TEMA 3a: La Transformada de Laplace.

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t)\,dt \Rightarrow F(s)$$

$t, f \in \mathbb{R}$ $\qquad$ $F \in \mathbb{R}$, $s \in \mathbb{C}$

$$\mathcal{L}^{-1}\{F(s)\} = f(t)$$ no es única

1) $\mathcal{L}\{af(t)+bg(t)\}=aF(s)+bG(s)$

$a,b\in\mathbb{R}$

2) $\mathcal{L}\{f(at)\}=\frac{1}{a}F\left(\frac{s}{a}\right)$

3) $\mathcal{L}\{f'(t)\}=sF(s)-f(0)$

$\mathcal{L}\{f''(t)\}=s^2F(s)-sf(0)-f'(0)$

$\mathcal{L}\{f'''(t)\}=s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

4) $\mathcal{L}^{-1}\{F'(s)\}=-tf(t)$

$\mathcal{L}^{-1}\{F^{(n)}(s)\}=(-1)^n t^n f(t)$

5) $\mathcal{L}\left\{ \int_0^t f(\tau)\,d\tau \right\} = \dfrac{F(s)}{s}$

6) $\mathcal{L}^{-1}\left\{ \int_s^{\infty} F(\sigma)\,d\sigma \right\} = \dfrac{f(t)}{t}$

7) $f_u(t-a) = \begin{cases} 0 & ; t < a \\ f(t-a) & ; t \geq a \end{cases}$

$\mathcal{L}\{ f_u(t-a) \} = e^{-sa} F(s)$

8) $\mathcal{L}\{ e^{at} f(t) \} = F(s-a)$

9) $f(t) * g(t) = \int^t f(\tau) \cdot g(t-\tau)\,d\tau.$

operación convolución.

$\mathcal{L}^{-1}\{ F(s) \cdot G(s) \} = f(t) * g(t)$

$f(t)$	$F(s)$
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\cos(bt)$	$\frac{s}{s^2+b^2}$
$\operatorname{sen}(bt)$	$\frac{b}{s^2+b^2}$

$$\mathcal{L}\{t^2 e^{5t}\} = \frac{2}{(s-5)^3}$$

$$\mathcal{L}\{t^2\} = \frac{2!}{s^3}$$

$$\mathcal{L}\{e^{at} f(t)\} = F(s-a)$$

$$\mathcal{L}\{e^{-3t}\cos(4t)\} = \frac{(s+3)}{(s+3)^2+16}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+s+1}\right\}=\mathcal{L}^{-1}\left\{\frac{s}{(s^2+s+(\ \ ))+1-(\ \ )}\right\}$$

$$=\mathcal{L}^{-1}\left\{\frac{s}{\left(s^2+s+\frac{1}{4}\right)+\frac{3}{4}}\right\}$$

$$=\mathcal{L}^{-1}\left\{\frac{\left(s+\frac{1}{2}\right)-\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+s+1}\right\}=\mathcal{L}^{-1}\left\{\frac{\left(s+\frac{1}{2}\right)}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right\}-\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}\mathcal{L}^{-1}\left\{\frac{\frac{\sqrt{3}}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+s+1}\right\}=e^{-\frac{1}{2}t}\cos\left(\frac{\sqrt{3}}{2}t\right)-\frac{1}{\sqrt{3}}e^{-\frac{1}{2}t}\operatorname{sen}\left(\frac{\sqrt{3}}{2}t\right).$$

$$y'' + 3y' + 2y = 2e^{4x} + x^3 + \operatorname{sen}(2x)$$

$$y(0) = 4 \qquad y'(0) = -6$$

$$\mathcal{L}\{y'' + 3y' + 2y\} = \mathcal{L}\{2e^{4x} + x^3 + \operatorname{sen}(2x)\}$$

$$\left(s^2 Y(s) - s(4) - (-6)\right) + 3\left(s Y(s) - (4)\right) +$$

$$+ 2Y(s) = \frac{2}{s-4} + \frac{3!}{s^4} + \frac{2}{s^2+4}$$

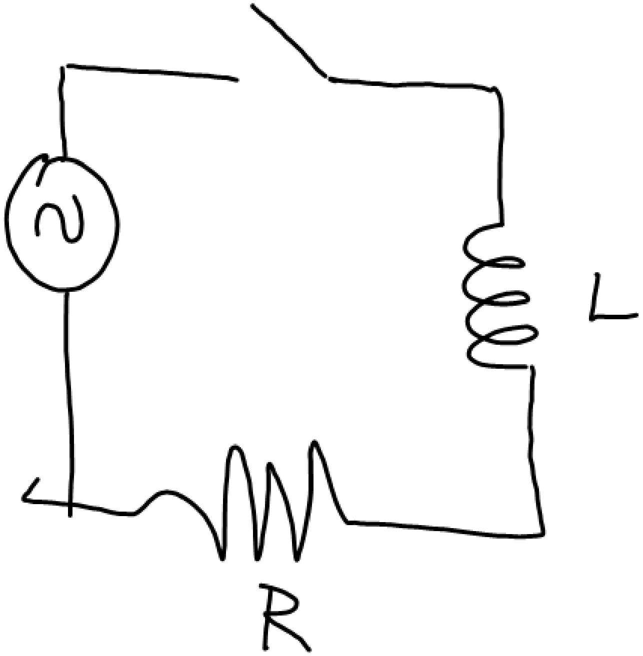
$$(s^2+3s+2)\,Y(s) - 4s - 6 = \frac{2s^4(s^2+4) + 6(s-4)(s^2+4) + 2s^4(s-4)}{(s-4)\,s^4(s^2+4)}$$

$$(s^2+3s+2)\,Y(s) = \frac{2s^6 + 8s^4 + 6s^3 - 24s^2 + 24s - 96 + 2s^5 - 8s^4}{(s-4)\,s^4(s^2+4)} + 4s + 6$$

$$L\frac{di}{dt} + Ri = E(t)\cdot u(t-2) \qquad i(0) = 0$$

$R = 10$ ohm

$L = 3$ henry



$$E(t) = 120\cos(60\pi t)$$

TEMA 3b.) Sistemas de Ecuaciones Diferenciales Lineales.

$$\frac{dx_1}{dt} = 2x_1 + 3x_2$$

$$\frac{dx_2}{dt} = x_1 + 4x_2$$

$$x_1 = \frac{dx_2}{dt} - 4x_2$$

$$\frac{dx_1}{dt} = \frac{d^2x_2}{dt^2} - 4\frac{dx_2}{dt}$$

$$\frac{d^2x_2}{dt^2} - 4\frac{dx_2}{dt} = 2\left(\frac{dx_2}{dt} - 4x_2\right) + 3x_2$$

$$\frac{d^2x_2}{dt^2} - 6\frac{dx_2}{dt} - 5x_2 = 0$$

$$(D^2 - 6D - 5)x_2 = 0$$

$$m^2 - 6m - 5 = 0 \qquad m = \frac{6 \pm \sqrt{36+20}}{2}$$

$$(m - 6.74)(m + 0.74) = 0 \qquad m = \frac{6 \pm \sqrt{56}}{2}$$

$$m = \frac{6 \pm 2\sqrt{14}}{2}$$

$$m = 3 \pm \sqrt{14}$$

$$m_1 = 3 + \sqrt{14} \qquad m_2 = 3 - \sqrt{14}$$

$$x_2(t) = C_1 e^{6.74t} + C_2 e^{-0.74t}$$

$$\frac{dx_2}{dt} = 6.74 C_1 e^{6.74t} - 0.74 C_2 e^{-0.74t}$$

$$-4x_2 = -4C_1 e^{6.74t} - 4C_2 e^{-0.74t}$$

$$\boxed{\begin{aligned} x_1(t) &= 2.74 C_1 e^{6.74t} - 4.74 C_2 e^{-0.74t} \\ x_2(t) &= C_1 e^{6.74t} + C_2 e^{-0.74t} \end{aligned}}$$

Matriz Exponencial

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

$$\bar{X} = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \qquad \frac{d}{dt}\bar{X} = \begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix}$$

$$\frac{d}{dt}\bar{X} = A\bar{X} \qquad \bar{X}(0)$$

$$\bar{X}(t) = \left[e^{At}\right]\bar{X}(0)$$

$$\frac{d}{dt}e^{At} = A \times e^{At}$$

$$e^{At}\Big|_{t=0} = I$$

$$\left[e^{At}\right]^{-1} = e^{At}\Big|_{t=-t}$$

$$e^{At} \times e^{A(-t)} = I$$