

$$\begin{aligned}
& \text{restart} \\
& SG := y(x) = C[1] \cdot \exp(2 \cdot x) + C[2] \cdot \cos(3 \cdot x) + C[3] \cdot \sin(3 \cdot x) \\
& \quad SG := y(x) = C_1 e^{2x} + C_2 \cos(3x) + C_3 \sin(3x) \tag{1} \\
& SPNH := y(x) = 5 \cdot x \cdot \exp(2 \cdot x) \\
& \quad SPNH := y(x) = 5x e^{2x} \tag{2} \\
& SGNHcero := y(x) = rhs(SG) + rhs(SPNH) \\
& \quad SGNHcero := y(x) = C_1 e^{2x} + C_2 \cos(3x) + C_3 \sin(3x) + 5x e^{2x} \tag{3} \\
& Sist := diff(SG, x), diff(SG, x$2), diff(SG, x$3) : Sist[1]; Sist[2]; Sist[3] \\
& \quad \frac{d}{dx} y(x) = 2 C_1 e^{2x} - 3 C_2 \sin(3x) + 3 C_3 \cos(3x) \\
& \quad \frac{d^2}{dx^2} y(x) = 4 C_1 e^{2x} - 9 C_2 \cos(3x) - 9 C_3 \sin(3x) \\
& \quad \frac{d^3}{dx^3} y(x) = 8 C_1 e^{2x} + 27 C_2 \sin(3x) - 27 C_3 \cos(3x) \tag{4} \\
& Para := simplify(solve({Sist}, {C[1], C[2], C[3]})) : Para[1]; Para[2]; Para[3] \\
& \quad C_1 = \frac{1}{26} \left(9 \left(\frac{d}{dx} y(x) \right) + \frac{d^3}{dx^3} y(x) \right) e^{-2x} \\
& \quad C_2 = \frac{2}{13} \cos(3x) \left(\frac{d}{dx} y(x) \right) - \frac{1}{9} \cos(3x) \left(\frac{d^2}{dx^2} y(x) \right) + \frac{2}{117} \cos(3x) \left(\frac{d^3}{dx^3} y(x) \right) \\
& \quad \quad - \frac{4}{39} \left(\frac{d}{dx} y(x) \right) \sin(3x) + \frac{1}{39} \sin(3x) \left(\frac{d^3}{dx^3} y(x) \right) \\
& \quad C_3 = \frac{4}{39} \cos(3x) \left(\frac{d}{dx} y(x) \right) - \frac{1}{39} \cos(3x) \left(\frac{d^3}{dx^3} y(x) \right) + \frac{2}{13} \left(\frac{d}{dx} y(x) \right) \sin(3x) \\
& \quad \quad - \frac{1}{9} \left(\frac{d^2}{dx^2} y(x) \right) \sin(3x) + \frac{2}{117} \sin(3x) \left(\frac{d^3}{dx^3} y(x) \right) \tag{5} \\
& EDO := simplify(subs(C[1] = rhs(Para[1]), C[2] = rhs(Para[2]), C[3] = rhs(Para[3]), SG)) \\
& \quad EDO := y(x) = \frac{1}{2} \frac{d}{dx} y(x) + \frac{1}{18} \frac{d^3}{dx^3} y(x) - \frac{1}{9} \frac{d^2}{dx^2} y(x) \tag{6} \\
& EDOfinal := rhs(EDO) \cdot 18 - lhs(EDO) \cdot 18 = 0 \\
& \quad EDOfinal := 9 \left(\frac{d}{dx} y(x) \right) + \frac{d^3}{dx^3} y(x) - 2 \left(\frac{d^2}{dx^2} y(x) \right) - 18 y(x) = 0 \tag{7} \\
& PNH := Q(x) = eval(subs(y(x) = rhs(SPNH), lhs(EDOfinal))) \\
& \quad PNH := Q(x) = 65 e^{2x} \tag{8} \\
& EDONH := lhs(EDOfinal) = rhs(PNH) \\
& \quad EDONH := 9 \left(\frac{d}{dx} y(x) \right) + \frac{d^3}{dx^3} y(x) - 2 \left(\frac{d^2}{dx^2} y(x) \right) - 18 y(x) = 65 e^{2x} \tag{9} \\
& SolucionGeneralNoHomogenea := dsolve(EDONH) \\
& \quad SolucionGeneralNoHomogenea := y(x) = 5x (e^x)^2 + _C1 \cos(3x) + _C2 e^{2x} + _C3 \sin(3x) \tag{10}
\end{aligned}$$

> *SGNHcero*

$$y(x) = C_1 e^{2x} + C_2 \cos(3x) + C_3 \sin(3x) + 5x e^{2x} \quad (11)$$

> *EDOhomogenea* := lhs(*EDONH*) = 0

$$EDOhomogenea := 9 \left(\frac{d}{dx} y(x) \right) + \frac{d^3}{dx^3} y(x) - 2 \left(\frac{d^2}{dx^2} y(x) \right) - 18 y(x) = 0 \quad (12)$$

> *QQ* := rhs(*EDONH*)

$$QQ := 65 e^{2x} \quad (13)$$

> *EcuaCarac* := $m \cdot 3 - 2 \cdot m \cdot 2 + 9 \cdot m - 18 = 0$

$$EcuaCarac := m^3 - 2m^2 + 9m - 18 = 0 \quad (14)$$

> *Raiz* := solve(*EcuaCarac*)

$$Raiz := 2, 3I, -3I \quad (15)$$

> *yy*[1] := exp(*Raiz*[1]·*x*); *yy*[2] := cos(Im(*Raiz*[2])·*x*); *yy*[3] := sin(Im(*Raiz*[2])·*x*)

$$yy_1 := e^{2x}$$

$$yy_2 := \cos(3x)$$

$$yy_3 := \sin(3x)$$

(16)

> *SGHuno* := $y(x) = C[1] \cdot yy[1] + C[2] \cdot yy[2] + C[3] \cdot yy[3]$

$$SGHuno := y(x) = C_1 e^{2x} + C_2 \cos(3x) + C_3 \sin(3x) \quad (17)$$

> with(*linalg*) :

> *WW* := wronskian([*yy*[1], *yy*[2], *yy*[3]], *x*)

$$WW := \begin{bmatrix} e^{2x} & \cos(3x) & \sin(3x) \\ 2e^{2x} & -3\sin(3x) & 3\cos(3x) \\ 4e^{2x} & -9\cos(3x) & -9\sin(3x) \end{bmatrix} \quad (18)$$

> *BB* := array([0, 0, *QQ*])

$$BB := \begin{bmatrix} 0 & 0 & 65e^{2x} \end{bmatrix} \quad (19)$$

> *PARAuno* := simplify(linsolve(*WW*, *BB*)) : *Aprima* := *PARAuno*[1]; *Bprima* := *PARAuno*[2];
Dprima := *PARAuno*[3];

$$Aprima := 5$$

$$Bprima := \frac{5}{3} e^{2x} (2 \sin(3x) - 3 \cos(3x))$$

$$Dprima := -\frac{5}{3} (2 \cos(3x) + 3 \sin(3x)) e^{2x} \quad (20)$$

> *A* := int(*Aprima*, *x*) + *C*[1]; *BBB* := int(*Bprima*, *x*) + *C*[2]; *DD* := int(*Dprima*, *x*) + *C*[3]

$$A := 5x + C_1$$

$$BBB := -\frac{10}{13} e^{2x} \cos(3x) + \frac{20}{39} e^{2x} \sin(3x) - \frac{20}{13} (2 \cos(x) + 3 \sin(x)) e^{2x} \cos(x)^2$$

$$- \frac{24}{13} e^{2x} (2 \cos(x) + \sin(x)) + 6 e^{2x} \cos(x) + 3 e^{2x} \sin(x) + C_2$$

$$DD := -\frac{40}{39} (2 \cos(x) + 3 \sin(x)) e^{2x} \cos(x)^2 - \frac{16}{13} e^{2x} (2 \cos(x) + \sin(x)) + 4 e^{2x} \cos(x) \quad (21)$$

$$+ \frac{15}{13} e^{2x} \cos(3x) - \frac{10}{13} e^{2x} \sin(3x) + 2 e^{2x} \sin(x) + C_3$$

$$\begin{aligned} & \text{> } SGNHuno := y(x) = \text{expand}(A \cdot yy[1] + BBB \cdot yy[2] + DD \cdot yy[3]) \\ SGNHuno &:= y(x) = 5x (e^x)^2 + (e^x)^2 C_1 - \frac{320}{13} (e^x)^2 \cos(x)^6 + \frac{480}{13} (e^x)^2 \cos(x)^4 \\ & - \frac{180}{13} (e^x)^2 \cos(x)^2 + 4 C_2 \cos(x)^3 - 3 C_2 \cos(x) - \frac{320}{13} (e^x)^2 \sin(x)^2 \cos(x)^4 \\ & + \frac{160}{13} (e^x)^2 \sin(x)^2 \cos(x)^2 - \frac{20}{13} (e^x)^2 \sin(x)^2 + 4 C_3 \sin(x) \cos(x)^2 - C_3 \sin(x) \end{aligned} \quad (22)$$

$$\begin{aligned} & \text{> } \text{comprob} := \text{simplify}(\text{rhs}(SGNHcero) - \text{rhs}(SGNHuno)) = 0 \\ \text{comprob} &:= C_2 \cos(3x) + C_3 \sin(3x) - 4 C_2 \cos(x)^3 + 3 C_2 \cos(x) + \frac{20}{13} e^{2x} \\ & - 4 C_3 \sin(x) \cos(x)^2 + C_3 \sin(x) = 0 \end{aligned} \quad (23)$$

$$\begin{aligned} & \text{> } g := \text{Int}(x \cdot 2, x) = \text{int}(x \cdot 2, x) \\ & g := \int x^2 dx = \frac{1}{3} x^3 \end{aligned} \quad (24)$$

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