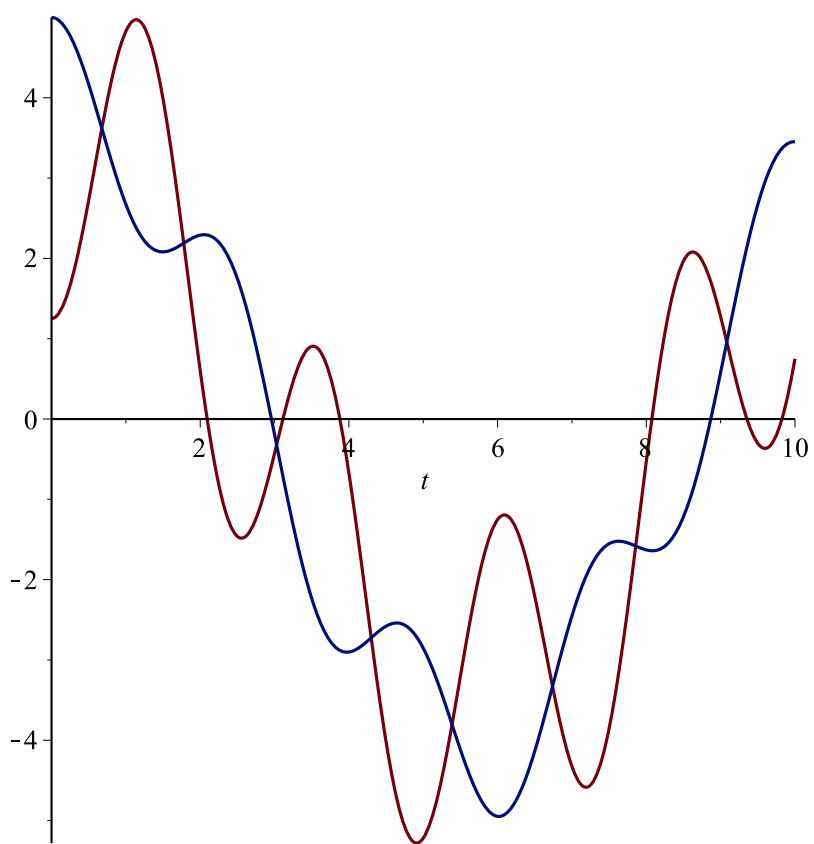
$$+ 14) \sqrt{41} \cos\left(\frac{1}{2} \sqrt{2 \sqrt{41} + 14} t\right) + \frac{25}{10496} (-2 \sqrt{41} + 14) \sqrt{41} (-81$$

$$+ 11 \sqrt{41}) \cos\left(\frac{1}{2} \sqrt{-2 \sqrt{41} + 14} t\right), x_3(t)$$

$$= \frac{5}{164} \frac{(11 \sqrt{41} + 81) \sqrt{41} \sin\left(\frac{1}{2} \sqrt{2 \sqrt{41} + 14} t\right)}{\sqrt{2 \sqrt{41} + 14}}$$

$$\begin{aligned}
& + \frac{5}{164} \frac{\sqrt{41} (-81 + 11 \sqrt{41}) \sin\left(\frac{1}{2} \sqrt{-2 \sqrt{41} + 14} t\right)}{\sqrt{-2 \sqrt{41} + 14}}, x_4(t) = \\
& - \frac{5}{32} \frac{(11 \sqrt{41} + 81) \sin\left(\frac{1}{2} \sqrt{2 \sqrt{41} + 14} t\right)}{\sqrt{2 \sqrt{41} + 14}} \\
& + \frac{5}{32} \frac{(-81 + 11 \sqrt{41}) \sin\left(\frac{1}{2} \sqrt{-2 \sqrt{41} + 14} t\right)}{\sqrt{-2 \sqrt{41} + 14}} \\
& + \frac{15}{1312} \frac{(11 \sqrt{41} + 81) \sqrt{41} \sin\left(\frac{1}{2} \sqrt{2 \sqrt{41} + 14} t\right)}{\sqrt{2 \sqrt{41} + 14}} \\
& + \frac{15}{1312} \frac{\sqrt{41} (-81 + 11 \sqrt{41}) \sin\left(\frac{1}{2} \sqrt{-2 \sqrt{41} + 14} t\right)}{\sqrt{-2 \sqrt{41} + 14}} \Bigg\}
\end{aligned}$$

> `plot([rhs(Sol[1]), rhs(Sol[2])], t=0..10)`



```
=  
> plot([rhs(Sol[3]), rhs(Sol[4])], t=0..10)
```

