Capítulo II.- Ecuación Lineal

EDO (n) L $\underline{\underline{CC}}$ $\begin{cases} \text{Hom.} \\ \text{No Hom.} \end{cases}$

$$a_0(x)\frac{d^n y}{dx^n} + a_1(x)\frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_{n-1}(x)\frac{dy}{dx} + a_n(x)\,y = Q(x)$$

LINEAL

$$\boxed{\frac{d^n y}{dx^n} + a_1\frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_{n-1}\frac{dy}{dx} + a_n\,y = Q(x).}$$

$$\frac{dy}{dx} + y = 0$$

$$\frac{dy}{dx} = -y$$

$$\int \frac{dy}{y} = -\int dx$$

$$\ln y + C_1 = -x + C_2$$

$$\ln y = -x + (C_2 - C_1)$$

$$\ln y = -x + C_3$$

$$y = e^{-x + C_3}$$

$$y = C_4 e^{-x}$$

$$\boxed{e^{mx}}$$

$$\frac{dy}{dx} + y = 0 \qquad y_s = e^{mx}$$

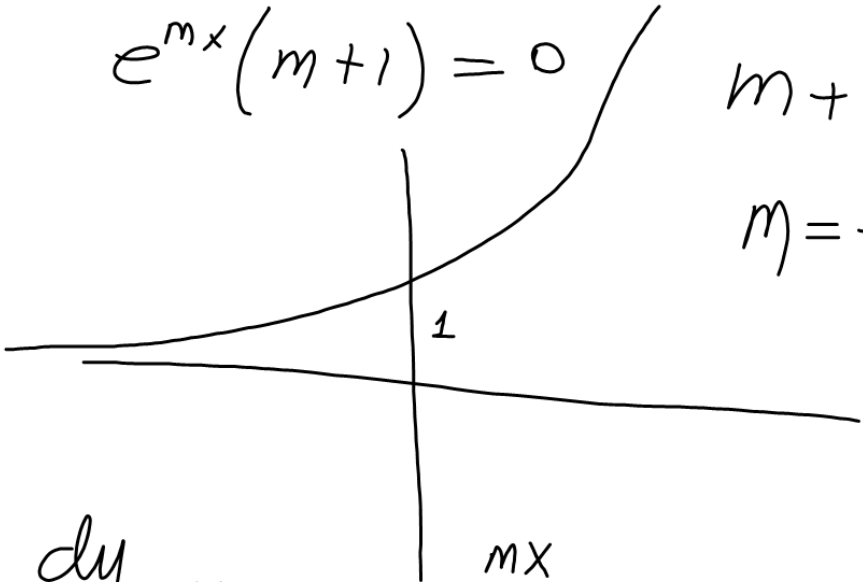
$$\frac{dy}{dx} = e^{mx} \cdot (m)$$

$$me^{mx} + e^{mx} = 0$$

$$e^{mx}(m+1) = 0$$

$$m+1 = 0$$

$$m = -1$$



$$\frac{dy}{dx} + y = 0 \qquad e^{mx} \qquad m = -1$$

$$y = C_1 e^{-x}$$

$$\frac{dy}{dx} - \sqrt{2}\, y = 0 \qquad EDO(1)\,L\,CC\,H.$$

$$y = e^{mx} \quad \rightarrow \frac{dy}{dx} = me^{mx}$$

$$me^{mx} - \sqrt{2}\, e^{mx} = 0$$

$$e^{mx}\left(m - \sqrt{2}\right) = 0$$

$$m - \sqrt{2} = 0$$

ECUACIÓN (ALGEBRAICA) CARACTERÍSTICA.

$$m = \sqrt{2}$$

$$y_g = C_1 e^{\sqrt{2}x}$$

SOLUCIÓN PARTICULAR FUNDAMENTAL.

EDO(2) L cc H.

CASO I:

$m_1 \neq m_2$ $\quad \dfrac{d^2y}{dx^2} + a_1 \dfrac{dy}{dx} + a_2 y = 0$

$y = e^{mx}$

$\dfrac{dy}{dx} = m e^{mx}$

$\dfrac{d^2y}{dx^2} = m^2 e^{mx}$

$$m^2 e^{mx} + a_1 m e^{mx} + a_2 e^{mx} = 0$$

$$(m^2 + a_1 m + a_2) e^{mx} = 0$$

$$m^2 + a_1 m + a_2 = 0$$ ECUACIÓN CARACTERÍSTICA

$m_1 \quad m_2$

$e^{m_1 x} \quad e^{m_2 x}$

$$\boxed{y_g = C_1 e^{m_1 x} + C_2 e^{m_2 x}}$$

$$W \Rightarrow \begin{vmatrix} e^{m_1 x} & e^{m_2 x} \\ m_1 e^{m_1 x} & m_2 e^{m_2 x} \end{vmatrix} \neq 0$$

$$m_2 e^{m_2 x} e^{m_1 x} - m_1 e^{m_1 x} e^{m_2 x} \neq 0$$

$$(m_2 - m_1) e^{m_1 x} e^{m_2 x} \neq 0$$

$$(m_2 - m_1) \neq 0$$

$$m_2 \neq m_1$$

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

$$m^2 - 5m + 6 = 0$$

$$(m-2)(m-3) = 0$$

$m_1 = 2$
$m_2 = 3$

$$\boxed{y_g = C_1 e^{2x} + C_2 e^{3x}}$$

CASO II.-

$$\frac{d^2y}{dx^2}+a_1\frac{dy}{dx}+a_2y=0$$

$$m^2+a_1m+a_2=0 \qquad m_1=m_2$$

$$y_g=C_1e^{m_1x}+C_2\boxed{}$$

$$m^2+a_1m+a_2=0 \qquad m_1=m_2$$

$$\frac{d}{dm}$$

$$2m+a_1=0 \qquad m_1$$

$$(m-m_1)^2=0$$

$$\frac{d}{dm}$$

$$2(m-m_1)=0$$

$$m^2+a_1m+a_2=0 \qquad m_1\neq m_2$$

$$\frac{d}{dm}$$

$$2m+a_1=0 \qquad \cancel{m_1} \quad \cancel{m_1}$$

$$(m-m_1)(m-m_2)=0$$

$$\frac{d}{dm}$$

$$(m-m_1)+(m-m_2)\neq 0$$

$$\frac{d^2y}{dx^2}+a_1\frac{dy}{dx}+a_2y=0$$

$$m^2+a_1m+a_2=0 \qquad m_1=m_2$$

$$\frac{d}{dm}\left(\begin{array}{l} e^{mx} \xrightarrow{m=m_1} e^{m_1x} \\ xe^{mx} \xrightarrow{m=m_1} xe^{m_1x} \end{array}\right.$$

$$y_g=C_1e^{m_1x}+C_2xe^{m_1x}$$

$$y_p=xe^{m_1x}$$

$$\frac{dy}{dx}=m_1xe^{m_1x}+e^{m_1x}$$

$$\frac{d^2y}{dx^2}=m_1^2xe^{m_1x}+m_1e^{m_1x}+m_1e^{m_1x}$$

$$=m_1^2xe^{m_1x}+2m_1e^{m_1x}$$

$$\frac{d^2y}{dx^2} \Leftrightarrow m_1^2xe^{m_1x}+2m_1e^{m_1x}$$

$$+$$

$$a_1\frac{dy}{dx} \Leftrightarrow a_1m_1xe^{m_1x}+a_1e^{m_1x}$$

$$+$$

$$a_2y \Leftrightarrow a_2xe^{m_1x}$$

$$=$$

$$0 \Leftrightarrow (m_1^2+a_1m_1+a_2)xe^{m_1x}$$

$$+(2m_1+a_1)e^{m_1x}$$

CASO III.- Raíces complejas

$$\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = 0$$

$$m^2 + a_1 m + a_2 = 0$$

$$m_1 \neq m_2 \qquad \begin{array}{ll} m_1 = a + bi & a \in \mathbb{R} \quad b \in \mathbb{R}^+ \\ m_2 = a - bi & a \in \mathbb{R} \quad b \in \mathbb{R}^+ \end{array}$$

$$y_g = c_1 e^{(a+bi)x} + c_2 e^{(a-bi)x}$$

$$= e^{ax}\left(c_1 e^{bxi} + c_2 e^{-bxi}\right)$$

$x \in \mathbb{R}$ $\quad c_1 \in \mathbb{C}$ $\quad c_2 \in \mathbb{C}$.

$y \in \mathbb{R}$