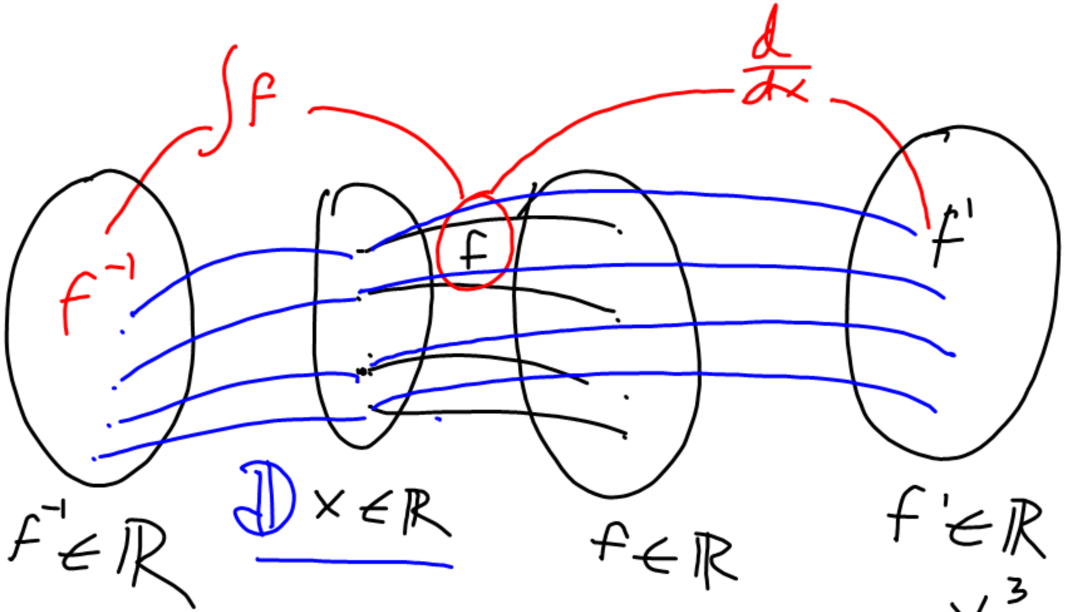
TEMA 3 :-

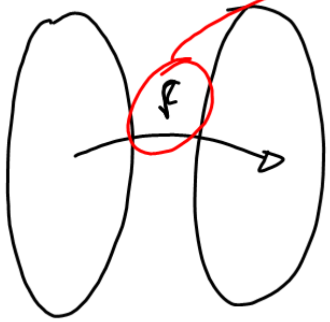
a) TRANSFORMADA DE LAPLACE

b) SISTEMAS DE ED's Y MATRIZ EXPONENCIAL.

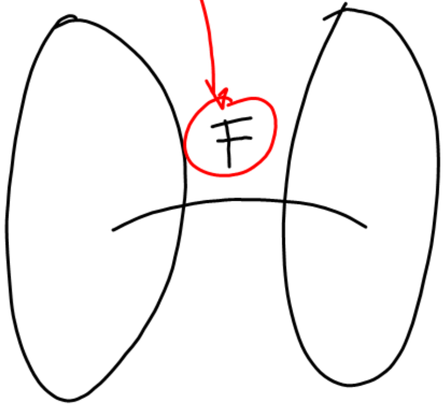


	x^2	$2x$	$\frac{x^3}{3}$
x	f	f'	f^{-1}
0	0	0	0
1	1	2	$\frac{1}{3}$
2	4	4	$\frac{8}{3}$
3	9	6	$\frac{27}{3}$

$T\{f(x)\} = F(s)$



$x \in \mathbb{R}$ $\quad f \in \mathbb{R}$



$s \in \mathbb{C}$ $\quad F \in \mathbb{R}$

complejo

$$T\{af+bg\} \longrightarrow aF(s)+bG(s)$$

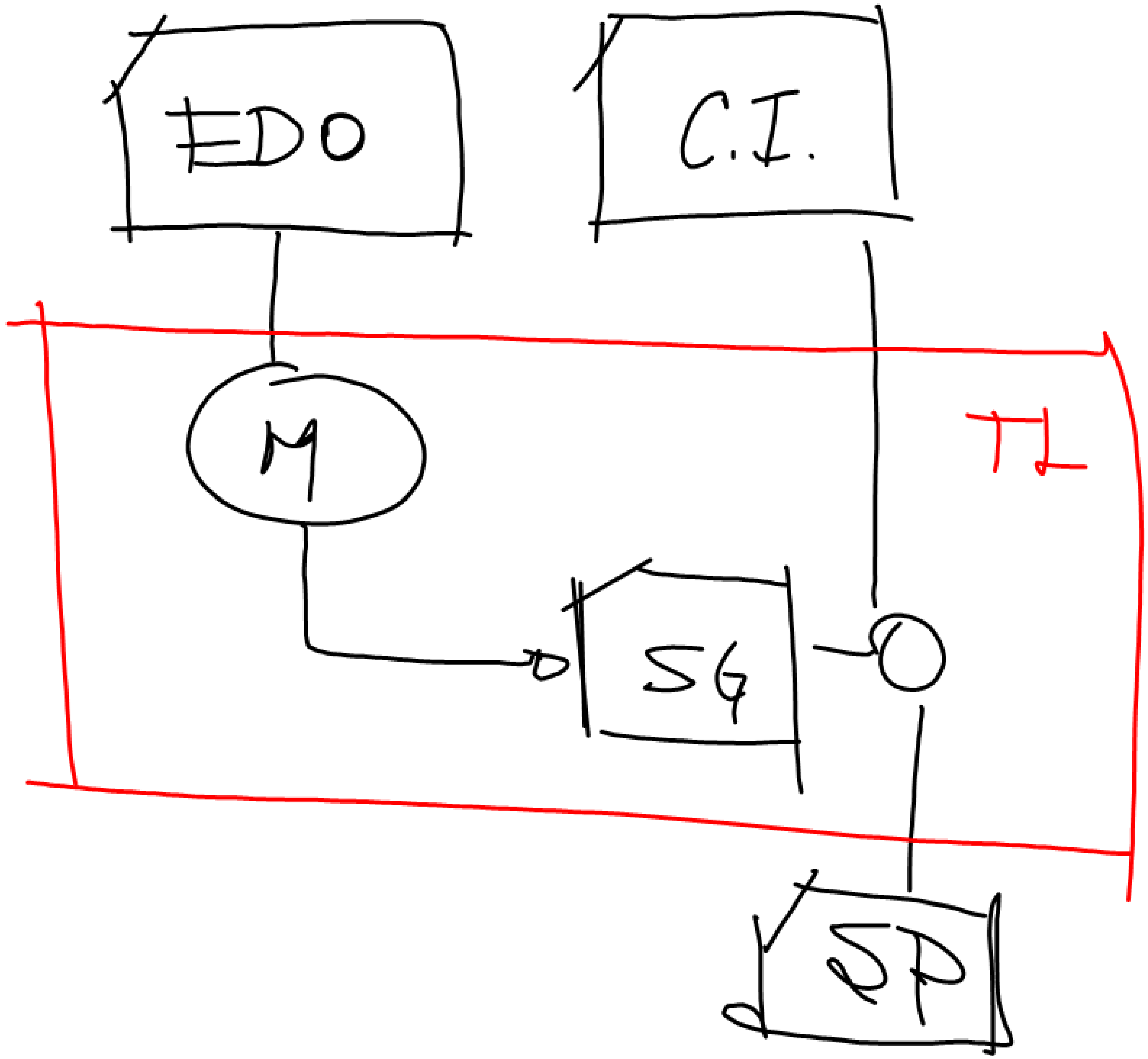
$f(x)$ $\quad g(x)$

$a,b \in \mathbb{R}$

$$T\{af\} = aT\{f\}$$

$$T\left\{\frac{df}{dt}\right\} \longrightarrow sF(s)$$

$$f \longleftarrow T^{-1}\left\{\frac{F(s)}{s}\right\}$$


EDO
C.I.
M
TL
SG
SP

$$T\left\{f(t)\right\} = \int_{-\infty}^{\infty} N(t,s)\, f(t)\, dt = F(s)$$

$f, t \in \mathbb{R}$ $\qquad$ $F \in \mathbb{R}$, $s \in \mathbb{C}$

Laplace $N(t,s) = \begin{cases} 0 & ; t < 0 \\ e^{-st} & ; t \geq 0 \end{cases}$

Si $f(t) = 1$

$$L\left\{1\right\} = \int_0^{\infty} e^{-st} \cdot (1) \cdot dt$$

$$= \left[\int e^{-st} dt\right]_0^{\infty}$$

$$= \left[-\frac{1}{s}\int e^{-st}(-s\,dt)\right]_0^{\infty}$$

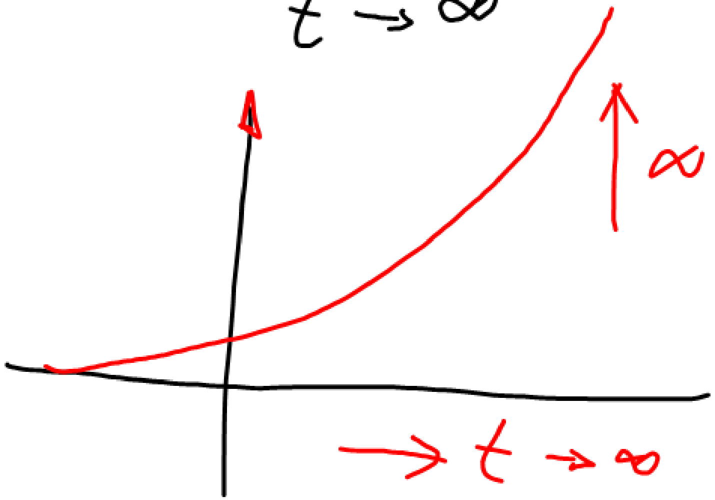
$$= \left[-\frac{1}{s} e^{-st}\right]_0^{\infty}$$

$$= -\frac{1}{s}\left[e^{-st}\right]_0^{\infty}$$

$$= -\frac{1}{s}\left(\lim_{t\to\infty} e^{-st} - 1\right)$$

$$\lim_{t \to \infty} e^{-st} = \lim_{t \to \infty} \left(\frac{1}{e^{st}} \right) = 0$$

$$\lim_{t \to \infty} \left(e^{st} \right) \to \infty \qquad = \lim_{a \to \infty} \left(\frac{1}{a} \right) = 0$$



$$\mathcal{L}\{1\} = -\frac{1}{s}\left((0) - 1 \right)$$

$$\mathcal{L}\{1\} = \frac{1}{s}$$

$$f = e^{5t} \qquad F = \mathcal{L}\{f\}$$

$$F = \int_0^\infty e^{-st}\left(e^{5t}\right)dt$$

$$= \int_0^\infty e^{-(s-5)t}\,dt$$

$$= \left[-\frac{1}{s-5}\int e^{-(s-5)t}\left(-(s-5)\,dt\right)\right]_0^\infty$$

$$= -\frac{1}{s-5}\left[e^{-(s-5)t}\right]_0^\infty$$

$$= -\frac{1}{s-5}\left[(0) - 1\right]$$

$$\mathcal{L}\{e^{5t}\} = \frac{1}{s-5}$$

$$f = t^{k}$$

$$F = \mathcal{L}\{f\} \Rightarrow \frac{k!}{s^{k+1}}$$

$$g = 1 \qquad G = \frac{1}{s}$$

$$h = t \qquad G = \frac{1}{s^2}$$

$$i = t^2 \qquad I = \frac{2!}{s^3}$$

$$m = t^3 \qquad M = \frac{3!}{s^4}$$

$$n = e^{at} \qquad N = \frac{1}{s-a}$$

$$r = te^{at} \qquad R = \frac{1}{(s-a)^2}$$