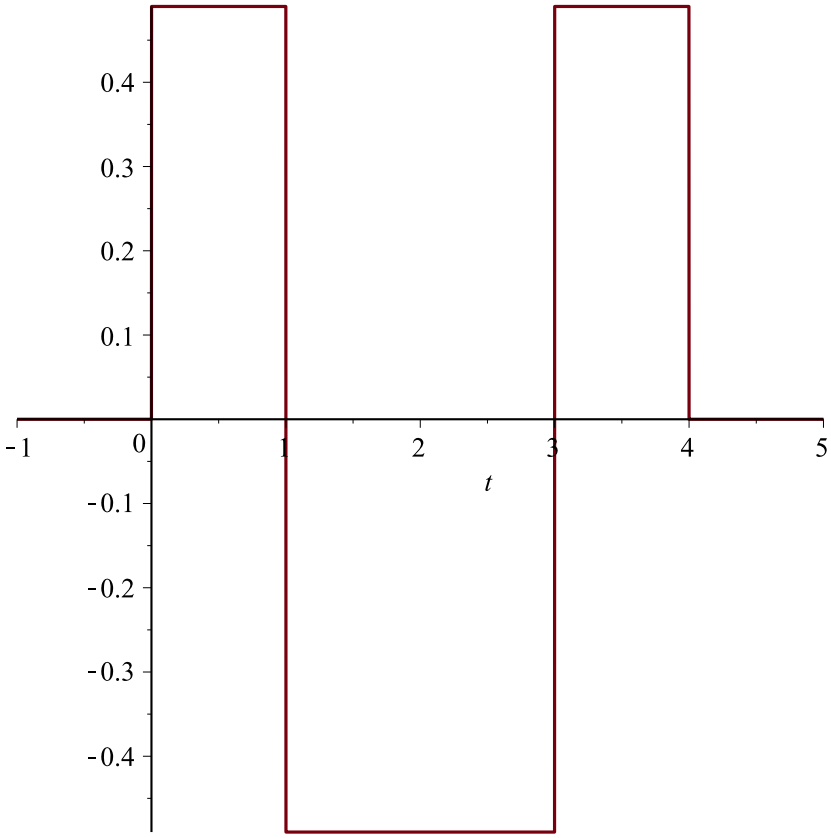
```
> restart
> S := 49/100 ·Heaviside(t) − 2·49/100 ·Heaviside(t − a) + 2·49/100 ·Heaviside(t − 3·a) − 49/100
      ·Heaviside(t − 4·a)
```

$$S := \frac{49}{100}\,\text{Heaviside}(t) - \frac{49}{50}\,\text{Heaviside}(t-a) + \frac{49}{50}\,\text{Heaviside}(t-3\,a) - \frac{49}{100}\,\text{Heaviside}(t-4\,a) \quad (1)$$

```
> plot(subs(a = 1, S), t = -1 ..5)
```




```
> Ecua := diff(y(t), t$3) = S
```

$$Ecua := \frac{d^3}{dt^3}\,y(t) = \frac{49}{100}\,\text{Heaviside}(t) - \frac{49}{50}\,\text{Heaviside}(t-a) + \frac{49}{50}\,\text{Heaviside}(t-3\,a) - \frac{49}{100}\,\text{Heaviside}(t-4\,a) \quad (2)$$

```
> CondIni := y(0) = 0, D(y)(0) = 0, D(D(y))(0) = 0
```

$$CondIni := y(0) = 0,\ D(y)(0) = 0,\ D^{(2)}(y)(0) = 0 \quad (3)$$

```
> with(inttrans) :
> EcuaTransLap := subs(CondIni, laplace(Ecua, t, s))
```

(4)

$$EcuaTransLap := s^3\, laplace(y(t), t, s) = \frac{49}{100\, s} - \frac{49}{50}\, laplace(\text{Heaviside}(t-a), t, s) + \frac{49}{50}\, laplace(\text{Heaviside}(t-3\,a), t, s) - \frac{49}{100}\, laplace(\text{Heaviside}(t-4\,a), t, s) \quad (4)$$

> $SolTransLap := isolate(EcuaTransLap, laplace(y(t), t, s))$

$$SolTransLap := laplace(y(t), t, s) = \frac{1}{s^3}\left(\frac{49}{100\, s} - \frac{49}{50}\, laplace(\text{Heaviside}(t-a), t, s) + \frac{49}{50}\, laplace(\text{Heaviside}(t-3\,a), t, s) - \frac{49}{100}\, laplace(\text{Heaviside}(t-4\,a), t, s)\right) \quad (5)$$

> $SolPart := invlaplace(SolTransLap, s, t)$

$$SolPart := y(t) = \frac{49}{600}\, t^3 - \frac{49}{50}\, \text{Heaviside}(-a)\, a^3 - \frac{49}{600}\, \text{Heaviside}(t-4\,a)\, (t-4\,a)^3 + \frac{49}{300}\, \text{Heaviside}(t-3\,a)\, (t-3\,a)^3 - \frac{49}{300}\, \text{Heaviside}(t-a)\, (t-a)^3 \quad (6)$$

> $SolReal := subs(t = 4\cdot a, rhs(SolPart) = 225)$

$$SolReal := \frac{392}{75}\, a^3 - \frac{49}{50}\, \text{Heaviside}(-a)\, a^3 + \frac{49}{300}\, \text{Heaviside}(a)\, a^3 - \frac{441}{100}\, \text{Heaviside}(3\,a)\, a^3 = 225 \quad (7)$$

> $Para := solve(subs(\text{Heaviside}(a) = 1, \text{Heaviside}(3\cdot a) = 0, \text{Heaviside}(-a) = 0, SolReal), a);$
> $evalf(\%, 5)$

$$Para := \frac{5}{7}\, 630^{1/3},\ -\frac{5}{14}\, 630^{1/3} + \frac{5}{14}\, I\sqrt{3}\, 630^{1/3},\ -\frac{5}{14}\, 630^{1/3} - \frac{5}{14}\, I\sqrt{3}\, 630^{1/3}$$

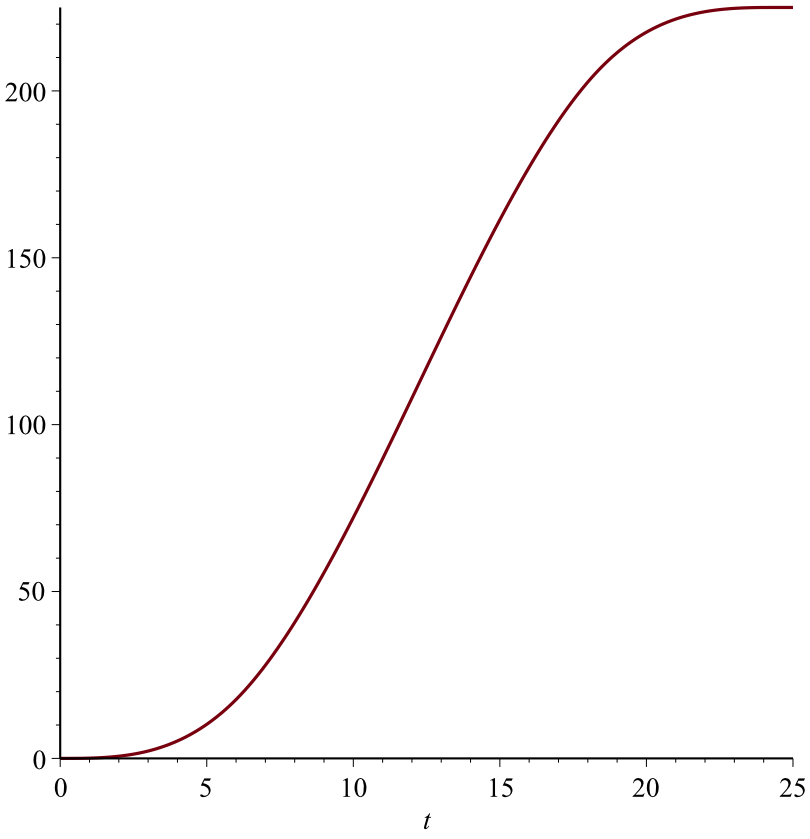
$$6.1233,\ -3.0616 + 5.3032\, I,\ -3.0616 - 5.3032\, I \quad (8)$$

> $TiempoFinal := Para[1]\cdot 4; evalf(\%, 5)$

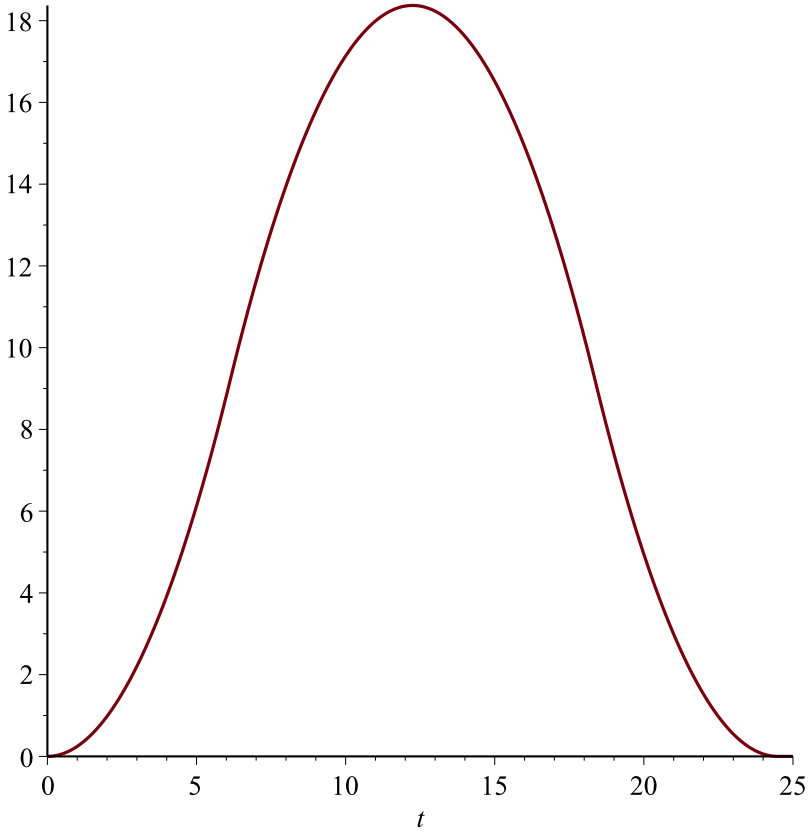
$$TiempoFinal := \frac{20}{7}\, 630^{1/3}$$

$$24.493 \quad (9)$$

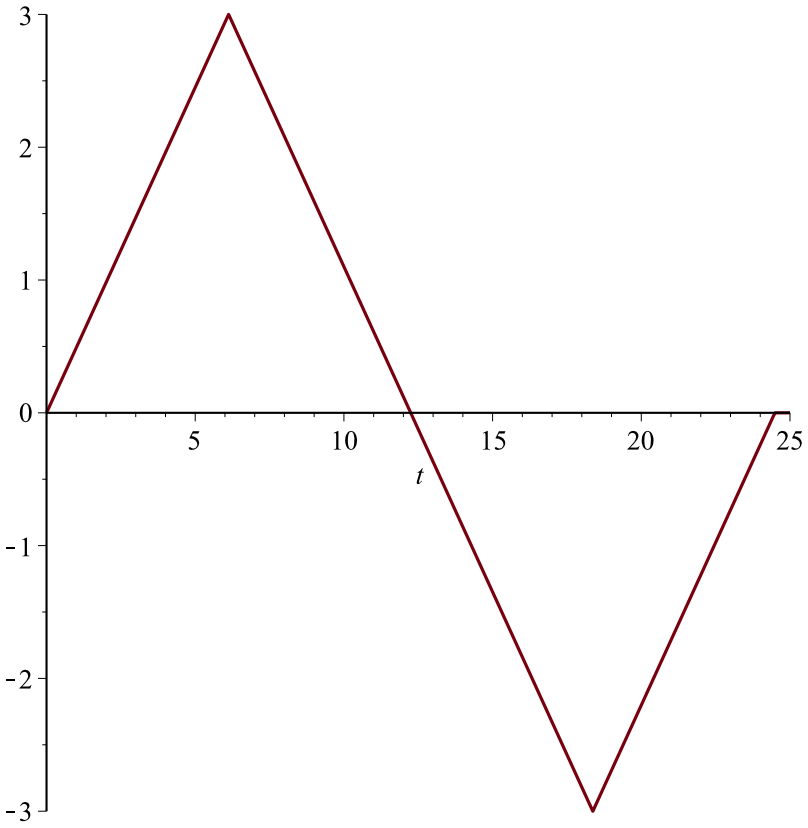
> $SolucionFinal := subs(a = Para[1], SolPart) : plot(rhs(SolucionFinal), t = 0..25)$



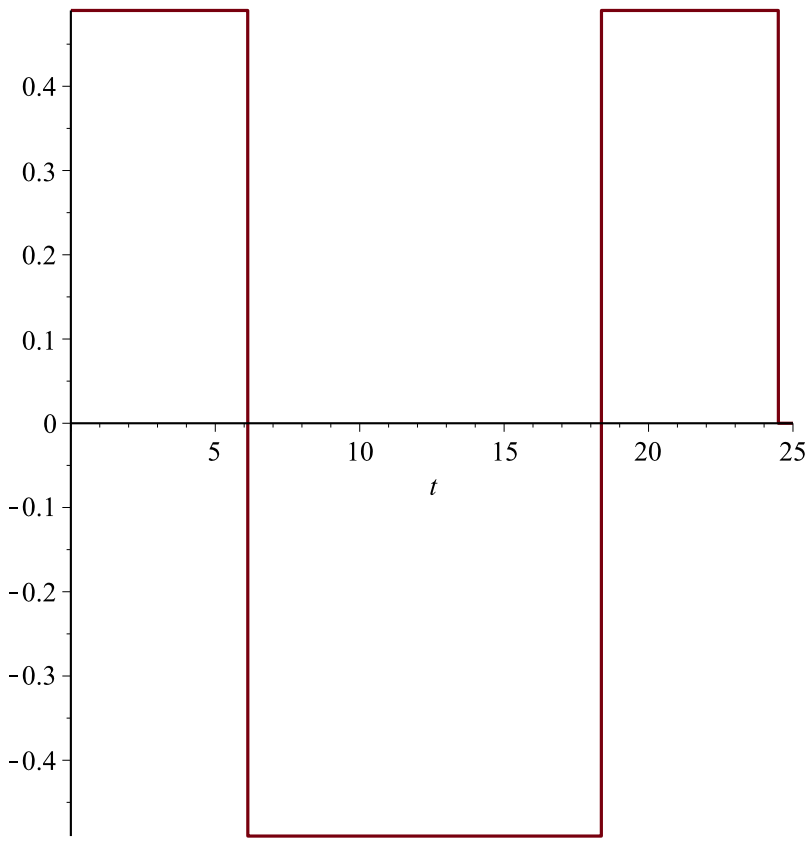
> $plot(rhs(diff(SolucionFinal, t)), t = 0..25)$



> $plot(rhs(diff(SolucionFinal, t\$2)), t = 0..25)$




```
> plot(rhs(diff(SolucionFinal, t$3)), t = 0..25)
```



> *restart*

> $Sistema := diff(h[1](t), t) = -\frac{4}{3}\cdot h[1](t) + \frac{4}{3}\cdot h[2](t) + 5,\ diff(h[2](t), t) = \frac{4}{24}\cdot h[1](t) - \frac{40}{24}\cdot h[2](t)$: $Sistema[1]$; $Sistema[2]$

$$\frac{d}{dt} h_1(t) = -\frac{4}{3} h_1(t) + \frac{4}{3} h_2(t) + 5$$

$$\frac{d}{dt} h_2(t) = \frac{1}{6} h_1(t) - \frac{5}{3} h_2(t) \quad \textbf{(10)}$$

> $AA := array\left(\left[\left[-\frac{4}{3}, \frac{4}{3}\right], \left[\frac{4}{24}, -\frac{40}{24}\right]\right]\right)$

$$AA := \begin{bmatrix} -\frac{4}{3} & \frac{4}{3} \\ \frac{1}{6} & -\frac{5}{3} \end{bmatrix} \quad \textbf{(11)}$$

> $Xcero := array([1, 1])$

$$Xcero := \begin{bmatrix} 1 & 1 \end{bmatrix} \quad \textbf{(12)}$$

```
> BB := array([5, 0])
```

$$BB := \begin{bmatrix} 5 & 0 \end{bmatrix} \tag{13}$$

```
> with(linalg) :
> MatExp := exponential(AA, t)
```

$$MatExp := \begin{bmatrix} \frac{1}{3}\,e^{-2t} + \frac{2}{3}\,e^{-t} & \frac{4}{3}\,e^{-t} - \frac{4}{3}\,e^{-2t} \\ \frac{1}{6}\,e^{-t} - \frac{1}{6}\,e^{-2t} & \frac{2}{3}\,e^{-2t} + \frac{1}{3}\,e^{-t} \end{bmatrix} \tag{14}$$

```
> SolHom := evalm(MatExp &* Xcero) : SolHom[1]; SolHom[2]
```

$$-e^{-2t} + 2\,e^{-t}$$

$$\frac{1}{2}\,e^{-t} + \frac{1}{2}\,e^{-2t} \tag{15}$$

```
> MatExpTau := map(rcurry(eval, t = t - tau), MatExp)
```

$$MatExpTau := \begin{bmatrix} \frac{1}{3}\,e^{-2t+2\tau} + \frac{2}{3}\,e^{-t+\tau} & \frac{4}{3}\,e^{-t+\tau} - \frac{4}{3}\,e^{-2t+2\tau} \\ \frac{1}{6}\,e^{-t+\tau} - \frac{1}{6}\,e^{-2t+2\tau} & \frac{2}{3}\,e^{-2t+2\tau} + \frac{1}{3}\,e^{-t+\tau} \end{bmatrix} \tag{16}$$

```
> ProdTau := evalm(MatExpTau &* BB) : ProdTau[1]; ProdTau[2]
```

$$\frac{5}{3}\,e^{-2t+2\tau} + \frac{10}{3}\,e^{-t+\tau}$$

$$\frac{5}{6}\,e^{-t+\tau} - \frac{5}{6}\,e^{-2t+2\tau} \tag{17}$$

```
> SolNoHom := map(int, ProdTau, tau = 0..t) : SolNoHom[1]; SolNoHom[2]
```

$$-\frac{5}{6}\,e^{-2t} - \frac{10}{3}\,e^{-t} + \frac{25}{6}$$

$$-\frac{5}{6}\,e^{-t} + \frac{5}{12}\,e^{-2t} + \frac{5}{12} \tag{18}$$

```
> SolFinal := evalm(SolHom + SolNoHom) : h[1](t) = SolFinal[1]; h[2](t) = SolFinal[2];
```

$$h_1(t) = -\frac{11}{6}\,e^{-2t} - \frac{4}{3}\,e^{-t} + \frac{25}{6}$$

$$h_2(t) = -\frac{1}{3}\,e^{-t} + \frac{11}{12}\,e^{-2t} + \frac{5}{12} \tag{19}$$

```
>
```