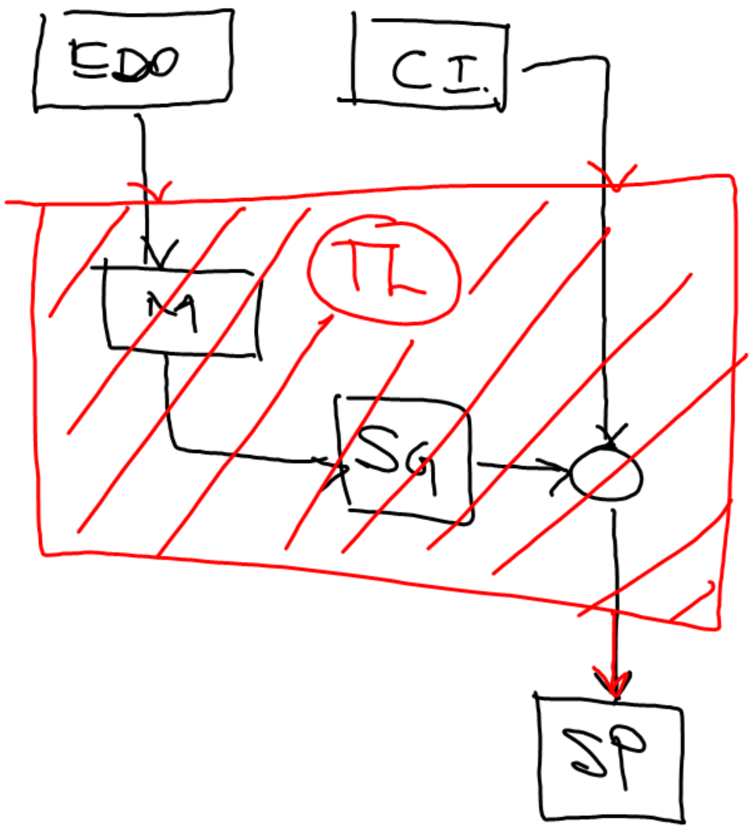
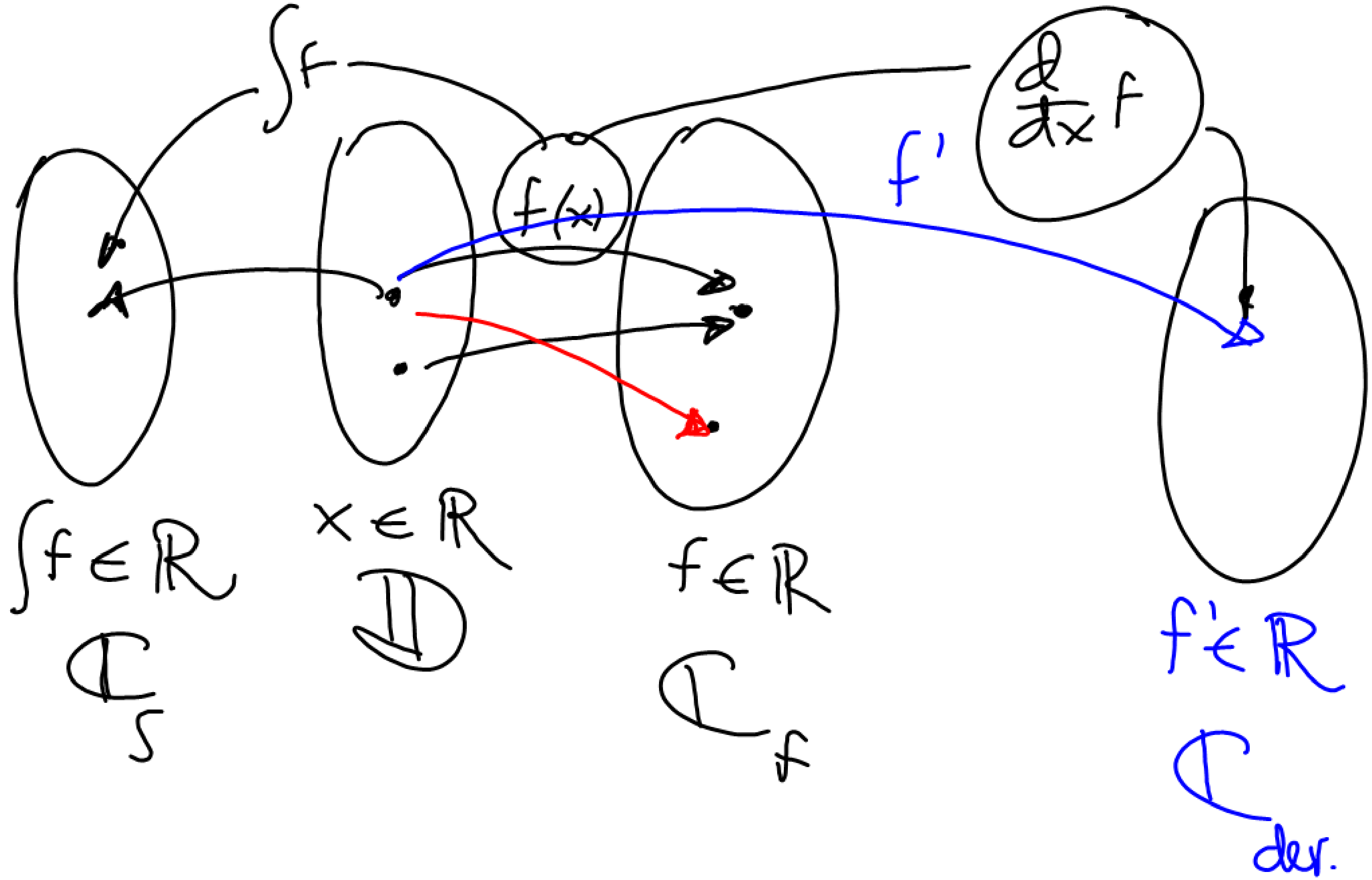
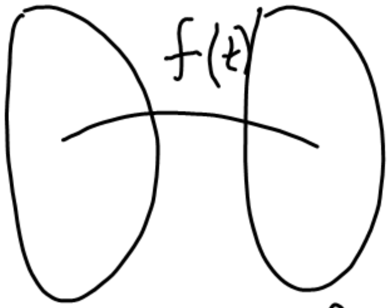
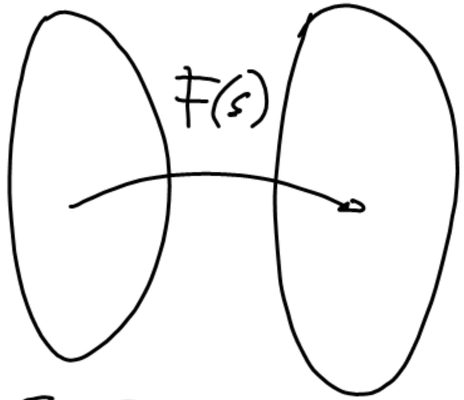
CAP. 3.- Transformada de Laplace y sistemas de EDOL's




$\int f$
$f(x)$
$\frac{d}{dx} f$
f'
$\int f \in \mathbb{R}$
$\mathbb{C}_{\int}$
$x \in \mathbb{R}$
$\mathbb{D}$
$f \in \mathbb{R}$
$\mathbb{C}_f$
$f' \in \mathbb{R}$
$\mathbb{C}_{der.}$

$$T\{f\} = F(s)$$





$t \in \mathbb{R}$ $\quad f \in \mathbb{R}$ $\qquad s \in \mathbb{C}$ $\quad F \in \mathbb{R}$

$\mathbb{D}_f$ $\quad \mathbb{C}_f$ $\qquad \mathbb{D}_T$ $\quad \mathbb{C}_T$

$$T\{af+bg\} \longrightarrow aF(s)+bG(s)$$

$$T\{f'\} \longrightarrow sF(s)$$

$$T\{\textstyle\int f\} \longrightarrow \frac{F(s)}{s}$$

operador

$$T\{f(t)\} = \int_{-\infty}^{\infty} N(s,t)\, f(t)\, dt$$

núcleo

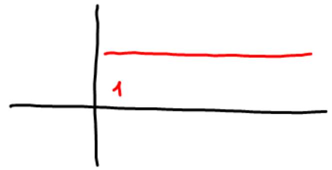
argumento

Laplace

$$N_{Lap}(s,t) = \begin{cases} 0 & t < 0 \\ e^{-st} & t \geq 0 \end{cases}$$

$$\boxed{\mathcal{L}\{f(t)\} = \int_{0}^{\infty} e^{-st} f(t)\, dt \Rightarrow F(s)}$$

$f(t) = 1$



$$\mathcal{L}\{1\} = \int_0^\infty e^{-st} \cdot (1) \cdot dt$$

$$= \left[\int e^{-st} dt \right]_0^\infty$$

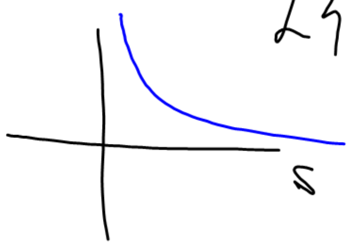
$$= \left[\frac{e^{-st}}{-s} \right]_0^\infty \Rightarrow \frac{1}{-s} \left(\lim_{b \to \infty} e^{-sb} - 1 \right)$$

$$\lim_{b \to \infty} e^{-sb} = \lim_{b \to \infty} \frac{1}{e^{sb}} \Rightarrow \lim_{a \to \infty} \frac{1}{a} = 0$$

$$\lim_{b \to \infty} e^{sb} \longrightarrow \infty$$

$$\mathcal{L}\{1\} = -\frac{1}{s}(0 - 1) \Rightarrow \frac{1}{s}$$

$$\mathcal{L}\{1\} = \frac{1}{s}$$



$$\mathcal{L}\{t\} = \int_0^\infty e^{-st} \cdot t \cdot dt$$

$$= \left[\int t e^{-st} dt \right]_0^\infty$$

$$\int u\,dv = uv - \int v\,du$$

$$u = t \qquad du = dt$$

$$dv = e^{-st} dt \qquad v = \frac{e^{-st}}{-s}$$

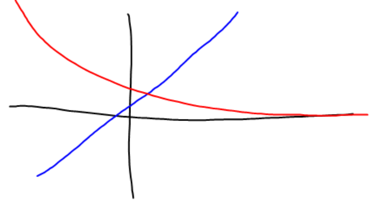
$$= \left[\frac{te^{-st}}{-s} + \frac{1}{s} \int e^{-st} dt \right]_0^\infty$$

$$= \left[\frac{te^{-st}}{-s} - \frac{1}{s^2} e^{-st} \right]_0^\infty$$

$$= -\frac{1}{s}\left(\lim_{b\to\infty} \overset{\infty}{b} \cdot \lim_{b\to\infty} \overset{0}{e^{-sb}} - 0 \right] - \frac{1}{s^2}\left[\lim_{b\to\infty} \overset{0}{e^{-sb}} - 1 \right]$$

$$\frac{|t| \leq Me^{At}}{}$$

$$e^{t^2}, \quad e^{t^3}, \quad e^{t^7} \Big\} \ \underline{\text{NO TL}}$$



$$\mathcal{L}\{t\} = -\frac{1}{s}[0-0] - \frac{1}{s^2}[0-1]$$

$$\mathcal{L}\{t\} = \frac{1}{s^2}$$

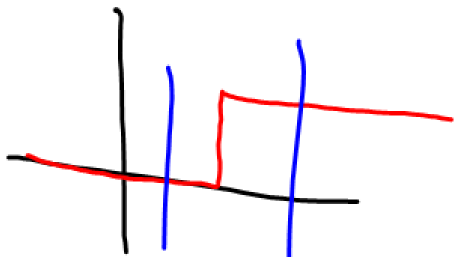
Teorema de Existencia y Unicidad de la transf. de Laplace

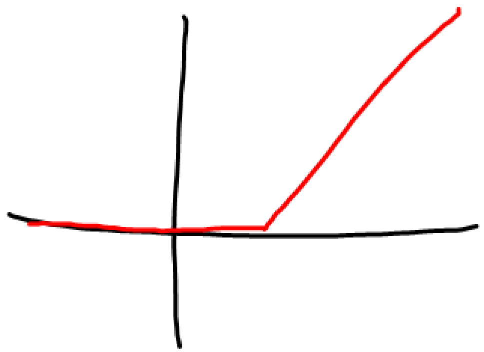
Sea $f(t)$

a) sea de orden exponencial

$$|f(t)| \leq M e^{At} \quad M, A \in \mathbb{R}$$

b) ser seccionalmente continua.





$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$$

$$\mathcal{L}\{\cos(bt\} = \frac{s}{s^2+b^2}$$

$$\mathcal{L}\{\operatorname{sen}(bt)\} = \frac{b}{s^2+b^2}$$

$$\mathcal{L}^{-1}\{F(s)\} = \int_{a-i\infty}^{a+i\infty} e^{st} F(s)\,ds$$

$s \in \mathbb{C}$
$t \in \mathbb{R}$

$$\mathcal{L}^{-1}\left\{\frac{4}{s^2+16}\right\} = \operatorname{sen}(4t)$$

Propiedades

(1) $$\mathcal{L}\{af+bg\} = aF + bG$$

$a, b \in \mathbb{R}$

(2) $$\mathcal{L}\{f(at)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a} \quad ?$$

$$\mathcal{L}\{e^{t}\} = \frac{1}{s-1}$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{a} \cdot \frac{1}{\left(\frac{s}{a}\right) - 1}$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{c\, b}$$

$$= \frac{1}{a} \cdot \frac{1}{\left(\frac{s-a}{a}\right)} \Rightarrow \frac{\cancel{a}}{\cancel{a}(s-a)} \Rightarrow \frac{1}{s-a}$$

③ $\mathcal{L}\{f'(t)\} = sF(s) - f(0)$

$\mathcal{L}\{f^{(n)}(t)\} = s^n F(s) - \left(s^{n-1}f(0) + s^{n-2}f'(0) + \dots + f^{(n-1)}(0)\right)$

$\mathcal{L}\{f''(t)\} = s^2 F(s) - sf(0) - f'(0)$

$\mathcal{L}\{f'''(t)\} = s^3 F(s) - s^2 f(0) - sf'(0) - f''(0)$

④ $\mathcal{L}^{-1}\{F'(s)\} = -tf(t)$

$\mathcal{L}^{-1}\{F^{(n)}(s)\} = (-1)^n t^n f(t)$

⑤ $\mathcal{L}\left\{\int_0^t f(\tau)\,d\tau\right\} = \dfrac{F(s)}{s}$

⑥ $\mathcal{L}^{-1}\left\{\int_s^\infty F(\sigma)\,d\sigma\right\} = \dfrac{f(t)}{t}$

$$\frac{d^2y}{dt^2} - 5\frac{dy}{dx} + 6y = 0 \qquad y(0)=2 \quad y'(0)=-3$$

$$\mathcal{L}\left\{\frac{d^2y}{dt^2} - 5\frac{dy}{d\cancel{x}t} + 6y\right\} = 0\mathcal{L}\{1\}$$

$$\mathcal{L}\left\{\frac{d^2y}{dt^2}\right\} - 5\mathcal{L}\left\{\frac{dy}{dt}\right\} + 6\mathcal{L}\{y\} = 0$$

$$\left[s^2Y(s) - s(2) - (-3)\right] - 5\left[sY(s) - (2)\right] + 6Y(s) = 0$$

$$\boxed{(s^2-5s+6)Y(s) - 2s + 13 = 0}$$

$$(s^2-5s+6)Y(s) = 2s-13$$

$$Y(s) = \frac{2s-13}{s^2-5s+6}$$

$$\frac{2s-13}{(s-2)(s-3)} = \frac{A}{s-2} + \frac{B}{s-3}$$

$$2s-13 = A(s-3) + B(s-2)$$

Si $\underline{s=3}$

$$2(3)-13 = A(0) + B(1) \qquad B=-7$$

Si $s=2$

$$2(2)-13 = A(-1) + B(0) \qquad A=9$$

$$\boxed{Y(s) = \frac{9}{s-2} - \frac{7}{s-3}}$$

$$\mathcal{L}^{-1}\{Y(s)\} \Rightarrow y(t) = 9\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} - 7\mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\}$$

$$\boxed{y(t) = 9e^{2t} - 7e^{3t}} \qquad y(0)=2$$

$$y'(t) = 18e^{2t} - 21e^{3t} \qquad y'(0)=-3$$