

$$(2xy^2 - 18x^2y + 4y^3) + (2x^2y - 6x^3 + 12x^2y^2) \frac{dy}{dx} = 0$$

$M(x,y)$ $N(x,y)$

$$\frac{\partial M}{\partial y} \equiv \frac{\partial N}{\partial x} \Rightarrow \text{exact form.}$$

$$\frac{\partial M}{\partial y} = 4xy - 18x^2 + 12y^2$$

$$\frac{\partial N}{\partial x} = 4xy - 18x^2 + 12y^2$$

$$\int M dx = 2y \left[xy - 18x \int y^2 dx + 4y^3 \right] dx$$

$$\hookrightarrow \int M dx = 2y^2 \left(\frac{x^2}{2} \right) - 18y \left(\frac{x^3}{3} \right) + 4y^3 (x)$$

$$= \cancel{X^2y^2} - 6x^3y + 4x^2y^3$$

$$\int N dy = 2x^2 \left\{ y dy - 6x^3 \int dy + 12x \int y^2 dy \right\}$$

$$= 2x^2 \left(\frac{y^2}{2} \right) - 6x^3(y) + 12x \left(\frac{y^3}{3} \right)$$

$$= \cancel{X^2y^2} - 6x^3y + 4x^2y^3$$

Solución general $\left[\int M dx \right] + \left[\int N dy \right] = C$



SOL
GRAL

$$x^2y^2 - 6x^3y + 4xy^3 = C$$

S₁: $\int M dx + \int \left[N - \frac{\partial}{\partial y} \{ M dx \} \right] dy = C$

S₂: $\int N dy + \int \left[M - \frac{\partial}{\partial x} \{ N dy \} \right] dx = C$

$$\frac{du}{dx} = - \frac{2u(4u^2 + u - 6)}{x(6u^2 + u - 3)}$$

$$x(6u^2 + u - 3) \frac{dy}{dx} = -2u(4u^2 + u - 6)$$

$$2u(4u^2 + u - 6) + x(6u^2 + u - 3) \frac{du}{dx} = 0$$

$$M(x, u) + N(x, u) \frac{du}{dx} = 0$$

$$P(x) = 1$$

$$Q(u) = 2u(4u^2 + u - 6)$$

$$R(x) = x$$

$$S(u) = 6u^2 + u - 3$$

$$\int \frac{P}{R} dx + \int \frac{S}{Q} du = C,$$

$$M = (x^3 + xy^2) \quad N = (x^2y + y^3)$$

$$\frac{\partial M}{\partial y} = 2xy \quad \frac{\partial N}{\partial x} = 2xy$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \therefore \text{is exact}$$

$$\int M dx = \int x^2 dx + y^2 \int x dx$$

$$= \frac{x^4}{4} + y^2 \frac{x^2}{2} \Rightarrow \frac{1}{4}x^4 + \frac{1}{2}x^2y^2$$

$$\int N dy = x^2 \int y dy + \int y^3 dy$$

$$= x^2 \left(\frac{y^2}{2} \right) + \left(\frac{y^4}{4} \right)$$

$$= \frac{1}{2}x^2y^2 + \frac{1}{4}y^4$$

$$\frac{1}{4}x^4 + \frac{1}{2}x^2y^2 + \frac{1}{4}y^4$$

Sol. Gral

$$\frac{1}{4}x^4 + \frac{1}{3}x^2y^2 + \frac{1}{4}y^4 = C,$$

$$x^4 + 2x^2y^2 + y^4 = 4C,$$
$$(x^2 + y^2)^2 = 4C,$$

$$\int M dx + \int \left[N - \frac{\partial}{\partial y} (M dx) \right] dy = C,$$

$$\int M dx = \frac{x^4}{4} + \frac{1}{2}x^2y^2$$

$$\frac{\partial}{\partial y} \{ M dx \} = x^2y$$

$$\frac{x^4}{4} + \frac{1}{2}x^2y^2 + \int \left[(\cancel{x^3y} + y^3) - \cancel{x^2y} \right] dy = C,$$

$$\frac{x^4}{4} + \frac{1}{2}x^2y^2 + \int y^3 dy = C,$$

$$\frac{x^4}{4} + \frac{1}{2}x^2y^2 + \frac{y^4}{4} = C,$$

$$M(x, y) + N(x, y) \frac{dy}{dx} = 0$$

$$N(x, y) \frac{dy}{dx} = -M(x, y)$$

$$\frac{dy}{dx} = -\frac{M(x, y)}{N(x, y)}.$$