```
> restart
```

PROBLEMA DE LA CUERDA DE GUITARRA

```
> EDP := diff(y(x,t),t$2) - c^^2*diff(y(x,t),x$2) = 0
```

$$EDP := \frac{\partial^2}{\partial t^2} y(x,t) - c^2 \left(\frac{\partial^2}{\partial x^2} y(x,t) \right) = 0 \quad (1)$$

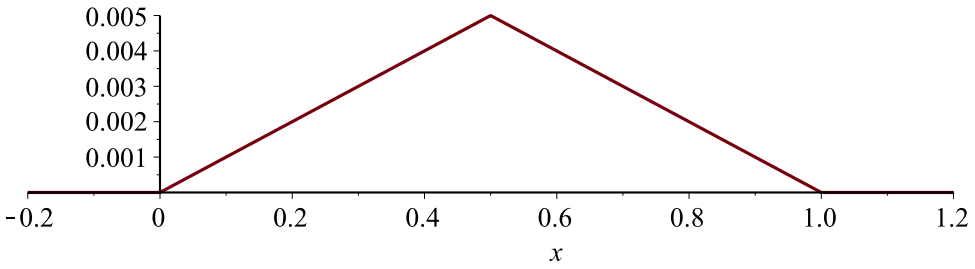
```
> CondFront := y(0,t) = 0, y(1,t) = 0
```

$$CondFront := y(0,t) = 0, y(1,t) = 0 \quad (2)$$

```
> CondIni := y(x,0) = (5/1000)/(5/10)*x*Heaviside(x) - 2*(5/1000)/(5/10)*(x - 5/10)*Heaviside(x
  - 5/10) + (5/1000)/(5/10)*(x - 1)*Heaviside(x - 1);
```

$$CondIni := y(x,0) = \frac{1}{100} x \,\mathrm{Heaviside}(x) - \frac{1}{50} \left(x - \frac{1}{2}\right) \mathrm{Heaviside}\left(x - \frac{1}{2}\right) + \frac{1}{100} (x - 1)\,\mathrm{Heaviside}(x - 1) \quad (3)$$

```
> plot(rhs(CondIni), x = -0.2 ..1.2)
```




```
> CondIniVel := DerYcero = 0
```

$$CondIniVel := DerYcero = 0 \quad (4)$$

MÉTODO DE SEPARACIÓN DE VARIABLES

```
> EcuaSep := eval(subs(y(x,t) = F(x)*G(t), EDP))
```

$$EcuaSep := F(x) \left(\frac{d^2}{dt^2} G(t) \right) - c^2 \left(\frac{d^2}{dx^2} F(x) \right) G(t) = 0 \quad (5)$$

```
> EcuaSeparada := (lhs(EcuaSep) + c^^2*diff(F(x),x$2)*G(t))/(F(x)*G(t))
  = (rhs(EcuaSep) + c^^2*diff(F(x),x$2)*G(t))/(F(x)*G(t))
```

$$EcuaSeparada := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = \frac{c^2 \left(\frac{d^2}{dx^2} F(x) \right)}{F(x)} \quad (6)$$

```
> EcuaX := rhs(EcuaSeparada) = alpha; EcuaT := lhs(EcuaSeparada) = alpha
```

$$EcuaX := \frac{c^2 \left(\frac{d^2}{dx^2} F(x) \right)}{F(x)} = \alpha$$

$$EcuaT := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = \alpha \quad (7)$$

> $EcuaXneg := subs(\text{alpha} = -n \cdot\cdot 2 \cdot \text{Pi} \cdot\cdot 2 \cdot c \cdot\cdot 2, EcuaX)$

$$EcuaXneg := \frac{c^2 \left(\frac{d^2}{dx^2} F(x) \right)}{F(x)} = -n^2 \pi^2 c^2 \quad (8)$$

> $SolX := dsolve(EcuaXneg)$

$$SolX := F(x) = \_C1 \sin(\pi n x) + \_C2 \cos(\pi n x) \quad (9)$$

> $EcuaTneg := subs(\text{alpha} = -n \cdot\cdot 2 \cdot \text{Pi} \cdot\cdot 2 \cdot c \cdot\cdot 2, EcuaT)$

$$EcuaTneg := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = -n^2 \pi^2 c^2 \quad (10)$$

> $SolT := dsolve(EcuaTneg)$

$$SolT := G(t) = \_C1 \sin(\pi c n t) + \_C2 \cos(\pi c n t) \quad (11)$$

> $SolPartX := subs(\_C2 = 0, \_C1 = 1, SolX)$

$$SolPartX := F(x) = \sin(\pi n x) \quad (12)$$

> $SolPart := y(x, t) = rhs(SolPartX) \cdot rhs(SolT)$

$$SolPart := y(x, t) = \sin(\pi n x) \left(\_C1 \sin(\pi c n t) + \_C2 \cos(\pi c n t) \right) \quad (13)$$

> $SolucionGeneral := y(x, t) = Sum(\sin(n \cdot \text{Pi} \cdot x) \cdot (b[n] \cdot \cos(n \cdot \text{Pi} \cdot c \cdot t) + a[n] \cdot \sin(n \cdot \text{Pi} \cdot c \cdot t)), n = 1 \,..infinity)$

$$SolucionGeneral := y(x, t) = \sum_{n=1}^{\infty} \sin(\pi n x) \left(b_n \cos(\pi c n t) + a_n \sin(\pi c n t) \right) \quad (14)$$

> $SolucionInicial := eval(subs(t = 0, SolucionGeneral))$

$$SolucionInicial := y(x, 0) = \sum_{n=1}^{\infty} \sin(\pi n x) \, b_n \quad (15)$$

> $b[n] := subs\left(\sin(n \cdot \text{Pi}) = 0, \frac{1}{\left(\frac{5}{10} \right)} \cdot int\left(rhs(CondIni) \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot x}{1} \right), x = 0 \,..1 \right) \right)$

$$b_n := \frac{1}{25} \frac{\sin\left(\frac{1}{2} \pi n \right)}{\pi^2 n^2} \quad (16)$$

> $a[n] := 0$

$$a_n := 0 \quad (17)$$

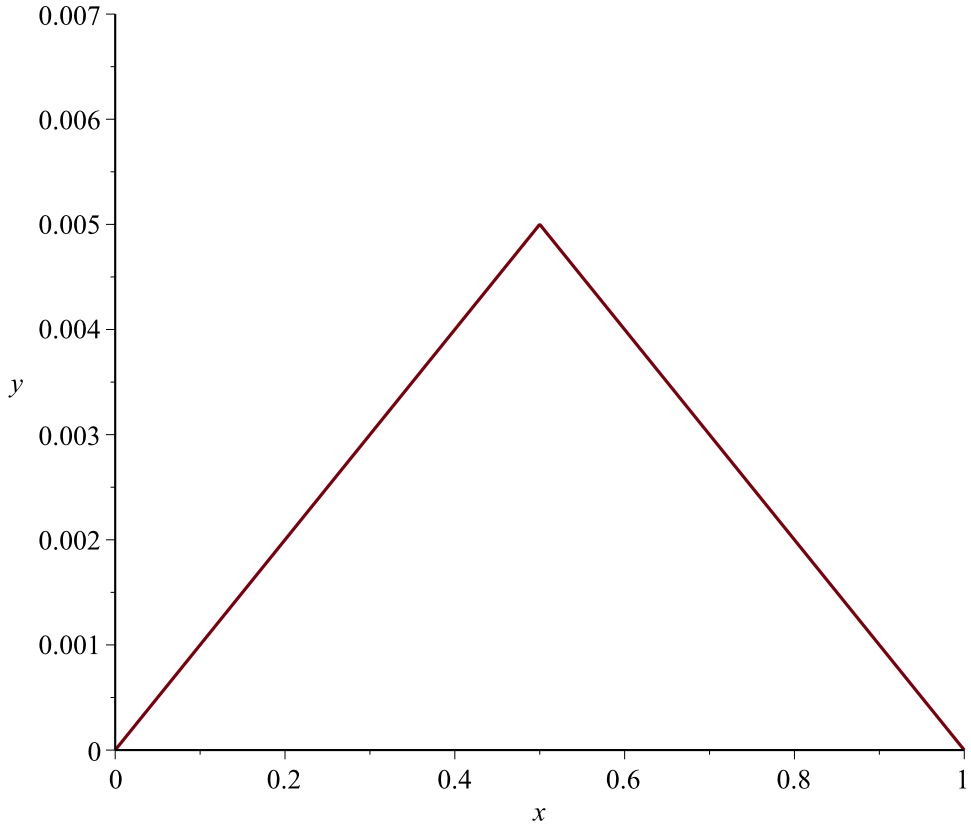
> $SolucionGeneral$

(18)

$$y(x,t) = \sum_{n=1}^{\infty} \frac{1}{25} \frac{\sin(\pi n x)\,\sin\left(\frac{1}{2}\pi n\right)\cos(\pi c n t)}{\pi^2 n^2} \tag{18}$$

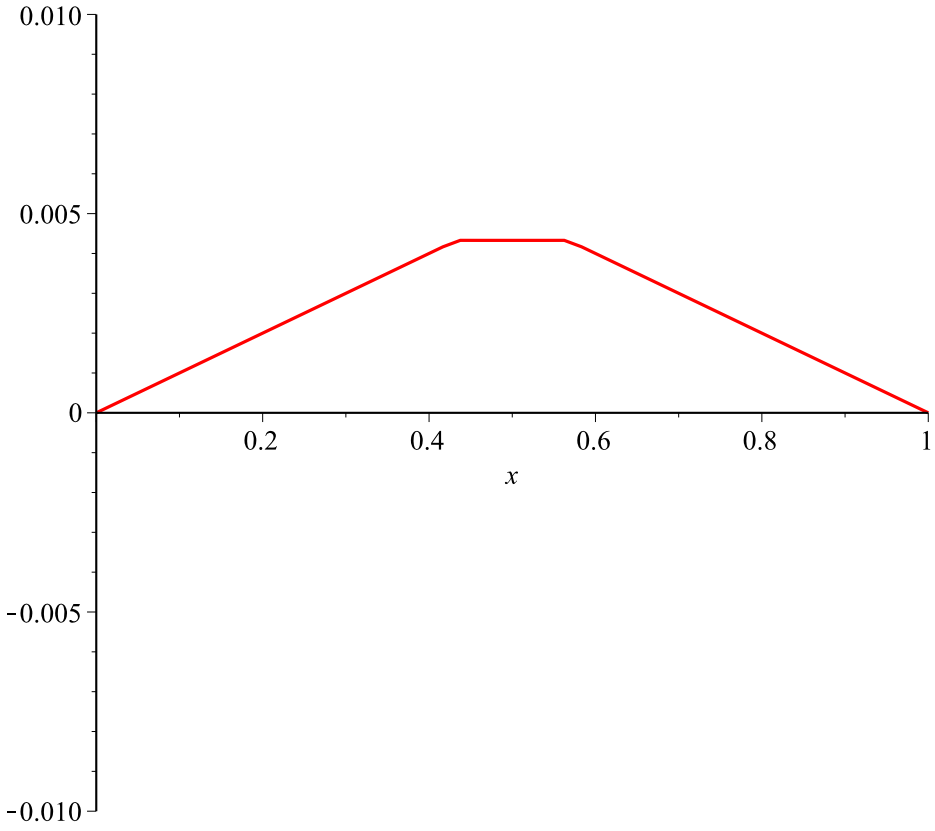
> $SolucionParticular := y(x,t) = \sum_{n=1}^{500} \frac{1}{25} \frac{\sin(\pi n x)\,\sin\left(\frac{1}{2}\pi n\right)\cos(\pi c n t)}{\pi^2 n^2}$:

> $plot(rhs(subs(c=1, t=0, SolucionParticular)), x=0..1, y=0..0.007)$



> $with(plots)$:

> $animate(rhs(subs(c=1, SolucionParticular)), x=0..1, t=0..4, frames=150, view=[0..1, -0.01..0.01])$


0.010
0.005
0
-0.005
-0.010
0.2
0.4
0.6
0.8
1
x