Ecuación No Lineal 1er orden

Método de la Ecuación Exacta.

SG: $x^3y + 4x^2y^2 - 6xy^3 = C_1 \qquad F(x,y) = C_1$

EDO(1)NL $\dfrac{\partial F}{\partial x} + \dfrac{\partial F}{\partial y} \cdot \dfrac{dy}{dx} = 0$

$$M(x,y) + N(x,y) \cdot \frac{dy}{dx} = 0$$

$$\underbrace{\left(3x^2y + 8xy^2 - 6y^3\right)}_{M(x,y)} + \underbrace{\left(x^3 + 8x^2y - 18xy^2\right)}_{N(x,y)} \cdot \frac{dy}{dx} = 0$$

$$\frac{\partial^2 F}{\partial x \partial y} \equiv \frac{\partial^2 F}{\partial y \partial x}$$

$$\left.\begin{aligned} \frac{\partial M}{\partial y} &\Rightarrow 3x^2 + 16xy - 18y^2 \\ \frac{\partial N}{\partial x} &\Rightarrow 3x^2 + 16xy - 18y^2 \end{aligned}\right\} \text{EDO(1)NL ES EXACTA.}$$

$$(3x^2y+8xy^2-6y^3)+(x^3+8x^2y-18xy^2)\frac{dy}{dx}=0$$

$$\int M\,dx \qquad \cap \qquad \int N\,dy$$

$$\boxed{SG} \Rightarrow \int M\,dx \cup \int N\,dx = C_1$$

$$\int M\,dx + \int N\,dy - \left(\int M\,dx \cap \int N\,dy\right) = C_1$$

$$\int M\,dx = \int (3x^2y+8xy^2-6y^3)\,dx$$

$$= 3y\int x^2\,dx + 8y^2\int x\,dx - 6y^3\int dx$$

$$= 3y\left(\frac{x^3}{3}\right) + 8y^2\left(\frac{x^2}{2}\right) - 6y^3x$$

$$= x^3y + 4x^2y^2 - 6xy^3$$

$$\int N\,dy = \int (x^3+8x^2y-18xy^2)\,dy$$

$$= x^3\int dy + 8x^2\int y\,dy - 18x\int y^2\,dy$$

$$= x^3y + 8x^2\left(\frac{y^2}{2}\right) - 18x\left(\frac{y^3}{3}\right)$$

$$\int M\,dx \cap \int N\,dy = x^3y + 4x^2y^2 - 6xy^3$$

$$0 + x^3y + 4x^2y^2 - 6xy^3 + 0$$

$\int M\,dx$ $\qquad$ $\int N\,dy$

$$x^3y + 4x^2y^2 - 6xy^3 = C_1$$