SERIE. 2020-1-3

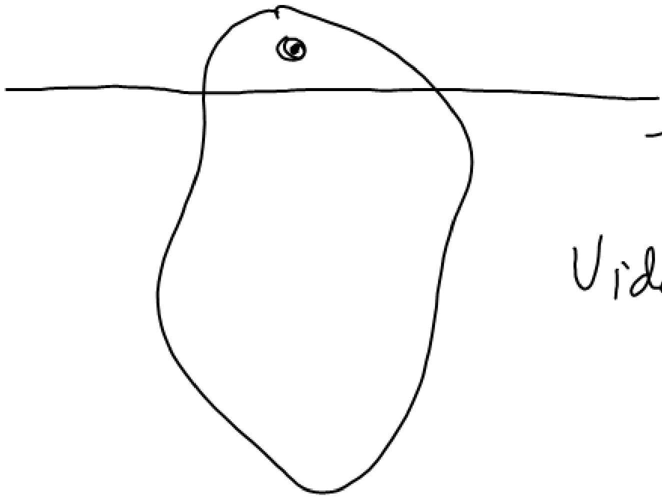
$$e^{At}$$

$$\frac{d}{dt} e^{At} = A e^{At}$$

$$\left[\frac{d}{dt} e^{At}\right]_{t=0} = A \left[e^{At}\right]_{t=0}$$

$$= A \times I.$$

TEMA IV.- "Una muy breve introducción a las ecuaciones diferenciales en derivadas parciales"



	EDO	ED en DP
Tem.	80%	20%
Vida Real	20%	80%

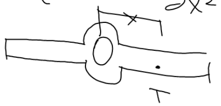
$$\frac{\partial^2 z(x,y)}{\partial x^2} + 5\frac{\partial^2 z(x,y)}{\partial x \partial y} + 6\frac{\partial^2 z(x,y)}{\partial y^2} = 0$$

$z(x,y)$ — incógnita

$x, y \in \mathbb{R}$ var. indep

EDenDP (z)

$$\frac{\partial T(x,t)}{\partial t} = k\frac{\partial^2 T(x,t)}{\partial x^2}$$



$z(x,y) = f(y+mx)$

$= f(u) \longrightarrow u = y+mx$

$\frac{\partial z}{\partial x} = f'(u) \cdot \frac{\partial u}{\partial x} \Rightarrow mf'(u)$

$\frac{\partial z}{\partial y} = f'(u) \cdot \frac{\partial u}{\partial y} \Rightarrow f'(u)$

$\frac{\partial^2 z}{\partial x^2} = mf''(u) \cdot \frac{\partial u}{\partial x} \Rightarrow m^2 f''(u)$

$\frac{\partial^2 z}{\partial x \partial y} = mf''(u) \cdot \frac{\partial u}{\partial y} \Rightarrow mf''(u)$

$\frac{\partial^2 z}{\partial y^2} = f''(u) \cdot \frac{\partial u}{\partial y} \Rightarrow f''(u)$

$m^2 f''(u) + 5mf''(u) + 6f''(u) = 0$

$(m^2 + 5m + 6) f''(u) = 0$

$f''(u) = 0 \quad f'(u) = C_1 \quad f(u) = C_1 u + C_2$

$z(x,y) = C_1(y+mx) + C_2$ solución inútil.

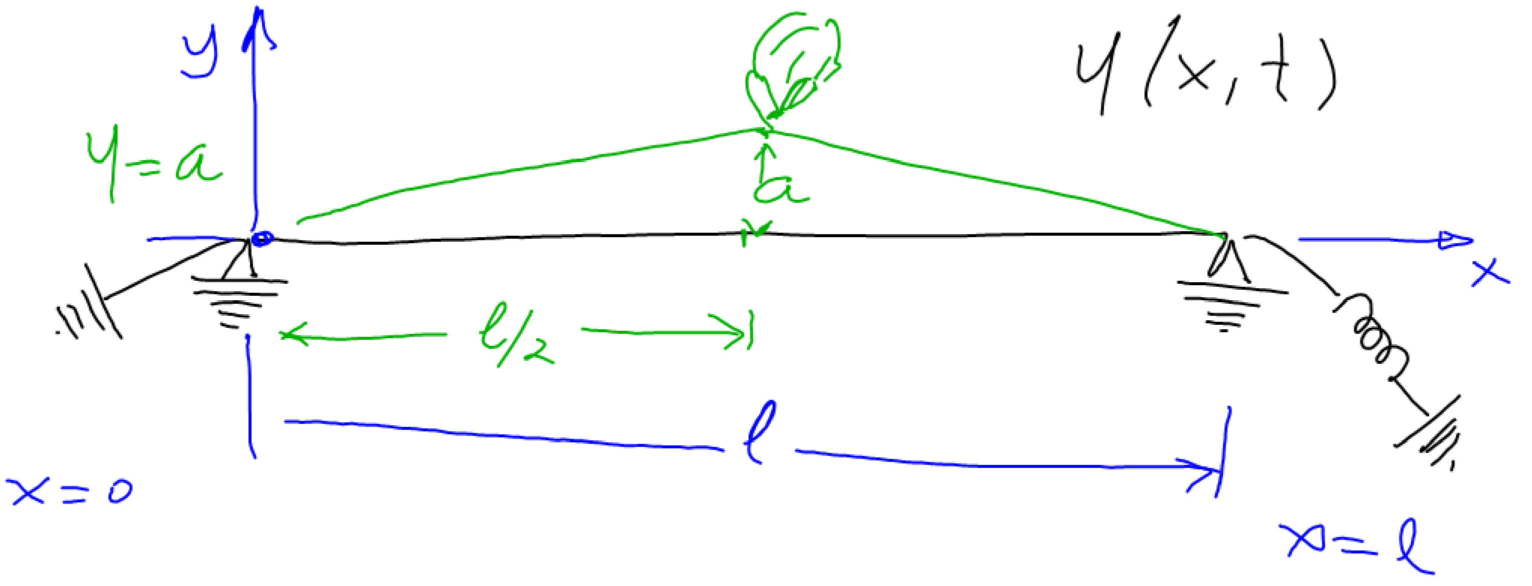
$m^2 + 5m + 6 = 0 \qquad m_1 = -2$

$(m+2)(m+3) = 0 \qquad m_2 = -3$

$z_1(x,y) = f_1(y-2x)$

$z_2(x,y) = f_2(y-3x)$

$z_g(x,y) = f_1(y-2x) + f_2(y-3x)$



Frontera

$$\forall t \quad y(0,t) = 0$$

$$y(\ell,t) = 0$$

Inicial

$$y(x,0) = \begin{cases} \frac{2a}{\ell}x & ; 0 \leq x \leq \ell/2 \\ 2a - \frac{2a}{\ell}x & ; \ell/2 < x \leq \ell \end{cases}$$

$$\frac{\partial y}{\partial t} = 0$$

- ED en DP(n)

$z(x,y)$
$f(x,y,z)$
$g(x,y,z,t)$

$$g(x,y,z,t) = G_1 + G_2 + G_3 + \dots + G_n$$

$$\frac{\partial^2 z}{\partial x^2} + 5\frac{\partial^2 z}{\partial x \partial y} + 6\frac{\partial^2 z}{\partial y^2} = 0$$

$$z_g(x,y) = f_1(y-2x) + f_2(y-3x)$$

$$z_p(x,y) = e^y e^{-2x} + \cos(y-3x)$$

$$\frac{\partial z}{\partial x} = e^y\left(-2e^{-2x}\right) + 3\,\mathrm{sen}(y-3x)$$

$$\frac{\partial^2 z}{\partial x^2} = e^y\left(4e^{-2x}\right) - 9\cos(y-3x)$$

$$\frac{\partial^2 z}{\partial x \partial y} = -2e^y e^{-2x} + 3\cos(y-3x)$$

$$\frac{\partial z}{\partial y} = e^y e^{-2x} - \mathrm{sen}(y-3x)$$

$$\frac{\partial^2 z}{\partial y^2} = e^y e^{-2x} - \cos(y-3x)$$

$$\left[4e^y e^{-2x} - 9\cos(y-3x)\right] + 5\left[-2e^y e^{-2x} + 3\cos(y-3x)\right] + 6\left[e^y e^{-2x} - \cos(y-3x)\right] = 0$$

$$(4-10+6)e^y e^{-2x} + (-9+15-6)\cos(y-3x) = 0$$

$$(0)e^y e^{-2x} + (0)\cos(y-3x) = 0$$

$$z_p(x,y) = (y-2x)^3 + (y-3x)^2$$

$$0 \equiv 0$$

Método de Separación de Variables.

- Prueba y Error

$$\frac{\partial \varphi(x,t)}{\partial x} = \frac{\partial^2 \psi(x,t)}{\partial t^2}$$

$$H_0 = \psi(x,t) = F(x) \cdot G(t)$$

$$\frac{\partial \psi}{\partial x} = F'(x) \cdot G(t)$$

$$\frac{\partial \psi}{\partial t} = F(x) \cdot G'(t)$$

$$\frac{\partial^2 \psi}{\partial t^2} = F(x) \cdot G''(t)$$

$$F'(x) \cdot G(t) = F(x) \cdot G''(t)$$

$$\frac{F'(x)}{F(x)} = \frac{G''(t)}{G(t)}$$

$$y(x,t) = F(x)\cdot G(t)$$

$$\frac{F''(x)}{F(x)} = \frac{G''(t)}{G(t)}$$

$$\frac{F''(x)}{F(x)} = \alpha \qquad \frac{G''(t)}{G(t)} = \alpha$$

$\alpha > 0 \qquad \alpha = 0 \qquad \alpha < 0$

$\alpha = 0$

$$\frac{F''(x)}{F(x)} = 0 \qquad F(x) \neq 0 \quad F''(x) = 0$$

$$\boxed{F(x) = k_1}$$

$$\frac{G''(t)}{G(t)} = 0 \qquad G(t) \neq 0 \quad G''(t) = 0$$

$$G'(t) = C_1 \qquad \boxed{G(t) = C_1 t + C_2}$$

$$\boxed{y_{\alpha=0}(x,t) = k_1(C_1 t + C_2)}$$

$$y_{\alpha=0} = C_{10} t + C_{20} \qquad \frac{\partial y}{\partial x} = 0$$

$$\boxed{\frac{\partial y}{\partial x} = \frac{\partial^2 y}{\partial t^2}} \qquad \frac{\partial y}{\partial t} = C_1 \quad \frac{\partial^2 y}{\partial t^2} = 0$$

$\alpha > 0 \quad \alpha = \beta^2 \quad \forall \beta \neq 0$

$\frac{F'(x)}{F(x)} = \beta^2 \qquad F'(x) = \beta^2 F(x)$

$F'(x) - \beta^2 F(x) = 0$

$(D - \beta^2) F = 0 \qquad$ EDOL(1)CCH.

$F(x) = k_1 e^{\beta^2 x}$

$\frac{G''(t)}{G(t)} = \beta^2 \qquad G''(t) = \beta^2 G(t)$

$G''(t) - \beta^2 G(t) = 0$

$(D^2 - \beta^2) G = 0 \qquad$ EDOL(2)CCH.

$G(t) = C_1 e^{\beta t} + C_2 e^{-\beta t}$

$m^2 + a^2 = (m-a)(m+a)$

$y_{\alpha>0}(x,t) = k_1 e^{\beta^2 x} \left(C_1 e^{\beta t} + C_2 e^{-\beta t} \right)$

$y_{g\,\alpha>0} = e^{\beta^2 x} C_{10} e^{\beta t} + e^{\beta^2 x} C_{20} e^{-\beta t}$

$\alpha < 0 \qquad \alpha = -\beta^2 \qquad \forall \beta \neq 0$

$\dfrac{F'(x)}{F(x)} = -\beta^2 \qquad F'(x) + \beta^2 F(x) = 0$

EDOL(1) c.c. H.

$F(x) = k_1 e^{-\beta^2 x}$

$\dfrac{G''(t)}{G(t)} = -\beta^2 \qquad G''(t) + \beta^2 G(t) = 0$

EDOL(2) cc H.

$(D^2 + \beta^2) G = 0$

$G(t) = C_1 \cos(\beta t) + C_2 \operatorname{sen}(\beta t)$

$y_{\alpha<0} = k_1 e^{-\beta^2 x} \left(C_1 \cos(\beta t) + C_2 \operatorname{sen}(\beta t) \right).$

$y_{\alpha<0} = e^{-\beta^2 x} \left(C_{10} \cos(\beta t) + C_{20} \operatorname{sen}(\beta t) \right).$