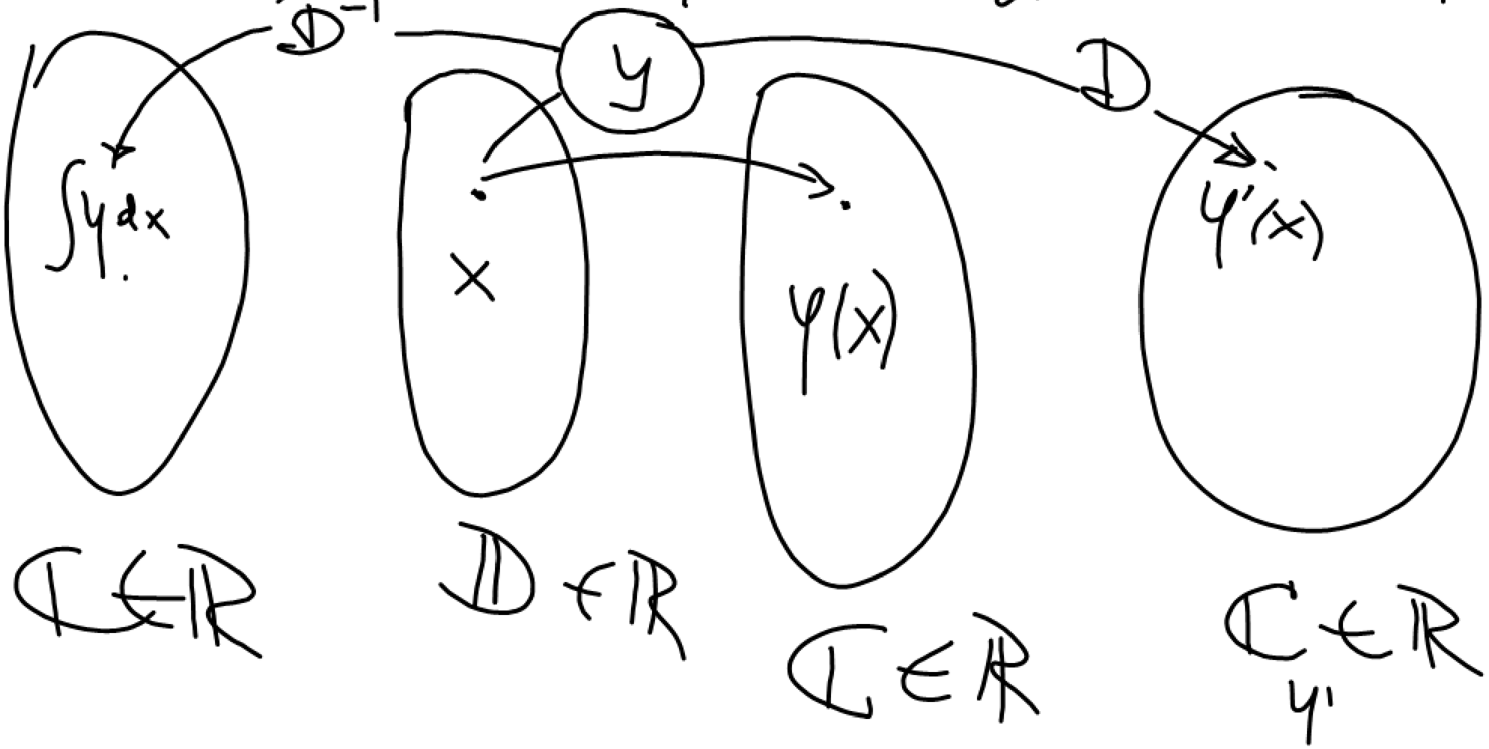
MÉTODO del OPERADOR DIFERENCIAL

$$y'(x) \iff \dot{y}(x) \iff \frac{dy}{dx} \iff Dy$$



$$y'' - 5y' + 6y = 0$$

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

$$D^2y - 5Dy + 6y = 0$$

$$(D^2 - 5D + 6)y = 0$$

EDO (2) LCCH. $(D-2)(D-3)\,y = 0$

E. (A) C.: $(m-2)(m-3) = 0$

$$y = c_1e^{2x} + c_2e^{3x}$$

$$(D-2)(D-3)\left[c_1e^{2x} + c_2e^{3x}\right] = 0$$

$$(D-2)\left[2c_1e^{2x} + \cancel{3c_2e^{3x}} - 3c_1e^{2x} - \cancel{3c_2e^{3x}}\right] = 0$$

$$(D-2)\left[-c_1e^{2x}\right] = 0$$

$$\left[-2\cancel{c_1}e^{2x} + 2\cancel{c_1}e^{2x}\right] = 0$$

$$0 \equiv 0$$

$$y'' - 4y' + 4y = 0$$

$$(D^2 - 4D + 4)y = 0$$

$$(D-2)^2 y = 0$$

$$(m-2)^2 = 0 \qquad \text{CASO II.}$$

$$y = C_1 e^{2x} + C_2 x e^{2x} \qquad m_1 = m_2$$

$$(D-2)^2 \left[C_1 e^{2x} + C_2 x e^{2x} \right] = 0$$

$$(D-2)(D-2)\left[C_1 e^{2x} + C_2 x e^{2x} \right] = 0$$

$$(D-2)\left[\cancel{2C_1 e^{2x}} + C_2\left(\cancel{2x e^{2x}} + e^{2x}\right) - \cancel{2C_1 e^{2x}} - \cancel{2C_2 x e^{2x}} \right] = 0$$

$$(D-2)\left[C_2 e^{2x} \right] = 0$$

$$\left[\cancel{2C_2 e^{2x}} - \cancel{2C_2 e^{2x}} \right] = 0$$

$$0 = 0$$

$$y'' - 2y' + 2y = 0$$

$$(D^2 - 2D + 2)y = 0$$

$$(m^2 - 2m + 2) = 0$$

$$m = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)}}{2}$$

$$m = \frac{2 \pm \sqrt{4-8}}{2}$$

$$m = \frac{2}{2} \pm \frac{\sqrt{-4}}{2}$$

$$m = 1 \pm i$$

$$y = C_1 e^x \cos(x) + C_2 e^x \operatorname{sen}(x)$$

$$(D^2 - 2D + 2)\left[C_1 e^x \cos(x) + C_2 e^x \operatorname{sen}(x)\right] = 0$$

$$D(D)\left[C_1 e^x \cos(x) + C_2 e^x \operatorname{sen}(x)\right] - 2D\left[C_1 e^x \cos(x) + C_2 e^x \operatorname{sen}(x)\right] +$$

$$+\left[2C_1 e^x \cos(x) + 2C_2 e^x \operatorname{sen}(x)\right] = 0$$

$$D\left[C_1\left(-e^x \operatorname{sen}(x) + e^x \cos(x)\right) + C_2\left(e^x \cos(x) + e^x \operatorname{sen}(x)\right)\right] -$$

$$-2\left[-C_1 e^x \operatorname{sen}(x) + C_1 e^x \cos(x) + C_2 e^x \cos(x) + C_2 e^x \operatorname{sen}(x)\right] +$$

$$+\left[2C_1 e^x \cos(x) + 2C_2 e^x \operatorname{sen}(x)\right] = 0$$

$$\left[C_1\left(\left(\cancel{-e^x \cos(x)} - \cancel{e^x \operatorname{sen}(x)}\right) + \left(\cancel{-e^x \operatorname{sen}(x)} + \cancel{e^x \cos(x)}\right)\right) +\right.$$

$$+ C_2\left(\left(\cancel{-e^x \operatorname{sen}(x)} + \cancel{e^x \cos(x)}\right) + \left(\cancel{e^x \cos(x)} + \cancel{e^x \operatorname{sen}(x)}\right)\right)\Big] +$$

$$+\left[\cancel{2C_1 e^x \operatorname{sen}(x)} - \cancel{2C_1 e^x \cos(x)} - \cancel{2C_2 e^x \cos(x)} - \cancel{2C_2 e^x \operatorname{sen}(x)}\right] +$$

$$+\left[\cancel{2C_1 e^x \cos(x)} + \cancel{2C_2 e^x \operatorname{sen}(x)}\right] = 0$$

$$0 = 0$$

ANIQUILADORES

$P(D)$	$f(x)$
D	C_1
D^2	x
D^3	x^2
$\vdots$	$\vdots$
D^{n+1}	x^n
$(D-a)$	e^{ax}
$(D-a)^2$	xe^{ax}
$(D-a)^3$	x^2e^{ax}
$\vdots$	$\vdots$
$(D-a)^{n+1}$	x^ne^{ax}
(D^2+b^2)	$\cos(bx)$, $\operatorname{sen}(bx)$
$((D-a)^2+b^2)$	$e^{ax}\cos(bx)$, $e^{ax}\operatorname{sen}(bx)$
$((D-a)^2+b^2)^2$	$xe^{ax}\cos(bx)$, $xe^{ax}\operatorname{sen}(bx)$
$\vdots$	$\vdots$
$((D-a)^2+b^2)^{n+1}$	$x^ne^{ax}\cos(bx)$, $x^ne^{ax}\operatorname{sen}(bx)$

$m^2+b^2=0$

$$y''' + y'' + y' + y = 5e^{-x} + x\cos(2x)$$

$$(D^3+D^2+D+1)y=0$$

$$(m^3+m^2+m+1)=0$$

$$(m+1)(m^2+1)=0$$

$$y = c_1e^{-x} + c_2\cos(x) + c_3\,\text{sen}(x)$$

$$(D+1)(D^2+1)y = \underbrace{5e^{-x} + \cos(2x)}_{Q(x)}$$

$$(D+1)(D^2+1)(D+1)_A(D^2+4)_B\,y=0$$

$$(D^2+1)(D+1)^2(D^2+4)\,y=0$$

$$y_{g/e} = c_1\cos(x) + c_2\,\text{sen}(x) + c_3e^{-x} + c_4xe^{-x} + c_5\cos(2x) + c_6\,\text{sen}(2x)$$

$$y_{g/NH} = c_1\cos(x) + c_2\,\text{sen}(x) + c_3e^{-x} + Axe^{-x} + B\cos(2x) + D\,\text{sen}(2x)$$

$$y_{p/Q} = Axe^{-x} + B\cos(2x) + D\,\text{sen}(2x)$$

$$y' = A(-xe^{-x}+e^{-x}) + (-2B\,\text{sen}(2x)) + D(2\cos(2x))$$

$$y'' = A(-(-xe^{-x}+e^{x}) + (-e^{-x})) + (-4B\cos(2x)) + D(-4\,\text{sen}(2x))$$

$$y'' = A(xe^{-x}-2e^{-x}) + (-4B\cos(2x)) + (-4D\,\text{sen}(2x))$$

$$y''' = A(-xe^{-x}+e^{-x}+2e^{-x}) + (8B\,\text{sen}(2x)) + (-8D\cos(2x))$$

Sol gral

$$\boxed{y(x) = c_1\cos(x) + c_2\,\text{sen}(x) + c_3e^{-x} + \frac{5}{2}xe^{-x} - \frac{1}{15}\cos(2x) - \frac{2}{15}\,\text{sen}(2x)}$$