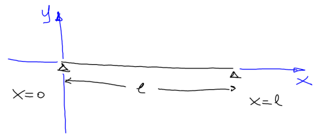
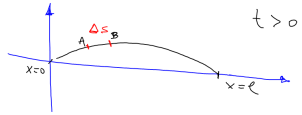
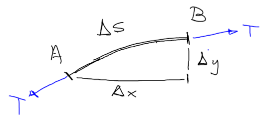
PROBLEMA DE LA CUERDA DE GUITARRA.







$y(x,t)$ $\qquad F = ma$

$$a = \frac{\partial^2 y(x,t)}{\partial t^2}$$

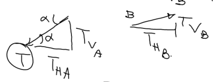
masa Δs

ρ densidad por unidad de longitud

$$m = \rho\,\Delta s$$

$$\Sigma F = \rho\,\Delta s \frac{\partial^2 y}{\partial t^2}$$

$$\Sigma F = T_{V_B} - T_{V_A}$$



$\alpha < 4^\circ$

$\operatorname{sen}\alpha \doteq \tan\alpha$

$\tan\alpha = \frac{\Delta y}{\Delta x}$

$$T_{V_A} = T \operatorname{sen}\alpha$$

$$T_{V_A} = T\frac{\Delta y}{\Delta x} \qquad \Delta x \to 0$$

$$T_{V_A} = T\frac{\partial y}{\partial x}$$

$$T_{V_B} = T\frac{\partial y}{\partial x} + \frac{\partial}{\partial x} T\left(\frac{\partial y}{\partial x}\right)\Delta x$$

$$= T\frac{\partial y}{\partial x} + T\frac{\partial^2 y}{\partial x^2}\Delta x$$

$$\Sigma F = T_{V_B} - T_{V_A}$$

$$\Sigma F = \cancel{T\frac{\partial y}{\partial x}} + T\frac{\partial^2 y}{\partial x^2}\Delta x - \left(\cancel{T\frac{\partial y}{\partial x}}\right)$$

$$T\frac{\partial^2 y}{\partial x^2}\Delta x = \rho\,\Delta s\,\frac{\partial^2 y}{\partial t^2} \qquad \begin{matrix}\Delta x \to 0\\ \Delta s \to 0\end{matrix}$$

$$T\frac{\partial^2 y}{\partial x^2} = \rho\frac{\partial^2 y}{\partial t^2}$$

$T > 0$

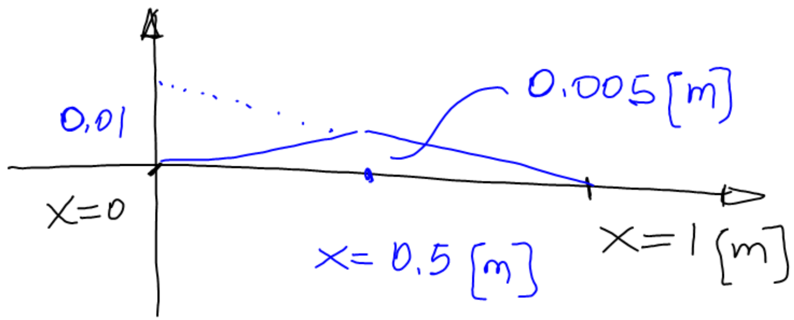
$\rho > 0$

$$\frac{T}{\rho}\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}$$

$c \in \mathbb{R}$ $c \neq 0$.

$$\boxed{c^2\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}}$$

$$\frac{\partial^2 y}{\partial t^2} - c^2\frac{\partial^2 y}{\partial x^2} = 0$$



FRONTERA

$\forall t > 0 \quad x = 0 \quad y(0,t) = 0$

$\qquad\quad x = 1[m] \quad y(1,t) = 0$

CONDICIONES INICIALES

$$t=0 \quad y(x,0) = f(x) = \begin{cases} \frac{0{,}005}{0{,}5}x & ; 0 \leq x \leq 0{,}5 \\ 0{,}01 - \frac{0{,}005}{0{,}5}x & ; 0{,}5 < x \leq 1 \end{cases}$$

$$\left.\frac{\partial y}{\partial t}\right|_{t=0} = 0$$

$$\frac{\partial^2 y}{\partial t^2} - c^2 \frac{\partial^2 y}{\partial x^2} = 0$$

MSV

$H_0 \Rightarrow \quad y(x,t) = F(x) \cdot G(t)$

$$\frac{\partial^2 y}{\partial x^2} = F'' \cdot G \qquad \frac{\partial^2 y}{\partial t^2} = F G''$$

$$F G'' - c^2 F'' G = 0$$

$$F G'' = c^2 F'' G$$

$$\boxed{\frac{G''}{G} = c^2 \frac{F''}{F}}$$

$$\frac{G''}{G} = c^2 \frac{F''}{F}$$

$$c^2 \frac{F''}{F} = \alpha \qquad \frac{G''}{G} = \alpha$$

para $\alpha = 0$

$$c^2 \frac{F''}{F} = 0 \qquad \frac{G''}{G} = 0$$

$$c^2 F'' = 0$$

$$F'' = 0$$

$$F' = C_1 \qquad \boxed{F(x) = C_1 x + C_2} \qquad G(t) \neq 0$$

$$\boxed{\begin{array}{l} y(0,t) = 0 \\ y(1,t) = 0 \end{array}}$$

$$y(0,t) = F(0) \cdot G(t) = 0$$

$$= F(0) = 0$$

$$y(1,t) = F(1) \cdot G(t) = 0$$

$$= F(1) = 0$$

$$F(0) = C_1(0) + C_2 = 0$$

$$C_2 = 0$$

$$F(1) = C_1(1) + (0) = 0$$

$$C_1 = 0$$

Se rechaza $\alpha = 0$

para $\alpha>0$ $\alpha=\beta^2$ $\forall\beta\neq 0$

$$c^2\frac{F''}{F}=\beta^2$$

$$F''=\frac{\beta^2}{c^2}F$$

$$F''-\frac{\beta^2}{c^2}F=0$$

$$\left(D^2-\frac{\beta^2}{c^2}\right)F(x)=0$$

$$m^2-\frac{\beta^2}{c^2}=0$$

$$\left(m-\frac{\beta}{c}\right)\left(m+\frac{\beta}{c}\right)=0$$

$$m_1=\frac{\beta}{c} \qquad m_2=-\frac{\beta}{c}$$

$$F(x)=C_1e^{\frac{\beta}{c}x}+C_2e^{-\frac{\beta}{c}x}$$

$F(0)=0$

$F(1)=0$

$$C_1e^{(0)}+C_2e^{-(0)}=0$$

$$C_1=-C_2$$

$$-C_2e^{\frac{\beta}{c}(1)}+C_2e^{-\frac{\beta}{c}(1)}=0$$

$\alpha>0$

no sirve

$$C_2\left(-e^{\frac{\beta}{c}}+\frac{1}{e^{\frac{\beta}{c}}}\right)=0$$

$$e^{\frac{\beta}{c}}=\frac{1}{e^{\frac{\beta}{c}}}$$

$$\left(e^{\frac{\beta}{c}}\right)^2=1 \qquad \frac{\beta}{c}=0 \qquad \beta=0$$

$\alpha < 0 \qquad \alpha = -\beta^2 \quad \forall \beta \neq 0$

$$c^2 \frac{F''}{F} = -\beta^2$$

$$F'' = -\frac{\beta^2}{c^2} F$$

$$F'' + \frac{\beta^2}{c^2} F = 0$$

$$\left(D^2 + \frac{\beta^2}{c^2}\right) F(x) = 0$$

$$m^2 + \frac{\beta^2}{c^2} = 0$$

$$m_1 = \sqrt{\frac{\beta^2}{c^2}}\, i \Rightarrow \frac{\beta}{c} i$$

$$m_2 = -\sqrt{\frac{\beta^2}{c^2}}\, i \Rightarrow -\frac{\beta}{c} i$$

$$F(x) = C_1 \cos\left(\frac{\beta}{c} x\right) + C_2 \operatorname{sen}\left(\frac{\beta}{c} x\right)$$

$F(0) = 0$
$F(1) = 0$

$$C_1 \cos(0) + C_2 \operatorname{sen}(0) = 0$$

$$C_1(1) + C_2(0) = 0 \qquad C_1 = 0$$

$$C_2 \operatorname{sen}\left(\frac{\beta}{c}(1)\right) = 0$$

$$\operatorname{sen}\left(\frac{\beta}{c}\right) = 0 \qquad \frac{\beta}{c} = n\pi.$$

$$\operatorname{sen}(n\pi) = 0 \qquad \beta = n\pi c$$

$$\beta^2 = n^2 \pi^2 c^2$$

$$F(x) = C_2 \operatorname{sen}(n\pi x) \qquad C_2 \neq 0$$

$$\left.\begin{aligned} \frac{G''}{G} &= -n^2\pi^2 c^2 \\ G'' &= -n^2\pi^2 c^2 G \\ G'' + n^2\pi^2 c^2 G &= 0 \end{aligned}\right\} \longrightarrow G(t) = k_1 \cos(n\pi c t) + k_2 \operatorname{sen}(n\pi c t).$$

$$y(x,t)_{\substack{\alpha<0 \\ \alpha = n^2\pi^2 c^2}} = C_2 \operatorname{sen}(n\pi x)\left(k_1 \cos(n\pi c t) + k_2 \operatorname{sen}(n\pi c t)\right)$$