TEMA 1.- EDO(1)NL

$$M(x,y) + N(x,y)\,\frac{dy}{dx} = 0.$$

a) MVS.

$$P(x)\cdot Q(y) + R(x)\cdot S(y)\,\frac{dy}{dx} = 0$$

$$\frac{P(x)}{R(x)}\,dx + \frac{S(y)}{Q(y)}\,dy = 0$$

$$\int \frac{P(x)}{R(x)}\,dx + \int \frac{S(y)}{Q(y)}\,dy = C_1$$

SOLUCIÓN GENERAL

$$F(x,y) = C_1$$

② COEFICIENTES HOMOGÉNEOS.

$$M(x,y) + N(x,y)\frac{dy}{dx} = 0$$

$$\boxed{\begin{aligned} M(\lambda x, \lambda y) &= \lambda^m M(x,y) \\ N(\lambda x, \lambda y) &= \lambda^n N(x,y) \end{aligned} \quad m = n}$$

$$\left.\begin{aligned} y(x) &= x \cdot u(x) \\ \frac{dy}{dx} &= x \cdot \frac{du}{dx} + u \end{aligned}\right\} \text{Variables Separables.}$$

$$u(x) = \frac{y(x)}{x}$$

$$M(x,y) + N(x,y)\frac{dy}{dx} = 0$$

$$x\frac{dy}{dx} = \sqrt{x^2-y^2} + y$$

$$M(x,y) = -\left(\sqrt{x^2-y^2} + y\right)$$

$$N(x,y) = x$$

$$M(\lambda x, \lambda y) = -\left(\sqrt{(\lambda x)^2 - (\lambda y)^2} + (\lambda y)\right)$$

$$= -\left(\sqrt{\lambda^2 x^2 - \lambda^2 y^2} + \lambda y\right)$$

$$= -\left(\sqrt{\lambda^2(x^2-y^2)} + \lambda y\right)$$

$$= -\left(\sqrt{\lambda^2}\sqrt{x^2-y^2} + \lambda y\right)$$

$$= -\left(\lambda\sqrt{x^2-y^2} + \lambda y\right)$$

$$= -\lambda\left(\sqrt{x^2-y^2} + y\right) \quad m=1$$

$$N(\lambda x, \lambda y) = (\lambda x)$$

$$= \lambda \cdot x \quad n=1$$

$$-\sqrt{x^2-y^2}-y+x\frac{dy}{dx}=0$$

$$y=x\cdot u$$

$$\frac{dy}{dx}=x\frac{du}{dx}+u$$

$$-\sqrt{x^2-(xu)^2}-(xu)+x\left(x\frac{du}{dx}+u\right)=0$$

$$-\sqrt{x^2-x^2u^2}-\cancel{xu}+\cancel{xu}+x^2\frac{du}{dx}=0$$

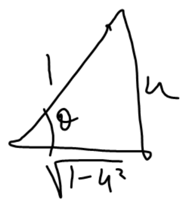
$$-\sqrt{x^2(1-u^2)}+x^2\frac{du}{dx}=0$$

$$-\sqrt{x^2}\sqrt{1-u^2}+x^2\frac{du}{dx}=0$$

$$-x\sqrt{1-u^2}+x^2\frac{du}{dx}=0$$

$$-\frac{x}{x^2}dx+\frac{du}{\sqrt{1-u^2}}=0$$

$$-\frac{dx}{x}+\frac{du}{\sqrt{1-u^2}}=0$$



$$-\frac{dx}{x}+\frac{\cos(\theta)\,d\theta}{\cos(\theta)}=0$$

$$\frac{u}{1}=\operatorname{sen}(\theta)$$

$$du=\cos(\theta)\,d\theta$$

$$\frac{\sqrt{1-u^2}}{1}=\cos(\theta)$$

$$\theta=\text{ang sen}(u)$$

$$-\int\frac{dx}{x}+\int d\theta=C$$

$$-L(x)+\theta=C_1$$

$$-L(x)+\text{ang sen}(u)=C_1$$

sust. $u=\dfrac{y}{x}$

$$-L(x)+\text{ang sen}\left(\frac{y}{x}\right)=C_1$$ Sol. gral.

(3) EDO(1)NL EXACTA.

(4) M. Factor Integrante.

$F(x,y)$

$$x^2y^2 - 6x^3y + 4xy^3 = C_1$$

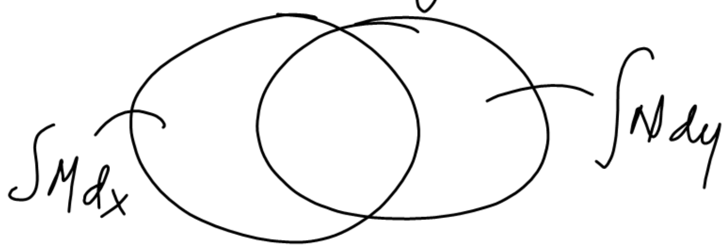
$$\left(2xy^2 - 18x^2y + 4y^3\right) + \left(2x^2y - 6x^3 + 12xy^2\right)\frac{dy}{dx} = 0$$

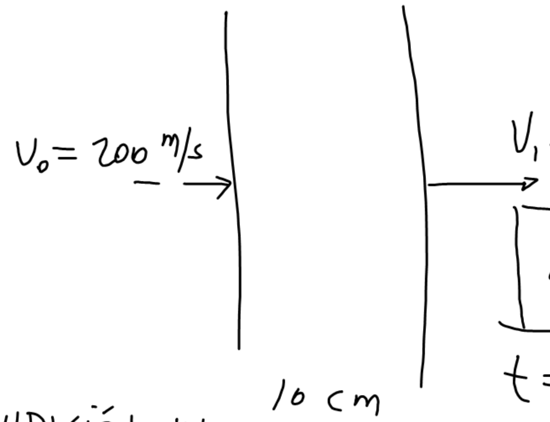
$$M = \frac{\partial F}{\partial x} \qquad N = \frac{\partial F}{\partial y}$$

$$\frac{\partial M}{\partial y} = 4xy - 18x^2 + 12y^2$$

$$\frac{\partial N}{\partial x} = 4xy - 18x^2 + 12y^2$$

$$\int M\,dx \;\cup\; \int N\,dy = C_1$$





$$\boxed{\frac{dV}{dt} = -kV^2}$$

$t = ?$ EDO(1)NL

$$\frac{dV}{V^2} = -k\,dt$$

$$\frac{dV}{V^2} + k\,dt = 0$$

$$\int \frac{dV}{V^2} + k\int dt = C_1$$

$$\frac{V^{-1}}{-1} + kt = C_1$$

$$-\frac{1}{V} + kt = C_1$$

$$-\frac{1}{V} = C_1 - kt$$

$$V = \frac{1}{kt - C_1}$$

SOL. GRAL.

CONDICIÓN INICIAL.

$$V(0) = 200 \text{ m/s}$$

$$V = \frac{1}{kt - C_1}$$ Sol. gral

$$200 = \frac{1}{k(0) - C_1}$$

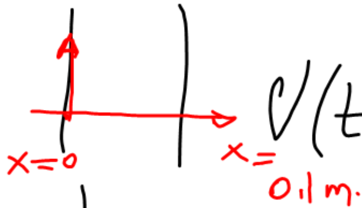
$$200 = -\frac{1}{C_1}$$

$$C_1 = -\frac{1}{200}$$

$$\boxed{V = \frac{1}{kt + \left(\frac{1}{200}\right)}}$$

SOLUCIÓN PARTICULAR

0.10 m

$V(0)=200$  $V(t_f)=80$

$$V=\frac{1}{kt+\frac{1}{200}}$$

$$\Longrightarrow \frac{dx}{dt}=\frac{1}{kt+\frac{1}{200}} \qquad dx=\frac{dt}{kt+\frac{1}{200}}$$

EDO(1)NL.

$u=kt+\frac{1}{200}$

$du=k\,dt$

$$\int dx-\frac{1}{k}\int\frac{k\,dt}{kt+\frac{1}{200}}=0$$

$$\int dx-\frac{1}{k}\int\frac{du}{u}=C_2$$

$$x-\frac{1}{k}\ln\left(kt+\frac{1}{200}\right)=C_2$$

SOLUCIÓN GENERAL

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$$V = \frac{1}{kt + \frac{1}{200}}$$

$$kt + \frac{1}{200} = \frac{1}{V}$$

$$kt = \frac{1}{V} - \frac{1}{200}$$

$$X - \frac{1}{k} L\left(kt + \frac{1}{200}\right) = C_2$$

$$t = \frac{\frac{1}{V} - \frac{1}{200}}{k}$$

$$\frac{1}{10} - \frac{1}{k} L\left(k\left(\frac{\frac{400}{3}}{k}\right) + \frac{1}{200}\right) = C_2$$

$$t_f = \frac{\frac{1}{80} - \frac{1}{200}}{k}$$

$$\frac{1}{10} - \frac{1}{k} L\left(\frac{400}{3} + \frac{1}{200}\right) = C_2$$

$$t_f = \frac{1}{k\left(\frac{1}{80} - \frac{1}{200}\right)}$$

$$-\frac{1}{k} L\left(\frac{80000 + 3}{600}\right) = C_2 - \frac{1}{10}$$

$$t_f = \frac{1}{k\left(\frac{200-80}{16000}\right)}$$

$$L\left(\frac{80003}{600}\right) = \left(\frac{1}{10} - C_2\right)k$$

$$t_f = \frac{1}{k\left(\frac{120}{16000}\right)}$$

$$4.89 = (0.1 - C_2)k$$

$$t_f = \frac{\frac{16000}{120}}{k}$$

$$t_f = \frac{\frac{400}{3}}{k}$$