```
> restart
> Ecuacion := y'' - 5 y' + 6 y = 4·exp(x)
```

$$Ecuacion := \frac{d^2}{dx^2} y(x) - 5 \left(\frac{d}{dx} y(x) \right) + 6 y(x) = 4 e^x \tag{1}$$

Ecuacion Diferencial Ordinaria segundo orden Lineal coeficientes constantes No Homogenea

```
> EcuacionHom := lhs(Ecuacion) = 0
```

$$EcuacionHom := \frac{d^2}{dx^2} y(x) - 5 \left(\frac{d}{dx} y(x) \right) + 6 y(x) = 0 \tag{2}$$

```
> Q := rhs(Ecuacion)
```

$$Q := 4 e^x \tag{3}$$

```
> EcuaCarac := m^2 - 5 m + 6 = 0
```

$$EcuaCarac := m^2 - 5 m + 6 = 0 \tag{4}$$

```
> Raiz := solve(EcuaCarac)
```

$$Raiz := 3, 2 \tag{5}$$

```
> yy[1] := exp(Raiz[1]·x); yy[2] := exp(Raiz[2]·x)
```

$$yy_1 := e^{3x}$$

$$yy_2 := e^{2x} \tag{6}$$

```
> SolHom := y(x) = _C1·yy[1] + _C2·yy[2]
```

$$SolHom := y(x) = \_C1 \, e^{3x} + \_C2 \, e^{2x} \tag{7}$$

MÉTODO DE PARÁMETROS VARIABLES

```
> SolNoHom := y(x) = A·yy[1] + B·yy[2]
```

$$SolNoHom := y(x) = A \, e^{3x} + B \, e^{2x} \tag{8}$$

```
> with(linalg) :
> WW := wronskian([yy[1], yy[2]], x)
```

$$WW := \begin{bmatrix} e^{3x} & e^{2x} \\ 3 e^{3x} & 2 e^{2x} \end{bmatrix} \tag{9}$$

```
> BB := array([0, Q])
```

$$BB := \begin{bmatrix} 0 & 4 e^x \end{bmatrix} \tag{10}$$

```
> ParamVar := simplify(linsolve(WW, BB))
```

$$ParamVar := \begin{bmatrix} 4 e^{-2x} & -4 e^{-x} \end{bmatrix} \tag{11}$$

```
> Aprima := ParamVar[1]
```

$$Aprima := 4 e^{-2x} \tag{12}$$

```
> Bprima := ParamVar[2]
```

$$Bprima := -4 e^{-x} \tag{13}$$

```
> A := (int(Aprima, x) + _C1)
```

$$A := -2 e^{-2x} + \_C1 \tag{14}$$

```
> B := (int(Bprima, x) + _C2)
```

$$B := 4 e^{-x} + \_C2 \tag{15}$$

> *SolNoHom*

$$y(x) = \left(-2\,e^{-2x} + \_C1\right) e^{3x} + \left(4\,e^{-x} + \_C2\right) e^{2x} \tag{16}$$

> *SolFinal* := *simplify*(*SolNoHom*)

$$SolFinal := y(x) = 2\,e^x + \_C2\,e^{2x} + \_C1\,e^{3x} \tag{17}$$

> *Solucion* := *dsolve*(*Ecuacion*)

$$Solucion := y(x) = e^{3x}\,\_C2 + e^{2x}\,\_C1 + 2\,e^x \tag{18}$$

MÉTODO DE COEFICIENTES INDETERMINADOS

> *Ecuacion*

$$\frac{d^2}{dx^2}\,y(x) - 5\left(\frac{d}{dx}\,y(x)\right) + 6\,y(x) = 4\,e^x \tag{19}$$

> *EcuacionHom*

$$\frac{d^2}{dx^2}\,y(x) - 5\left(\frac{d}{dx}\,y(x)\right) + 6\,y(x) = 0 \tag{20}$$

> *Q*

$$4\,e^x \tag{21}$$

> *yy*[1]; *yy*[2]

$$e^{3x}$$
$$e^{2x} \tag{22}$$

> *yy*[3] := *exp*(x)

$$yy_3 := e^x \tag{23}$$

> *SolNoHom* := y(x) = *rhs*(*SolHom*) + *AA*·*yy*[3]

$$SolNoHom := y(x) = \_C1\,e^{3x} + \_C2\,e^{2x} + AA\,e^x \tag{24}$$

> *SolPart* := y(x) = *AA*·*yy*[3]

$$SolPart := y(x) = AA\,e^x \tag{25}$$

> *Parametro* := *isolate*(*eval*(*subs*(y(x) = *rhs*(*SolPart*), *Ecuacion*)), *AA*)

$$Parametro := AA = 2 \tag{26}$$

> *SolucionUltima* := *subs*(*AA* = *rhs*(*Parametro*), *SolNoHom*)

$$SolucionUltima := y(x) = 2\,e^x + \_C2\,e^{2x} + \_C1\,e^{3x} \tag{27}$$

> *Solucion*

$$y(x) = e^{3x}\,\_C2 + e^{2x}\,\_C1 + 2\,e^x \tag{28}$$

> *SolFinal*

$$y(x) = 2\,e^x + \_C2\,e^{2x} + \_C1\,e^{3x} \tag{29}$$

> *restart*

> *Ecuacion* := $y''' + y'' + y' + y = 4\exp(2x) + 3\cos(5x)$

$$Ecuacion := \frac{d^3}{dx^3}\,y(x) + \frac{d^2}{dx^2}\,y(x) + \frac{d}{dx}\,y(x) + y(x) = 4\,e^{2x} + 3\cos(5x) \tag{30}$$

> *CondIni* := $y(0) = 6,\ D(y)(0) = -8,\ D(D(y))(0) = 3$

$$CondIni := y(0) = 6,\ D(y)(0) = -8,\ D^{(2)}(y)(0) = 3 \tag{31}$$

> *EcuacionHom* := *lhs*(*Ecuacion*) = 0

$$\tag{32}$$

$$EcuacionHom := \frac{d^3}{dx^3}\, y(x) + \frac{d^2}{dx^2}\, y(x) + \frac{d}{dx}\, y(x) + y(x) = 0 \tag{32}$$

> $Q := rhs(Ecuacion)$

$$Q := 4\, e^{2x} + 3\, \cos(5\, x) \tag{33}$$

> $EcuaCarac := m^3 + m^2 + m + 1 = 0$

$$EcuaCarac := m^3 + m^2 + m + 1 = 0 \tag{34}$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := -1, I, -I \tag{35}$$

> $yy[1] := \exp(Raiz[1]\,x)$

$$yy_1 := e^{-x} \tag{36}$$

> $yy[2] := \cos(Im(Raiz[2]) \cdot x)$

$$yy_2 := \cos(x) \tag{37}$$

> $yy[3] := \sin(Im(Raiz[2]) \cdot x)$

$$yy_3 := \sin(x) \tag{38}$$

> $SolHom := y(x) = \_C1 \cdot yy[1] + \_C2 \cdot yy[2] + \_C3 \cdot yy[3]$

$$SolHom := y(x) = \_C1\, e^{-x} + \_C2\, \cos(x) + \_C3\, \sin(x) \tag{39}$$

> $SolNoHom := y(x) = A \cdot yy[1] + B \cdot yy[2] + DD \cdot yy[3]$

$$SolNoHom := y(x) = A\, e^{-x} + B\, \cos(x) + DD\, \sin(x) \tag{40}$$

> $with(linalg):$
> $WW := wronskian([yy[1], yy[2], yy[3]], x)$

$$WW := \begin{bmatrix} e^{-x} & \cos(x) & \sin(x) \\ -e^{-x} & -\sin(x) & \cos(x) \\ e^{-x} & -\cos(x) & -\sin(x) \end{bmatrix} \tag{41}$$

> $BB := array([0, 0, Q])$

$$BB := \begin{bmatrix} 0 & 0 & 4\, e^{2x} + 3\, \cos(5\, x) \end{bmatrix} \tag{42}$$

> $ParamVar := simplify(linsolve(WW, BB))$

$$ParamVar := \left[\frac{1}{2}\, \left(4\, e^{2x} + 3\, \cos(5\, x)\right) e^x, -2\, \cos(x)\, e^{2x} - 2\, \sin(x)\, e^{2x} - \frac{3}{2}\, \cos(x)\, \cos(5\, x) \right. \tag{43}$$

$$\left. - \frac{3}{2}\, \sin(x)\, \cos(5\, x), 2\, \cos(x)\, e^{2x} - 2\, \sin(x)\, e^{2x} + \frac{3}{2}\, \cos(x)\, \cos(5\, x) \right.$$

$$\left. - \frac{3}{2}\, \sin(x)\, \cos(5\, x) \right]$$

> $Aprima := ParamVar[1]$

$$Aprima := \frac{1}{2}\, \left(4\, e^{2x} + 3\, \cos(5\, x)\right) e^x \tag{44}$$

> $Bprima := ParamVar[2]$

$$Bprima := -2\, \cos(x)\, e^{2x} - 2\, \sin(x)\, e^{2x} - \frac{3}{2}\, \cos(x)\, \cos(5\, x) - \frac{3}{2}\, \sin(x)\, \cos(5\, x) \tag{45}$$

> $Dprima := ParamVar[3]$

$$Dprima := 2\cos(x)\,e^{2x} - 2\sin(x)\,e^{2x} + \frac{3}{2}\cos(x)\cos(5x) - \frac{3}{2}\sin(x)\cos(5x) \tag{46}$$

> $A := simplify(int(Aprima, x) + \_C1)$

$$A := \frac{12}{13}\,e^x\cos(x)^5 + \frac{60}{13}\,e^x\cos(x)^4\sin(x) - \frac{15}{13}\,e^x\cos(x)^3 - \frac{45}{13}\,e^x\cos(x)^2\sin(x) + \frac{2}{3}\,e^{3x} \tag{47}$$

$$+ \frac{15}{52}\,e^x\cos(x) + \frac{15}{52}\,e^x\sin(x) + \_C1$$

> $B := simplify(int(Bprima, x) + \_C2)$

$$B := -\frac{2}{5}\cos(x)\,e^{2x} - \frac{6}{5}\sin(x)\,e^{2x} - \frac{3}{16}\sin(4x) - \frac{1}{8}\sin(6x) + \frac{1}{8}\cos(6x) \tag{48}$$

$$- \frac{3}{16}\cos(4x) + \_C2$$

> $DD := simplify(int(Dprima, x) + \_C3)$

$$DD := \frac{6}{5}\cos(x)\,e^{2x} - \frac{2}{5}\sin(x)\,e^{2x} + \frac{3}{16}\sin(4x) + \frac{1}{8}\sin(6x) + \frac{1}{8}\cos(6x) \tag{49}$$

$$- \frac{3}{16}\cos(4x) + \_C3$$

> $SolFinal := simplify(SolNoHom)$

$$SolFinal := y(x) = \frac{12}{13}\cos(x)^5 + \frac{60}{13}\cos(x)^4\sin(x) - \frac{15}{13}\cos(x)^3 - \frac{45}{13}\cos(x)^2\sin(x) \tag{50}$$

$$+ \frac{15}{52}\cos(x) + \frac{15}{52}\sin(x) + \frac{4}{15}\,e^{2x} - \frac{3}{16}\cos(x)\sin(4x) + \frac{3}{16}\sin(x)\sin(4x)$$

$$- \frac{1}{8}\cos(x)\sin(6x) + \frac{1}{8}\sin(x)\sin(6x) + \frac{1}{8}\cos(x)\cos(6x) + \frac{1}{8}\sin(x)\cos(6x)$$

$$- \frac{3}{16}\cos(x)\cos(4x) - \frac{3}{16}\sin(x)\cos(4x) + \_C2\cos(x) + \_C1\,e^{-x} + \_C3\sin(x)$$

> $Ecuacion$

$$\frac{d^3}{dx^3}y(x) + \frac{d^2}{dx^2}y(x) + \frac{d}{dx}y(x) + y(x) = 4\,e^{2x} + 3\cos(5x) \tag{51}$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolFinal), lhs(Ecuacion) - rhs(Ecuacion))))$
$$= 0$$

$$Comprobar := 0 = 0 \tag{52}$$

> $CondIni$
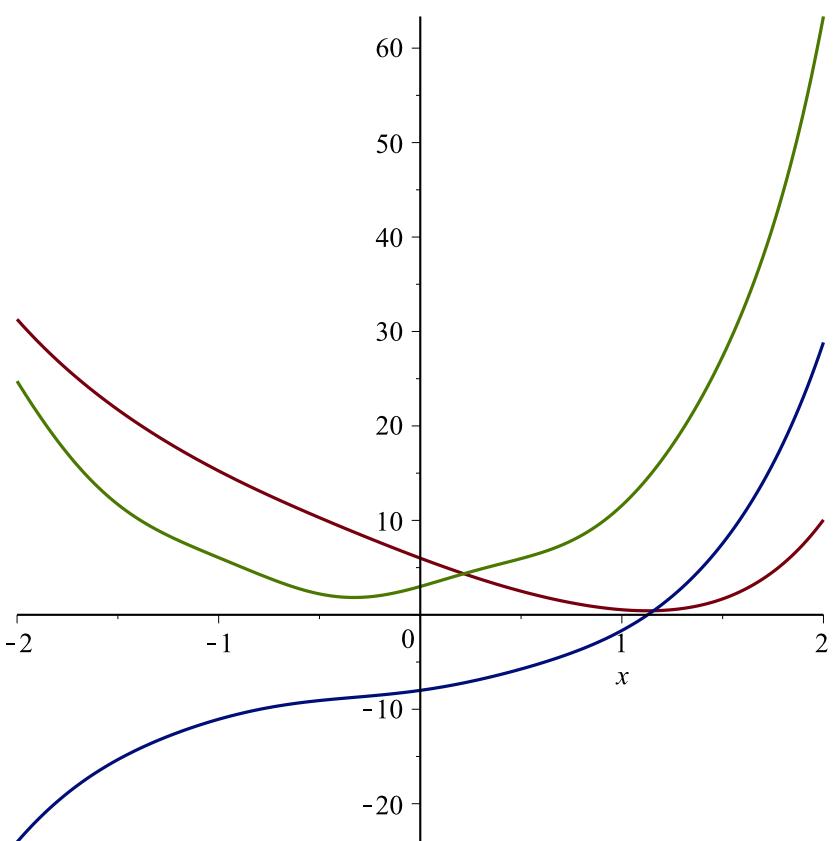
$$y(0) = 6,\ D(y)(0) = -8,\ D^{(2)}(y)(0) = 3 \tag{53}$$

>

> $SolFinal$

$$y(x) = \frac{12}{13}\cos(x)^5 + \frac{60}{13}\cos(x)^4\sin(x) - \frac{15}{13}\cos(x)^3 - \frac{45}{13}\cos(x)^2\sin(x) + \frac{15}{52}\cos(x) \tag{54}$$

$$+ \frac{15}{52}\sin(x) + \frac{4}{15}\,e^{2x} - \frac{3}{16}\cos(x)\sin(4x) + \frac{3}{16}\sin(x)\sin(4x)$$

$$- \frac{1}{8}\cos(x)\sin(6x) + \frac{1}{8}\sin(x)\sin(6x) + \frac{1}{8}\cos(x)\cos(6x) + \frac{1}{8}\sin(x)\cos(6x)$$

$$- \frac{3}{16}\cos(x)\cos(4x) - \frac{3}{16}\sin(x)\cos(4x) + \_C2\cos(x) + \_C1\,e^{-x} + \_C3\sin(x)$$

> $Cond[1] := simplify(subs(x = 0, rhs(SolFinal) = 6))$

$$Cond_1 := \frac{817}{3120} + \_C2 + \_C1 = 6 \tag{55}$$

> $Cond[2] := simplify(subs(x = 0, rhs(diff(SolFinal, x)) = -8))$

$$Cond_2 := \frac{1289}{3120} + \_C3 - \_C1 = -8 \tag{56}$$

> $Cond[3] := simplify(subs(x = 0, rhs(diff(SolFinal, x\$2)) = 3))$

$$Cond_3 := \frac{3703}{3120} + \_C1 - \_C2 = 3 \tag{57}$$

> $Para := solve([Cond[1], Cond[2], Cond[3]])$

$$Para := \left\{ \_C1 = \frac{589}{156}, \_C2 = \frac{157}{80}, \_C3 = -\frac{371}{80} \right\} \tag{58}$$

> $SolParticular := subs(\_C1 = rhs(Para[1]), \_C2 = rhs(Para[2]), \_C3 = rhs(Para[3]), SolFinal)$

$$SolParticular := y(x) = \frac{12}{13}\cos(x)^5 + \frac{60}{13}\cos(x)^4\sin(x) - \frac{15}{13}\cos(x)^3 - \frac{45}{13}\cos(x)^2\sin(x) \tag{59}$$

$$+ \frac{2341}{1040}\cos(x) - \frac{4523}{1040}\sin(x) + \frac{4}{15}e^{2x} - \frac{3}{16}\cos(x)\sin(4x) + \frac{3}{16}\sin(x)\sin(4x)$$

$$- \frac{1}{8}\cos(x)\sin(6x) + \frac{1}{8}\sin(x)\sin(6x) + \frac{1}{8}\cos(x)\cos(6x) + \frac{1}{8}\sin(x)\cos(6x)$$

$$- \frac{3}{16}\cos(x)\cos(4x) - \frac{3}{16}\sin(x)\cos(4x) + \frac{589}{156}e^{-x}$$

> $evalf(\%, 2)$

$$y(x) = 0.92\cos(x)^5 + 4.6\cos(x)^4\sin(x) - 1.2\cos(x)^3 - 3.5\cos(x)^2\sin(x) + 2.3\cos(x) \tag{60}$$

$$- 4.3\sin(x) + 0.27e^{2.x} - 0.19\cos(x)\sin(4.x) + 0.19\sin(x)\sin(4.x)$$

$$- 0.12\cos(x)\sin(6.x) + 0.12\sin(x)\sin(6.x) + 0.12\cos(x)\cos(6.x)$$

$$+ 0.12\sin(x)\cos(6.x) - 0.19\cos(x)\cos(4.x) - 0.19\sin(x)\cos(4.x) + 3.8e^{-1.x}$$

> $plot(\{rhs(SolParticular), rhs(diff(SolParticular, x)), rhs(diff(SolParticular, x\$2))\}, x = -2 ..2)$