TEMA 2.- EDO(n)L CC $\begin{cases} H \\ NH. \end{cases}$

EDO(1)L CV NH.

a_1

$$\frac{dy}{dx} + p(x)y = q(x)$$

$$y = C_1 e^{-\int p\,dx} + e^{-\int p\,dx} \int e^{\int p\,dx} q\,dx.$$

$$\frac{dy}{dx} + a_1 y = q(x)$$

$$y = C_1 e^{-\int a_1 dx} + e^{-\int a_1 dx} \int e^{\int a_1 dx} q\,dx$$

$$y = C_1 e^{-a_1 x} + e^{-a_1 x} \int e^{a_1 x} q(x)\,dx$$

$$y_{g/nh} = \left(C_1 + \int e^{a_1 x} q(x)\,dx \right) e^{-a_1 x}$$

$$y_{g/h.} = C_1 e^{-a_1 x}$$

RECETA

$$y_{g/nh} = A(x) e^{-a_1 x}$$

EDO (2) L cc H.

$$a_0 \frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0$$

$$\frac{d^2y}{dx^2} + \frac{a_1}{a_0} \frac{dy}{dx} + \frac{a_2}{a_0} y = 0$$

$$\frac{d^2y}{dx^2} + b_1 \frac{dy}{dx} + b_2 y = 0$$

$$\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = 0$$

Suponiendo que $y_{p/f} = e^{mx}$

$$y = e^{mx}$$

$$\frac{dy}{dx} = me^{mx}$$

$$\frac{d^2y}{dx^2} = m^2e^{mx}$$

$$(m^2e^{mx}) + a_1(me^{mx}) + a_2(e^{mx}) = 0$$

$$(m^2 + a_1 m + a_2)e^{mx} = 0$$

para $e^{mx} = 0$

$m \to -\infty$

$e^{mx} \neq 0$

$$m^2 + a_1 m + a_2 = 0$$

ecuación característica de la EDO(2)LCCH.

raíces. m_1 m_2

CASO $\underline{I} \longrightarrow m_1, m_2 \in \mathbb{R}$ $m_1 \neq m_2$

CASO $\underline{II} \longrightarrow m_1 m_2 \in \mathbb{R}$ $m_1 = m_2$

CASO $\underline{III} \longrightarrow m_1 m_2 \in \mathbb{C}$ $m_1 \neq m_2$

$m_1 = a + bi$

$m_2 = a - bi$

$a \in \mathbb{R}$ $b \in \mathbb{R}^+$

$$\frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0$$

$$m^2 + a_1 m + a_2 \cdot 1 = 0$$

CASO I.- $m_1, m_2 \in \mathbb{R}$ $\quad m_1 \neq m_2$

$$(m - m_1)(m - m_2) = 0$$

$$y_1 = e^{m_1 x} \qquad y_2 = e^{m_2 x}$$

$$y_g = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$

$$W = \begin{vmatrix} e^{m_1 x} & e^{m_2 x} \\ m_1 e^{m_1 x} & m_2 e^{m_2 x} \end{vmatrix} \neq 0$$

$$m_2 e^{m_2 x} e^{m_1 x} - m_1 e^{m_2 x} e^{m_1 x} \neq 0$$

$$(m_2 - m_1) e^{m_1 x} e^{m_2 x} \neq 0$$

$$m_2 - m_1 \neq 0$$

$$m_2 \neq m_1$$

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

e^{mx} EDO(2)LCCH.

$$m^2 - 5m + 6 = 0$$

E(A)C.

$$(m-2)(m-3) = 0 \qquad m_1 \neq m_2$$

$$y_g = C_1 e^{2x} + C_2 e^{3x}$$

$$y = C_1 e^{-x} + C_2 e^{x} \quad \text{SGH.}$$

EDO(2) LCC H.

$$(m+1)(m-1) = 0 \quad \text{EC.}$$

$$m^2 - m + m - 1 = 0$$

$$m^2 - 1 = 0 \quad \text{EC}$$

$$\frac{d^2y}{dx^2} - y = 0$$

CASO III $\quad m_1, m_2 \in \mathbb{C} \quad m_1 \neq m_2$

$$\frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_1 y = 0$$

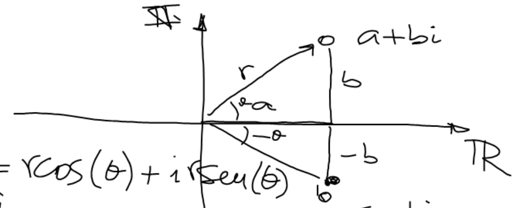
$$m^2 + a_1 m + a_2 = 0 \qquad m_1 \neq m_1$$

$$\big(m - (a+bi)\big)\big(m - (a-bi)\big) = 0 \qquad m_1, m_2 \in \mathbb{C}.$$

$$e^{(a+bi)x} \qquad e^{(a-bi)x}$$

$$y = C_1 e^{(a+bi)x} + C_2 e^{(a-bi)x} \qquad \begin{cases} x \in \mathbb{R} \\ \times \\ y \in \mathbb{R} \end{cases}$$

EULER $\quad e^{\pi i} = -1.$



$$re^{\theta i} = r\cos(\theta) + i r \operatorname{sen}(\theta)$$

$$re^{-\theta i} = r\cos(\theta) - i r(\operatorname{sen}(\theta)).$$

$$e^{\theta i} = \cos(\theta) + i(\operatorname{sen}\theta)$$

$$e^{-\theta i} = \cos(\theta) - i(\operatorname{sen}\theta)$$

$$y = C_1 e^{(a+bi)x} + C_2 e^{(a-bi)x}$$

$$= C_1 e^{ax} e^{(bx)i} + C_2 e^{ax} e^{(-bx)i}$$

$$y = C_1 e^{ax}\big(\cos(bx) + i(\operatorname{sen}(bx))\big) + $$

$$+ C_2 e^{ax}\big(\cos(bx) - i(\operatorname{sen}(bx))\big)$$

$$y = (C_1 + C_2) e^{ax} \cos(bx) +$$

$$+ (C_1 i - C_2 i) e^{ax} \operatorname{sen}(bx)$$

$$\Rightarrow y = C_{10} e^{ax} \cos(bx) + C_{20} e^{ax} \operatorname{sen}(bx) \qquad \begin{cases} x \in \mathbb{R} \\ y \in \mathbb{R} \end{cases}$$

CASO II: $m_1, m_2 \in \mathbb{R}$ $m_1 = m_2$

$$\frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0$$

$$m^2 + a_1 m + a_2 = 0$$

$$(m - m_1)^2 = 0 \quad m_1 = m_2$$

$$y_1 = e^{m_1 x} \quad y_2 = e^{m_2 x}$$

$$y = c_1 e^{m_1 x} + c_2 e^{m_1 x}$$

$$= (c_1 + c_2) e^{m_1 x} + \ldots\ldots$$

$$m^2 + a_1 m + a_2 = 0 \qquad 2m + a_1 = 0$$

$$(m - m_1)(m - m_2) = 0$$

$$(m - m_1) + (m - m_2) = 0$$

$$m^2 - a_1 m + a_2 = 0 \qquad 2m + a_1 = 0$$

$$(m - m_1)^2 = 0$$

$$2(m - m_1) = 0$$

$$\frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0$$

$$m^2 + a_1 m + a_2 = 0$$

$$(m - m_1)^2 = 0 \quad m_1 = m_2$$

$$y = e^{mx} \xrightarrow{m = m_1} e^{m_1 x} \quad \checkmark$$

$$\frac{d}{dm}$$

$$y = xe^{mx} \xrightarrow{m = m_1} xe^{m_1 x}$$

$$y = xe^{m_1 x}$$

$$\frac{dy}{dx} = m_1 x e^{m_1 x} + e^{m_1 x}$$

$$\frac{d^2y}{dx^2} = m_1^2 x e^{m_1 x} + 2m_1 e^{m_1 x}$$

$$\left(m_1^2 x e^{m_1 x} + 2m_1 e^{m_1 x}\right) + a_1\left(m_1 x e^{m_1 x} + e^{m_1 x}\right) + a_2\left(xe^{m_1 x}\right) = 0$$

$$\underbrace{(m_1^2 + am_1 + a)}_{0} xe^{m_1 x} + (2m_1 + a_1)e^{m_1 x} = 0$$

$$y_g = c_1 e^{m_1 x} + c_2 x e^{m_1 x}$$

$$y = C_1 e^{2x} \cos(3x) + C_2 e^{2x} \operatorname{sen}(3x) +$$

$$+ C_3 x e^{2x} \cos(3x) + C_4 x e^{2x} \operatorname{sen}(3x)$$

$$\left(m - (2+3i)\right)^2 \cdot \left(m - (2-3i)\right) = 0$$

$$m^4 - 8m^3 + 42m^2 - 104m + 169 = 0$$

$$y^{IV} - 8y^{III} + 42y'' - 104y' + 169y = 0$$