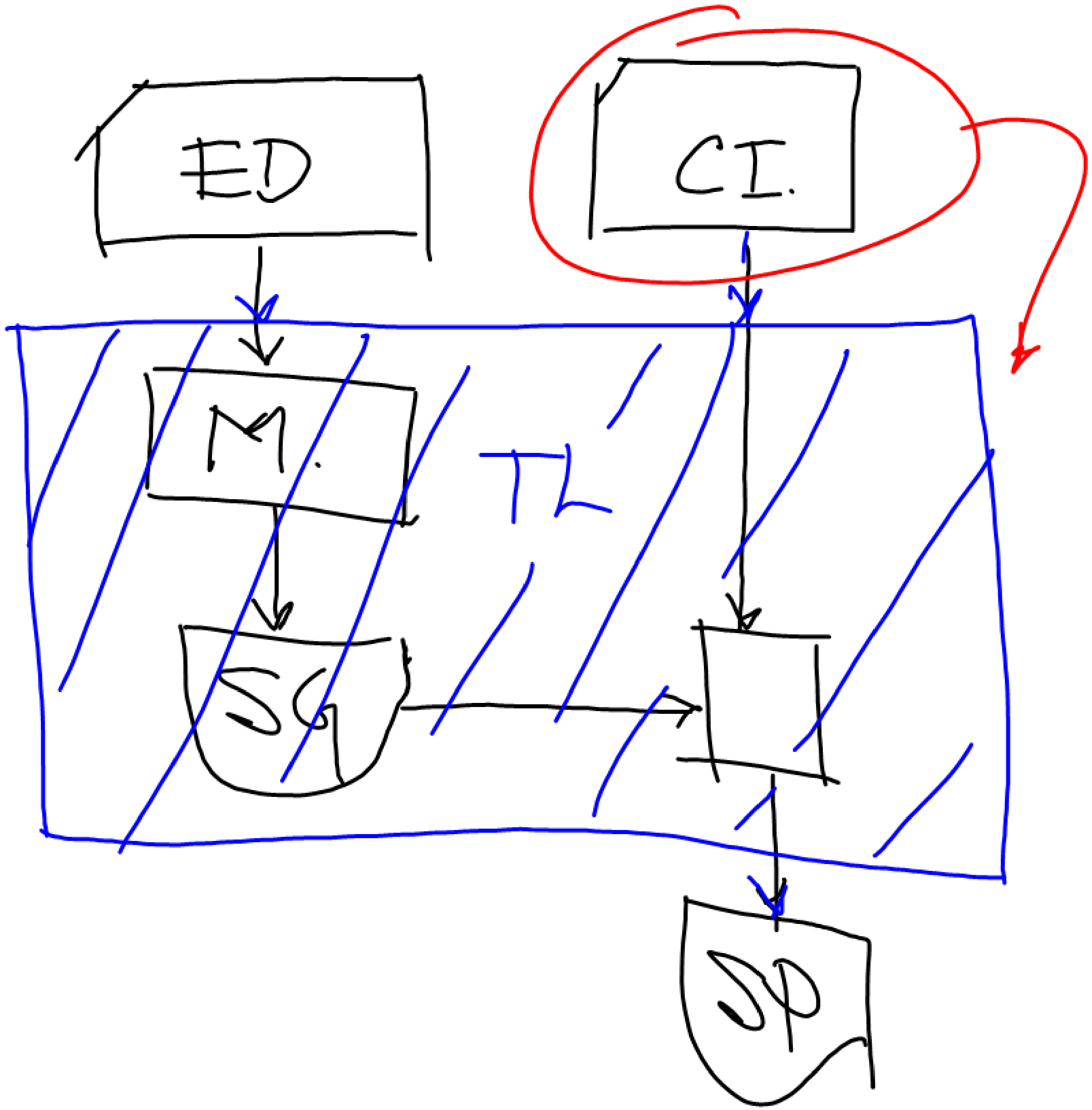
EXAMEN TEMAS 1 y 2

Jueves 3 noviembre J205 A&B.

Tema 3.- Transformada de Laplace (como método de solución de problemas de Ecuaciones Diferenciales con condiciones iniciales).


ED
CI.
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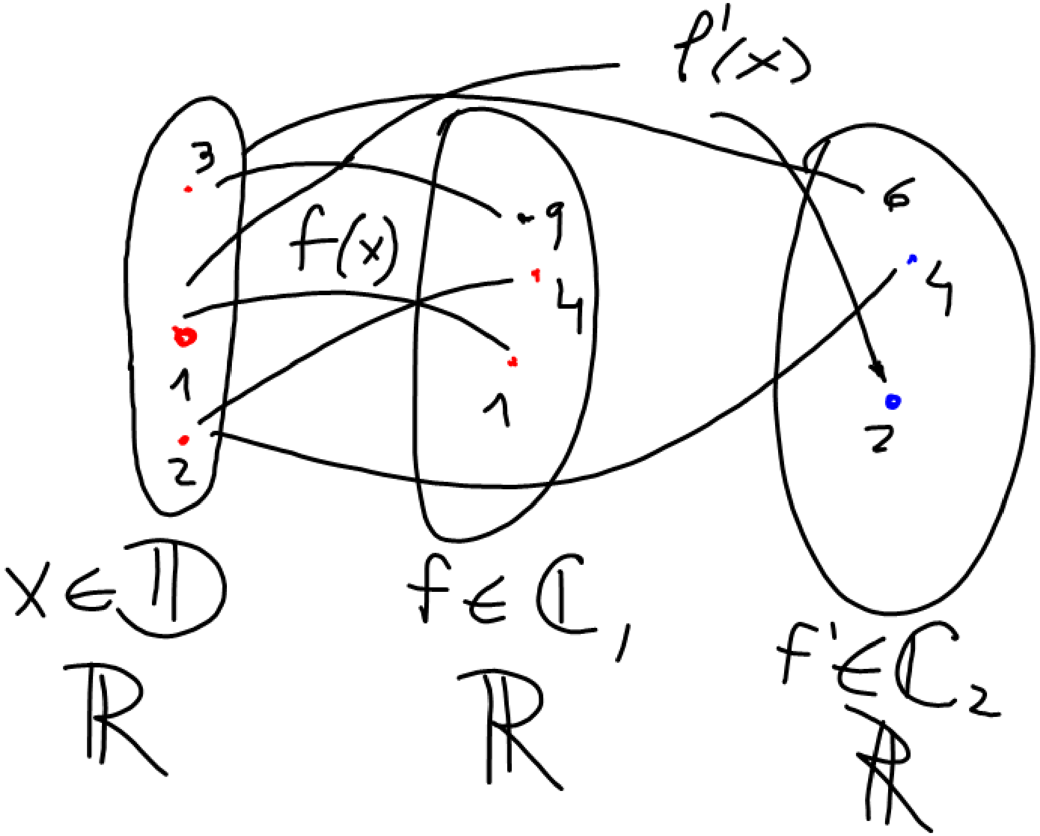
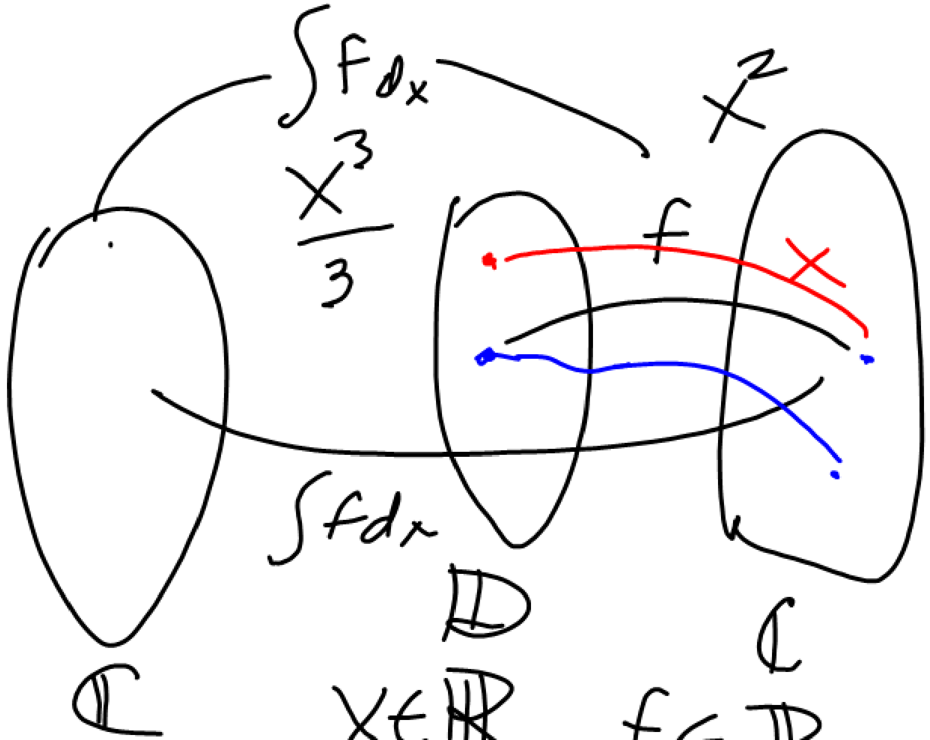
TL
SG
SP



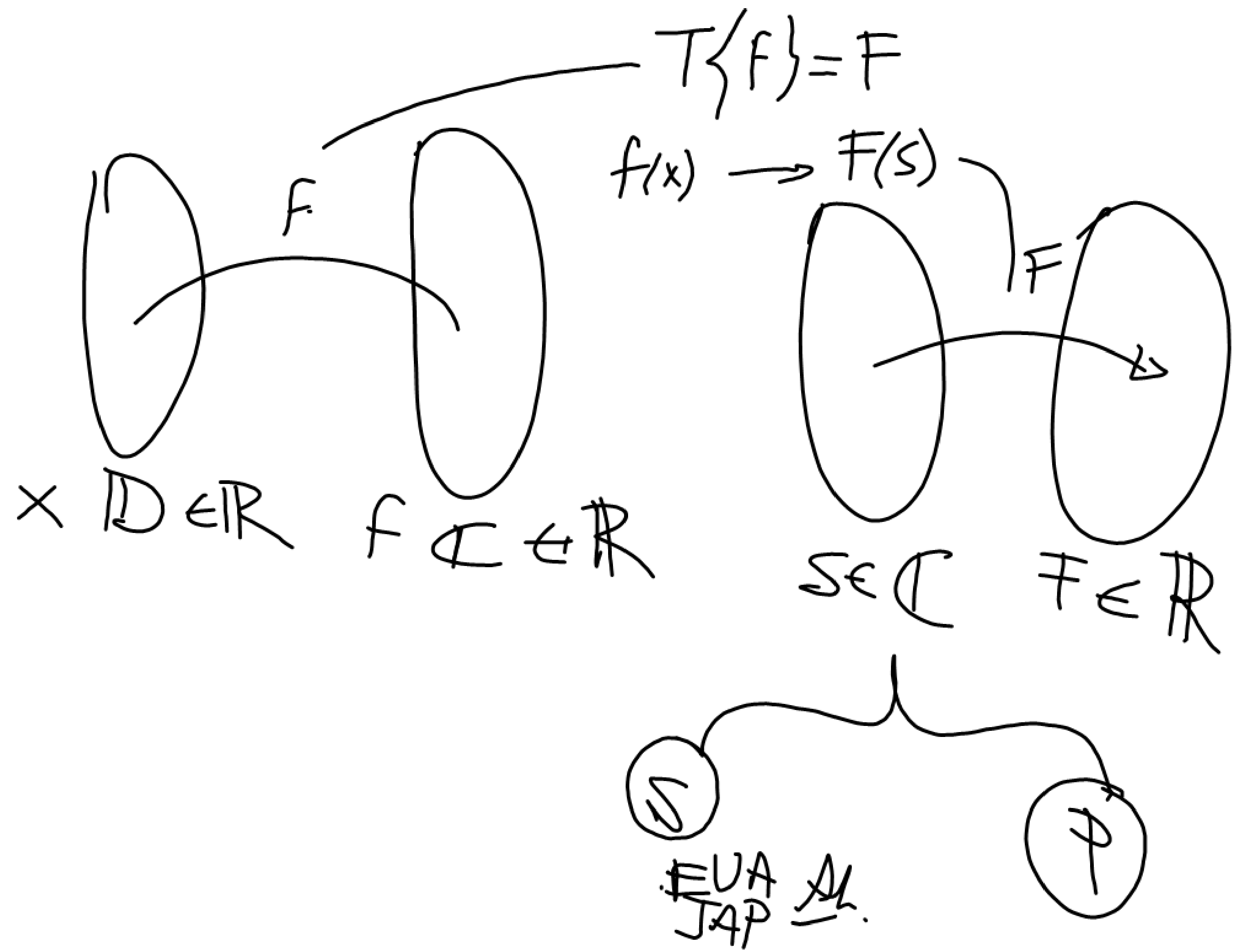
$\frac{d}{dx}$

x	x^2	$2x$
1	1	2
2	4	4
3	9	6

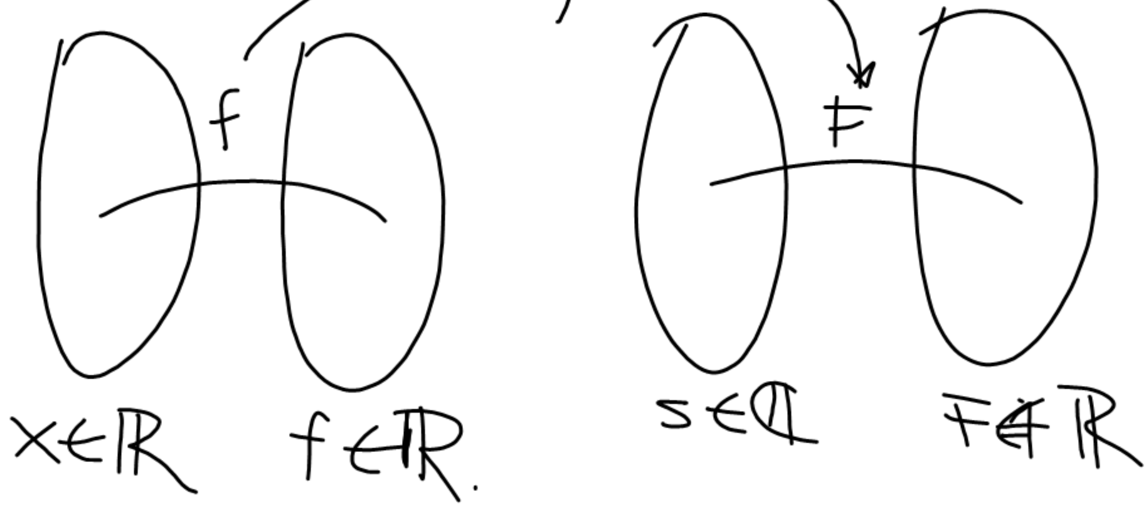
$\sqrt{x}$



$\int f \in \mathbb{R}$ $\quad x \in \mathbb{R}$ $\quad f \in \mathbb{R}$


$T\{f\}=F$
$f(x) \longrightarrow F(s)$
f
F
x D∈ℝ
f ∈ℝ
s∈ℂ
F∈ℝ
EUA Al.
JAP

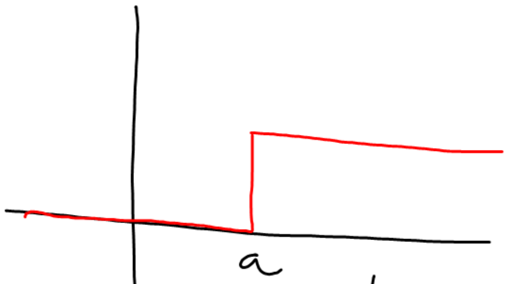
$$\mathcal{L}\{f(x)\} = F(s)$$



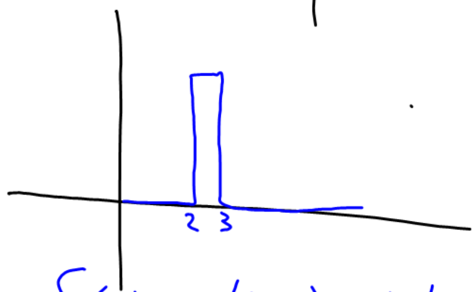
$$c_1 f_1(x) + c_2 g_2(x) \longrightarrow c_1 F_1(s) + c_2 G_2(s)$$

$$f''(x) + 3f'(x) = 0 \longrightarrow s^2 F(s) - (\;) + 3sF(s) = 0$$

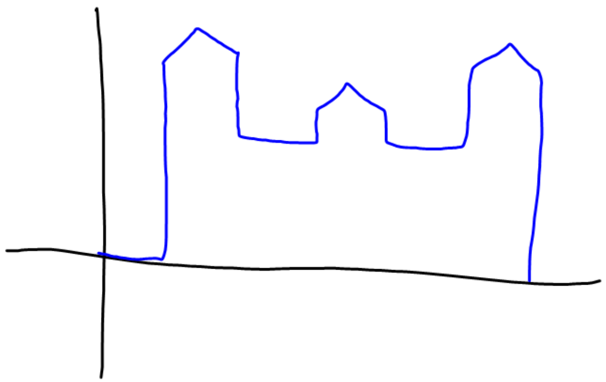
$$f(x) = \ldots\ldots \xleftarrow[\mathcal{L}^{-1}\{F(s)\}]{} F(s) = \ldots\ldots\ldots$$



$$u(t-a)\begin{cases} 0; & t<a \\ 1; & t\geq a \end{cases}$$



$$\delta(t)=u(t-2)-u(t-3)$$



núcleo   producto

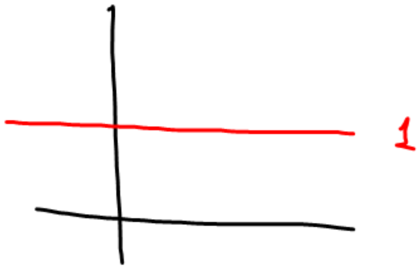
$$T\{f(x)\} = \int_{-\infty}^{\infty} N(x,s)\, f(x)\, dx = F(s)$$

operador   argumento

$$N(x,s) = \begin{cases} 0 \; ; & t < 0 \\ e^{-st} \; ; & t \geq 0 \end{cases}$$

$$\mathcal{L}\{f(t)\} = \int_{0}^{\infty} e^{-st} f(t)\, dt = F(s)$$

$$f(t) = 1$$



$$\mathcal{L}\{1\} = \int_0^{\infty} e^{-st} \cdot (1) \cdot dt$$

$$= \left[\int e^{-st} dt \right]_0^{\infty}$$

$$= \left[-\frac{1}{s} \int e^{-st} (-s\,dt) \right]_0^{\infty}$$

$$= \left[-\frac{1}{s} e^{-st} \right]_0^{\infty}$$

$$\mathcal{L}\{1\} = -\frac{1}{s} \left(\lim_{t \to \infty} e^{-st} - 1 \right)$$

$$\lim_{t \to \infty} \frac{1}{e^{st}} = \lim_{b \to \infty} \left(\frac{1}{b} \right) = 0$$

$$\boxed{\mathcal{L}\{1\} = \frac{1}{s}}$$

$$\mathcal{L}\{t\} = \int_0^{\infty} e^{-st} \cdot t \cdot dt$$

$$= \left[\int t e^{-st} dt \right]_0^{\infty}$$

$$\int t e^{-st} dt = -\frac{1}{s} t e^{-st} + \frac{1}{s} \int e^{-st} dt$$

$u = t \quad dV = e^{-st} dt$

$du = dt \quad V = -\frac{1}{s} e^{-st}$

$$= -\frac{1}{s} t e^{-st} + \frac{1}{s} \left[-\frac{1}{s} e^{-st} \right]$$

$$= -\frac{1}{s} t e^{-st} - \frac{1}{s^2} e^{-st}$$

$$\left[\int e^{-st} t \, dt \right]_0^{\infty} = -\frac{1}{s} \left[\lim_{t \to \infty} t e^{-st} - (0) e^{(0)} \right] +$$

$$- \frac{1}{s^2} \left[\lim_{t \to \infty} e^{-st} - 1 \right]$$

$$\mathcal{L}\{t\} = \frac{1}{s^2}$$

$$\mathcal{L}\{e^{at}\} = \int_0^\infty e^{-st} e^{at} dt$$

$$= \left[\int e^{-(s-a)t} dt \right]_0^\infty$$

$$= \left[-\frac{1}{s-a} \int e^{-(s-a)t} (-(s-a)dt) \right]_0^\infty$$

$$= -\frac{1}{s-a} \left[e^{-(s-a)t} dt \right]_0^\infty$$

$$= -\frac{1}{s-a} \left[\lim_{t \to \infty} e^{-(s-a)t} - 1 \right]$$

$$0$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$$

$$\mathcal{L}\{x'(t)\} = sF(s) - x(0)$$

$$\mathcal{L}\{x''(t)\} = s^2F(s) - sx(0) - x'(0)$$

$$\frac{d^2x}{dt^2} - 5\frac{dx}{dt} + 6x = 0 \qquad x(0)=2 \quad x'(0)=-3$$

$$\left[s^2F(s) - s\cdot(2) - (-3)\right] - 5\left[sF(s) - (2)\right] + 6F(s) = 0$$

$$(s^2 - 5s + 6)F(s) - 2s + 3 + 10 = 0$$

$$(s^2 - 5s + 6)F(s) = 2s - 13$$

$$F(s) = \frac{2s-13}{s^2-5s+6}$$

$$\frac{2s-13}{(s-3)(s-2)} = \frac{A}{s-3} + \frac{B}{s-2}$$

$$2s - 13 = A(s-2) + B(s-3)$$

$$= (A+B)s + (-2A-3B)$$

$$A + B = 2$$

$$\oplus\ -2A - 3B = -13$$

$$3A + 3B = 6$$

$$A + (0) = -7$$

$$A = -7$$

$$B = 2 + 7 \Rightarrow 9$$

$$F(s) = \frac{-7}{s-3} + \frac{9}{s-2}$$

$$f(t) = -7\mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\} + 9\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\}$$

$$x(t) = -7e^{3t} + 9e^{2t} \qquad x(0)=2$$

$$x'(t) = -21e^{3t} + 18e^{2t} \qquad x'(0)=-3$$