EDO(1)NL Método EXACTA.

$$\underbrace{\left(\frac{x}{\sqrt{x^2+y^2}}+\frac{1}{x}+\frac{1}{y}\right)}_{M}+\underbrace{\left(\frac{y}{\sqrt{x^2+y^2}}+\frac{1}{y}-\frac{x}{y^2}\right)}_{N}\frac{dy}{dx}=0$$

$$\frac{\partial M}{\partial y}=x\frac{d}{dy}\left(\frac{1}{\sqrt{x^2+y^2}}\right)+(0)+\frac{d}{dy}\left(\frac{1}{y}\right)$$

$$=x\left(-\frac{1}{2}\left(\sqrt{x^2+y^2}\cdot 2y\right)\right)-y^{-2}$$

$$\boxed{\frac{\partial M}{\partial y}=-xy\sqrt{x^2+y^2}-\frac{1}{y^2}}$$

$$\frac{\partial N}{\partial x}=y\frac{d}{dx}\left(\left(x^2+y^2\right)^{-\frac{1}{2}}\right)+(0)-\frac{1}{y^2}$$

$$=y\left(-\frac{1}{2}\sqrt{x^2+y^2}\cdot 2x\right)-\frac{1}{y^2}$$

$$\boxed{\frac{\partial N}{\partial x}=-xy\sqrt{x^2+y^2}-\frac{1}{y^2}}$$

$$\frac{\partial M}{\partial y}\equiv\frac{\partial N}{\partial x}$$ $\therefore$ EDO(1)NL EXACTA.

$$\left[\int M\,dx\right] \cup \left[\int N\,dy\right] = C_1$$

$$\left(\frac{x}{\sqrt{x^2+y^2}} + \frac{1}{x} + \frac{1}{y}\right) + \left(\frac{y}{\sqrt{x^2+y^2}} + \frac{1}{y} - \frac{x}{y^2}\right)\frac{dy}{dx} = 0$$

$$\int \frac{x}{\sqrt{x^2+y^2}}\,dx + \int \frac{1}{x}\,dx + \frac{1}{y}\int dx$$

$u = x^2+y^2$

$\frac{du}{\cancel{dx}} = 2x\,dx$

$$\frac{1}{2}\int \frac{2x}{\sqrt{x^2+y^2}}\,dx + Lx + \frac{x}{y}$$

$$\frac{1}{2}\int \frac{du}{\sqrt{u}} + Lx + \frac{x}{y}$$

$$\frac{1}{2}\int u^{-\frac{1}{2}}\,du$$

$$\frac{1}{2}\int \frac{u^{\frac{1}{2}}}{\frac{1}{2}}\,du$$

$$\int M\,dx = \sqrt{x^2+y^2} + Lx + \frac{x}{y}$$

$$\int N\,dy = \int y\,(x^2+y^2)^{-1/2}\,dy + \int \frac{dy}{y} - x\int \frac{dy}{y^2}$$

$$\int N\,dy = \sqrt{x^2+y^2} + Ly + \frac{x}{y}$$

$$\left[\int M\,dx\right] \cap \left[\int N\,dy\right] = \sqrt{x^2+y^2} + \frac{x}{y}$$

$$\boxed{SG = Lx + \sqrt{x^2+y^2} + \frac{x}{y} + Ly = C_1}$$

$\int M\,dx$ $\qquad$ $\int N\,dy$

$$Lx + \sqrt{x^2+y^2} + \frac{x}{y} + Ly$$

$$\underbrace{x^4y^2 + x^3y^3 + x^2}_{F(x,y)} = C_1$$

$$F(x,y) = C_1$$

$$\frac{\partial F}{\partial x} + \frac{\partial F}{\partial y}\cdot\frac{dy}{dx} = 0$$

$$\left(4x^3y^2 + 3x^2y^3 + 2x\right) + \left(2x^4y + 3x^3y^2 + (0)\right)\frac{dy}{dx} = 0$$

$$x\left(4x^2y^2 + 3xy^3 + 2\right) + x\left(2x^3y + 3x^2y^2\right)\frac{dy}{dx} = 0$$

$$\left(4x^2y^2 + 3xy^3 + 2\right) + \left(2x^3y + 3x^2y^2\right)\frac{dy}{dx} = 0$$

$$M(x,y)+N(x,y)\frac{dy}{dx}=0$$

$$\frac{\partial M}{\partial y}\neq\frac{\partial N}{\partial x}$$ NO ES EXACTA.

$$F(x,y)M(x,y)+F(x,y)N(x,y)\frac{dy}{dx}=0$$

Factor Integrante $\Rightarrow F(x,y)$

$$\frac{\partial}{\partial y}\Big(F(x,y)M(x,y)\Big)=\frac{\partial}{\partial x}\Big(F(x,y)N(x,y)\Big).$$

$$\frac{\partial F}{\partial y}\cdot M+\frac{\partial M}{\partial y}\cdot F=\frac{\partial F}{\partial x}\cdot N+\frac{\partial N}{\partial x}\cdot F$$

$F(x,y)=f(x)$

$$\frac{\partial M}{\partial y}f=\frac{df}{dx}\cdot N+\frac{\partial N}{\partial x}f.$$

$$\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)f=\frac{df}{dx}N$$

$$\left(\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N}\right)f=\frac{df}{dx}$$

$$\frac{1}{f}\frac{df}{dx}=\left(\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N}\right)$$

$$\frac{df}{f}=\left(\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N}\right)dx$$

$$\left(4x^2y^2+3xy^3+2\right)+\left(2x^3y+3x^2y^2\right)\frac{dy}{dx}=0$$

$$\frac{\partial M}{\partial y}=8x^2y+9xy^2+(0)$$

$$\frac{\partial N}{\partial x}=6x^2y+6xy^2$$

$$\frac{df}{f}=\left(\frac{8x^2y+9xy^2-6x^2y-6xy^2}{2x^3y+3x^2y^2}\right)dx$$

$$\frac{df}{f}=\left(\frac{2x^2y+3xy^2}{2x^3y+3x^2y^2}\right)dx$$

$$=\left(\frac{1}{x}\left(\frac{\cancel{2x^2y+3xy^2}}{\cancel{2x^2y+3xy^2}}\right)\right)dx$$

$$\int\frac{df}{f}=\int\frac{dx}{x}$$

$$\ln f=\ln x$$

$$f=x$$

$$\frac{\partial M}{\partial y}\cdot F+\frac{\partial F}{\partial y}\cdot M=\frac{\partial N}{\partial x}\cdot F+\frac{\partial F}{\partial x}\cdot N$$

$$F(x,y)=g(y)$$

$$\frac{\partial M}{\partial y}\cdot g+\frac{dg}{dy}\cdot M=\frac{\partial N}{\partial x}\cdot g$$

$$\frac{dg}{dy}M=\frac{\partial N}{\partial x}\cdot g-\frac{\partial M}{\partial y}\cdot g$$

$$\frac{dg}{dy}\cdot\frac{1}{g}=\left(\frac{\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}}{M}\right)$$

$$\frac{dg}{g}=\left(\frac{\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}}{M}\right)dy$$