> restart

> Ecua := y''+4·y'+8·y=exp(2x)·cos(2x)+exp(2x)·sin(2x)

$$Ecua := \frac{d^2}{dx^2}\,y(x)+4\left(\frac{d}{dx}\,y(x)\right)+8\,y(x)=e^{2x}\cos(2x)+e^{2x}\sin(2x) \quad (1)$$

> SolHom := y(x)=_C1·exp(−2x)·cos(2x)+_C2·exp(−2x)·sin(2x)

$$SolHom := y(x)=\_C1\,e^{-2x}\cos(2x)+\_C2\,e^{-2x}\sin(2x) \quad (2)$$

> Q := rhs(Ecua)

$$Q := e^{2x}\cos(2x)+e^{2x}\sin(2x) \quad (3)$$

> SolPart := y(x)=A·exp(2x)·cos(2x)+B·exp(2x)·sin(2x)

$$SolPart := y(x)=A\,e^{2x}\cos(2x)+B\,e^{2x}\sin(2x) \quad (4)$$

> Para := simplify(eval(subs(y(x)=rhs(SolPart), Ecua)))

$$Para := -16\,e^{2x}\,(A\sin(2x)-A\cos(2x)-B\sin(2x)-B\cos(2x))=e^{2x}\,(\sin(2x)+\cos(2x)) \quad (5)$$

> Raiz := solve([16·A+16B=1, −16·A+16B=1])

$$Raiz := \left\{A=0,\,B=\frac{1}{16}\right\} \quad (6)$$

> SolFinal := y(x)=_C1·exp(−2x)·cos(2x)+_C2·exp(−2x)·sin(2x)+$\frac{1}{16}$·exp(2x)·sin(2x)

$$SolFinal := y(x)=\_C1\,e^{-2x}\cos(2x)+\_C2\,e^{-2x}\sin(2x)+\frac{1}{16}\,e^{2x}\sin(2x) \quad (7)$$

Método del Operador Diferencial (Aniquilador)

> Ecua

$$\frac{d^2}{dx^2}\,y(x)+4\left(\frac{d}{dx}\,y(x)\right)+8\,y(x)=e^{2x}\cos(2x)+e^{2x}\sin(2x) \quad (8)$$

> Comprobar := simplify(eval(subs(y(x)=rhs(SolFinal), lhs(Ecua)−rhs(Ecua)=0)))

$$Comprobar := 0=0 \quad (9)$$

> SolHom

$$y(x)=\_C1\,e^{-2x}\cos(2x)+\_C2\,e^{-2x}\sin(2x) \quad (10)$$

> SolNoHom := y(x)=A(x)·exp(−2x)·cos(2x)+B(x)·exp(−2x)·sin(2x)

$$SolNoHom := y(x)=A(x)\,e^{-2x}\cos(2x)+B(x)\,e^{-2x}\sin(2x) \quad (11)$$

Método Parámetros Variables

> yy[1] := exp(−2x)·cos(2x); yy[2] := exp(−2x)·sin(2x)

$$yy_1 := e^{-2x}\cos(2x)$$

$$yy_2 := e^{-2x}\sin(2x) \quad (12)$$

> with(linalg) :

> WW := wronskian([yy[1], yy[2]], x)

$$WW := \begin{bmatrix} e^{-2x}\cos(2x) & e^{-2x}\sin(2x) \\ -2\,e^{-2x}\cos(2x)-2\,e^{-2x}\sin(2x) & -2\,e^{-2x}\sin(2x)+2\,e^{-2x}\cos(2x) \end{bmatrix} \quad (13)$$

> BB := array([0, Q])

$$BB := \begin{bmatrix} 0 & e^{2x}\cos(2x) + e^{2x}\sin(2x) \end{bmatrix} \tag{14}$$

> $ParaVar := simplify(linsolve(WW, BB))$

$$ParaVar := \begin{bmatrix} -\frac{1}{2}\,e^{4x}\,(\sin(2x) + \cos(2x))\,\sin(2x) & \frac{1}{2}\,e^{4x}\cos(2x)\,(\sin(2x) + \cos(2x)) \end{bmatrix} \tag{15}$$

> $Aprima := ParaVar[1]; Bprima := ParaVar[2]$

$$Aprima := -\frac{1}{2}\,e^{4x}\,(\sin(2x) + \cos(2x))\,\sin(2x)$$

$$Bprima := \frac{1}{2}\,e^{4x}\cos(2x)\,(\sin(2x) + \cos(2x)) \tag{16}$$

> $A(x) := simplify(int(Aprima, x)) + \_C1; B(x) := simplify(int(Bprima, x)) + \_C2$

$$A(x) := \frac{1}{16}\,e^{4x}\,(-1 + \cos(4x)) + \_C1$$

$$B(x) := \frac{1}{16}\,\left(8\cos(x)^3\sin(x) - 4\sin(x)\cos(x) + 1\right)e^{4x} + \_C2 \tag{17}$$

> $SolFinalDos := expand(simplify(\ SolNoHom))$

$$SolFinalDos := y(x) = \left(e^{x}\right)^2\cos(x)^4\sin(x)^2 - \frac{1}{2}\left(e^{x}\right)^2\cos(x)^2\sin(x)^2 + \left(e^{x}\right)^2\cos(x)^6 - \frac{3}{2}\left(e^{x}\right)^2\cos(x)^4 + \frac{1}{2}\left(e^{x}\right)^2\cos(x)^2 + \frac{1}{8}\left(e^{x}\right)^2\cos(x)\sin(x) + \frac{2\,\_C2\sin(x)\cos(x)}{\left(e^{x}\right)^2} + \frac{2\,\_C1\cos(x)^2}{\left(e^{x}\right)^2} - \frac{\_C1}{\left(e^{x}\right)^2} \tag{18}$$

> $ComprobarDos := simplify(eval(subs(y(x) = rhs(SolFinalDos), lhs(Ecua) - rhs(Ecua) = 0)))$

$$ComprobarDos := e^{2x}\left(2\cos(x)^2 + 2\sin(x)\cos(x) - \cos(2x) - \sin(2x) - 1\right) = 0 \tag{19}$$

> $ComprobarZero := subs(\sin(2\cdot x) = 2\cdot\sin(x)\cdot\cos(x), ComprobarDos)$

$$ComprobarZero := e^{2x}\left(2\cos(x)^2 - \cos(2x) - 1\right) = 0 \tag{20}$$

> $ComprobarUno := simplify\left(subs\left(\cos(2\cdot x) = \cos(x)^2 - \sin(x)^2, ComprobarZero\right)\right)$

$$ComprobarUno := 0 = 0 \tag{21}$$

> $SolFinal$

$$y(x) = \_C1\,e^{-2x}\cos(2x) + \_C2\,e^{-2x}\sin(2x) + \frac{1}{16}\,e^{2x}\sin(2x) \tag{22}$$

> $restart$

> $Ecua := y'''' + 4\cdot y'' + 4\cdot y = x\cdot\sin(\mathrm{sqrt}(2\ )\cdot x)$

$$Ecua := \frac{d^4}{dx^4}\,y(x) + 4\left(\frac{d^2}{dx^2}\,y(x)\right) + 4\,y(x) = x\sin\left(\sqrt{2}\,x\right) \tag{23}$$

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^4}{dx^4}\,y(x) + 4\left(\frac{d^2}{dx^2}\,y(x)\right) + 4\,y(x) = 0 \tag{24}$$

> $Q := rhs(Ecua)$

$$Q := x\sin\left(\sqrt{2}\,x\right) \tag{25}$$

> $EcuaCarac := m^4 + 4\cdot m^2 + 4 = 0$

$$EcuaCarac := m^4 + 4\, m^2 + 4 = 0 \tag{26}$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := \mathrm{I}\sqrt{2},\ -\mathrm{I}\sqrt{2},\ \mathrm{I}\sqrt{2},\ -\mathrm{I}\sqrt{2} \tag{27}$$

>

> $yy[1] := \cos(\mathrm{Im}(Raiz[1])\cdot x);\ yy[2] := \sin(\mathrm{Im}(Raiz[1])\cdot x);\ yy[3] := x\cdot\cos(\mathrm{Im}(Raiz[1])\cdot x);\ yy[4] := x\cdot\sin(\mathrm{Im}(Raiz[1])\cdot x);$

$$yy_1 := \cos(\sqrt{2}\, x)$$

$$yy_2 := \sin(\sqrt{2}\, x)$$

$$yy_3 := x\cos(\sqrt{2}\, x)$$

$$yy_4 := x\sin(\sqrt{2}\, x) \tag{28}$$

> $SolHom := y(x) = \_C1\cdot yy[1] + \_C2\cdot yy[2] + \_C3\cdot yy[3] + \_C4\cdot yy[4]$

$$SolHom := y(x) = \_C1\cos(\sqrt{2}\, x) + \_C2\sin(\sqrt{2}\, x) + \_C3\, x\cos(\sqrt{2}\, x) + \_C4\, x\sin(\sqrt{2}\, x) \tag{29}$$

> $SolNoHom := y(x) = A(x)\cdot yy[1] + B(x)\cdot yy[2] + \mathrm{D}(x)\cdot yy[3] + E(x)\cdot yy[4]$

$$SolNoHom := y(x) = A(x)\cos(\sqrt{2}\, x) + B(x)\sin(\sqrt{2}\, x) + \mathrm{D}(x)\, x\cos(\sqrt{2}\, x) + E(x)\, x\sin(\sqrt{2}\, x) \tag{30}$$

> $EcuaHom$

$$\frac{d^4}{dx^4} y(x) + 4\left(\frac{d^2}{dx^2} y(x)\right) + 4\, y(x) = 0 \tag{31}$$

> $Comprueba := simplify(eval(subs(y(x) = rhs(SolHom), EcuaHom)))$

$$Comprueba := 0 = 0 \tag{32}$$

>

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2], yy[3], yy[4]], x)$

$$WW := [[\cos(\sqrt{2}\, x), \sin(\sqrt{2}\, x), x\cos(\sqrt{2}\, x), x\sin(\sqrt{2}\, x)],\ [-\sin(\sqrt{2}\, x)\sqrt{2}, \cos(\sqrt{2}\, x)\sqrt{2}, \cos(\sqrt{2}\, x) - x\sin(\sqrt{2}\, x)\sqrt{2}, \sin(\sqrt{2}\, x) + x\cos(\sqrt{2}\, x)\sqrt{2}],\ [-2\cos(\sqrt{2}\, x), -2\sin(\sqrt{2}\, x), -2\sin(\sqrt{2}\, x)\sqrt{2} - 2\, x\cos(\sqrt{2}\, x), 2\cos(\sqrt{2}\, x)\sqrt{2} - 2\, x\sin(\sqrt{2}\, x)],\ [2\sin(\sqrt{2}\, x)\sqrt{2}, -2\cos(\sqrt{2}\, x)\sqrt{2}, -6\cos(\sqrt{2}\, x) + 2\, x\sin(\sqrt{2}\, x)\sqrt{2}, -6\sin(\sqrt{2}\, x) - 2\, x\cos(\sqrt{2}\, x)\sqrt{2}]] \tag{33}$$

> $BB := array([0, 0, 0, Q])$

$$BB := \begin{bmatrix} 0 & 0 & 0 & x\sin(\sqrt{2}\, x) \end{bmatrix} \tag{34}$$

> $ParaVar := simplify(linsolve(WW, BB))$

$$ParaVar := \left[\frac{1}{8}\, x\sin(\sqrt{2}\, x)\,(2\, x\cos(\sqrt{2}\, x) - \sin(\sqrt{2}\, x)\sqrt{2}),\ \frac{1}{8}\,(x\sin(\sqrt{2}\, x)\sqrt{2} + \cos(\sqrt{2}\, x))\sqrt{2}\sin(\sqrt{2}\, x)\, x,\ -\frac{1}{4}\, x\sin(\sqrt{2}\, x)\cos(\sqrt{2}\, x),\ \frac{1}{4}\, x\,(\cos(\sqrt{2}\, x)^2 \right. \tag{35}$$

$$-1)\Big]$$

> $Aprima := ParaVar[1]; Bprima := ParaVar[2]; Dprima := ParaVar[3]; Eprima := ParaVar[4]$

$$Aprima := \frac{1}{8}\, x \sin\left(\sqrt{2}\, x\right) \left(2\, x \cos\left(\sqrt{2}\, x\right) - \sin\left(\sqrt{2}\, x\right) \sqrt{2}\right)$$

$$Bprima := \frac{1}{8} \left(x \sin\left(\sqrt{2}\, x\right) \sqrt{2} + \cos\left(\sqrt{2}\, x\right)\right) \sqrt{2} \sin\left(\sqrt{2}\, x\right) x$$

$$Dprima := -\frac{1}{4}\, x \sin\left(\sqrt{2}\, x\right) \cos\left(\sqrt{2}\, x\right)$$

$$Eprima := \frac{1}{4}\, x \left(\cos\left(\sqrt{2}\, x\right)^2 - 1\right) \tag{36}$$

> $A(x) := simplify(int(Aprima, x)) + \_C1; B(x) := simplify(int(Bprima, x)) + \_C2; \mathrm{D}(x) := simplify(int(Dprima, x)) + \_C3; E(x) := simplify(int(Eprima, x)) + \_C4$

$$A(x) := \frac{1}{32} \cos\left(\sqrt{2}\, x\right) \left(-2 \cos\left(\sqrt{2}\, x\right) \sqrt{2}\, x^2 + 4\, x \sin\left(\sqrt{2}\, x\right) + \cos\left(\sqrt{2}\, x\right) \sqrt{2}\right) + \_C1$$

$$B(x) := -\frac{1}{16}\, x^2 \cos\left(\sqrt{2}\, x\right) \sqrt{2} \sin\left(\sqrt{2}\, x\right) - \frac{1}{8} \cos\left(\sqrt{2}\, x\right)^2 x + \frac{1}{24}\, x^3 + \frac{1}{32} \sin\left(\sqrt{2}\, x\right) \cos\left(\sqrt{2}\, x\right) \sqrt{2} + \frac{1}{16}\, x + \_C2$$

$$\mathrm{D}(x) := \frac{1}{16} \sqrt{2}\, x \cos\left(\sqrt{2}\, x\right)^2 - \frac{1}{32} \cos\left(\sqrt{2}\, x\right) \sin\left(\sqrt{2}\, x\right) - \frac{1}{32} \sqrt{2}\, x + \_C3$$

$$E(x) := \frac{1}{16}\, x \sin\left(\sqrt{2}\, x\right) \cos\left(\sqrt{2}\, x\right) \sqrt{2} - \frac{1}{16}\, x^2 + \frac{1}{32} \cos\left(\sqrt{2}\, x\right)^2 + \_C4 \tag{37}$$

> $SolFinal := simplify(SolNoHom)$

$$SolFinal := y(x) = \frac{1}{32} \cos\left(\sqrt{2}\, x\right) \sqrt{2} - \frac{1}{48} \sin\left(\sqrt{2}\, x\right) x^3 - \frac{1}{32} \cos\left(\sqrt{2}\, x\right) \sqrt{2}\, x^2 + \_C4\, x \sin\left(\sqrt{2}\, x\right) + \_C3\, x \cos\left(\sqrt{2}\, x\right) + \_C2 \sin\left(\sqrt{2}\, x\right) + \frac{1}{16}\, x \sin\left(\sqrt{2}\, x\right) + \_C1 \cos\left(\sqrt{2}\, x\right) \tag{38}$$

> $Ecua$

$$\frac{d^4}{dx^4}\, y(x) + 4 \left(\frac{d^2}{dx^2}\, y(x)\right) + 4\, y(x) = x \sin\left(\sqrt{2}\, x\right) \tag{39}$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolFinal), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comprobar := 0 = 0 \tag{40}$$

> $SolNoHomDos := y(x) = \_C1 \cdot \cos(\mathrm{sqrt}(2) \cdot x) + \_C2 \cdot \sin(\mathrm{sqrt}(2) \cdot x) + \_C3 \cdot x \cdot \cos(\mathrm{sqrt}(2) \cdot x) + \_C4 \cdot x \cdot \sin(\mathrm{sqrt}(2) \cdot x) + A \cdot x^2 \cdot \cos(\mathrm{sqrt}(2) \cdot x) + B \cdot x^2 \cdot \sin(\mathrm{sqrt}(2) \cdot x) + \mathrm{D} \cdot x^3 \cdot \cos(\mathrm{sqrt}(2) \cdot x) + E \cdot x^3 \cdot \sin(\mathrm{sqrt}(2) \cdot x)$

$$SolNoHomDos := y(x) = \_C1 \cos\left(\sqrt{2}\, x\right) + \_C2 \sin\left(\sqrt{2}\, x\right) + \_C3\, x \cos\left(\sqrt{2}\, x\right) + \_C4\, x \sin\left(\sqrt{2}\, x\right) + A\, x^2 \cos\left(\sqrt{2}\, x\right) + B\, x^2 \sin\left(\sqrt{2}\, x\right) + \mathrm{D}\, x^3 \cos\left(\sqrt{2}\, x\right) + E\, x^3 \sin\left(\sqrt{2}\, x\right) \tag{41}$$

> $SolPart := y(x) = A \cdot x^2 \cdot \cos(\mathrm{sqrt}(2) \cdot x) + B \cdot x^2 \cdot \sin(\mathrm{sqrt}(2) \cdot x) + \mathrm{D} \cdot x^3 \cdot \cos(\mathrm{sqrt}(2) \cdot x) + E \cdot x^3 \cdot \sin(\mathrm{sqrt}(2) \cdot x)$

$$SolPart := y(x) = A\,x^2 \cos(\sqrt{2}\,x) + B\,x^2 \sin(\sqrt{2}\,x) + \mathrm{D}\,x^3 \cos(\sqrt{2}\,x) + E\,x^3 \sin(\sqrt{2}\,x) \tag{42}$$

> *Ecua*

$$\frac{d^4}{dx^4}\,y(x) + 4\left(\frac{d^2}{dx^2}\,y(x)\right) + 4\,y(x) = x \sin(\sqrt{2}\,x) \tag{43}$$

> *ParaDos* := *simplify*(*eval*(*subs*($y(x) = rhs(SolPart)$, *Ecua*)))

$$ParaDos := -48\,\mathrm{D}\,x\cos(\sqrt{2}\,x) - 48\,E\,x\sin(\sqrt{2}\,x) - 24\,\mathrm{D}\sin(\sqrt{2}\,x)\sqrt{2} - 16\,B\sin(\sqrt{2}\,x) - 16\,A\cos(\sqrt{2}\,x) + 24\,E\cos(\sqrt{2}\,x)\sqrt{2} = x\sin(\sqrt{2}\,x) \tag{44}$$

> *ParaTres* := *solve*([$-48E = 1, -48\,\mathrm{D} = 0, -24\cdot\mathrm{sqrt}(2)\cdot\mathrm{D} - 16\cdot B = 0, 24\cdot\mathrm{sqrt}(2)\cdot E - 16\,A = 0$])

$$ParaTres := \left\{A = -\frac{1}{32}\sqrt{2},\ B = 0,\ \mathrm{D} = 0,\ E = -\frac{1}{48}\right\} \tag{45}$$

> *SolPartTres* := *subs*($A = rhs(ParaTres[1])$, $B = rhs(ParaTres[2])$, $\mathrm{D} = rhs(ParaTres[3])$, $E = rhs(ParaTres[4])$, *SolPart*)

$$SolPartTres := y(x) = -\frac{1}{32}\cos(\sqrt{2}\,x)\sqrt{2}\,x^2 - \frac{1}{48}\sin(\sqrt{2}\,x)\,x^3 \tag{46}$$

> *SolFinalTres* := *subs*($A = rhs(ParaTres[1])$, $B = rhs(ParaTres[2])$, $\mathrm{D} = rhs(ParaTres[3])$, $E = rhs(ParaTres[4])$, *SolNoHomDos*)

$$SolFinalTres := y(x) = \_C1\cos(\sqrt{2}\,x) + \_C2\sin(\sqrt{2}\,x) + \_C3\,x\cos(\sqrt{2}\,x) + \_C4\,x\sin(\sqrt{2}\,x) - \frac{1}{32}\cos(\sqrt{2}\,x)\sqrt{2}\,x^2 - \frac{1}{48}\sin(\sqrt{2}\,x)\,x^3 \tag{47}$$

> *Ecua*

$$\frac{d^4}{dx^4}\,y(x) + 4\left(\frac{d^2}{dx^2}\,y(x)\right) + 4\,y(x) = x \sin(\sqrt{2}\,x) \tag{48}$$

> *CompruebaTres* := *simplify*(*eval*(*subs*($y(x) = rhs(SolFinalTres)$, $lhs(Ecua) - rhs(Ecua) = 0$)))

$$CompruebaTres := 0 = 0 \tag{49}$$

>