TEMA 3a) TRANSFORMADA DE LAPLACE.

OPERADOR

NUCLEO

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t)\,dt \Rightarrow F(s)$$

ÚNICA.

$f, t \in \mathbb{R}$

ARGUMENTO

RESULTADO

$F \in \mathbb{R}$ $\underline{\underline{s \in \mathbb{C}}}$

$$\mathcal{L}\{1\} = \frac{1}{s} \qquad \mathcal{L}\{e^{at}\} = \frac{1}{s-a}$$

$$\mathcal{L}\{t\} = \frac{1}{s^2}$$

$$\mathcal{L}\{\cos(bt)\} = \frac{s}{s^2+b^2}$$

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}\{\mathrm{sen}(bt)\} = \frac{b}{s^2+b^2}$$

$$\mathcal{L}^{-1}\left\{F(s)\right\} = f(t)$$

TRANSFORMADA INVERSA DE LAPLACE.

$$f(t) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{st}F(s)\,ds$$

NO ES ÚNICA

$f(t)$	$F(s)$
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{(s-a)}$
$\cos(bt)$	$\frac{s}{s^2+b^2}$
$\text{sen}(bt)$	$\frac{b}{s^2+b^2}$

1- $\mathcal{L}\{af(t)+bg(t)\} = aF(s)+bG(s)$

$a, b \in \mathbb{R}$

2- $\mathcal{L}\{f(at)\} = \frac{1}{a}F\left(\frac{s}{a}\right)$

$\mathcal{L}\{t^2\} = \frac{2}{s^3}$ $\quad$ $\mathcal{L}\{(3t)^2\} = \frac{1}{3}\left(\frac{2!}{\left(\frac{s}{3}\right)^3}\right)$

$\mathcal{L}\{9t^2\} = 9\mathcal{L}\{t^2\}$ $\quad$ $= \frac{1}{3}\left(\frac{2\times 3^3}{s^3}\right)$

$= 9\left(\frac{2!}{s^3}\right)$ $\quad$ $= \frac{2\times 3^2}{s^3} \Rightarrow \frac{18}{s^3}$

$= \frac{2\times 9}{s^3} \Rightarrow \frac{18}{s^3}$

$\frac{\frac{a}{b}}{\frac{c}{d}} \Rightarrow \frac{a\cdot d}{b\cdot c}$

$\mathcal{L}\{\cos(t)\} = \frac{s}{s^2+1}$

$\mathcal{L}\{\cos(at)\} = \frac{s}{s^2+a^2}$

$\mathcal{L}\{\cos(at)\} = \frac{1}{a}\left(\frac{\frac{s}{a}}{\left(\frac{s}{a}\right)^2+1}\right)$

$= \frac{\frac{s}{a^2}}{\frac{s^2}{a^2}+1} \Rightarrow \frac{\frac{s}{a^2}}{\frac{s^2+a^2}{a^2}} \Rightarrow \frac{s}{s^2+a^2}$

3

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

$$\mathcal{L}\{f''(t)\} = s^2F(s) - sf(0) - f'(0)$$

$$\mathcal{L}\{f^{(n)}(t)\} = s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) -$$

$$- s^{n-3}f''(0) - \dots - f^{(n-1)}(0)$$

$$\frac{dy}{dt} - 6y = 0 \qquad y(0) = 4$$

$$\mathcal{L}\left\{\frac{dy}{dt} - 6y\right\} = 0\mathcal{L}\{1\}$$

$$= 0$$

$$\mathcal{L}\left\{\frac{dy}{dt}\right\} - 6\mathcal{L}\{y\} = 0$$

$$\left(s\mathcal{L}\{y\} - (4)\right) - 6\mathcal{L}\{y\} = 0$$

$$(s-6)\mathcal{L}\{y\} = 4$$

$$\mathcal{L}\{y\} = \frac{4}{s-6}$$

$$y = 4\mathcal{L}^{-1}\left\{\frac{1}{s-6}\right\}$$

SOL PART. $\boxed{y = 4e^{6t}}$

4 $$\mathcal{L}^{-1}\{F'(s)\} = -t f(t)$$

$$\mathcal{L}^{-1}\{F^{(n)}(s)\} = (-1)^n t^n f(t)$$

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5 $$\mathcal{L}\left\{\int_0^t f(\tau)\,d\tau\right\} = \frac{F(s)}{s}$$

6 $$\mathcal{L}^{-1}\left\{\int_s^\infty F(\sigma)\,d\sigma\right\} = \frac{f(t)}{t}$$

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7 $$\mathcal{L}\{f(t-\tau)\} = e^{-s\tau}F(s)$$

8 $$\mathcal{L}\{e^{\lambda t}f(t)\} = F(s-\lambda)$$

9 $$\mathcal{L}^{-1}\{F(s)\cdot G(s)\} = f * g$$

convolución

$$f * g = \int_0^t f(\tau)\cdot g(t-\tau)\,d\tau.$$

Teorema de existencia y unidad de la transformada de Laplace.

$$\mathcal{L}\{f(t)\} = F(s)$$

→ $f(t)$ debe ~~de~~ ser una función de clase "A"

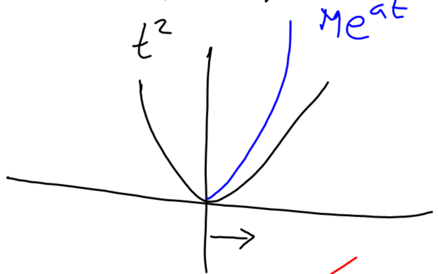
Clase "A"

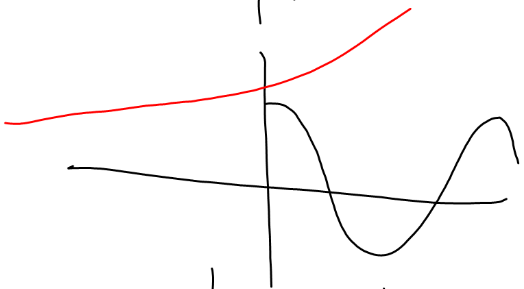
a) $f(t)$ sea de orden exponencial

b) sea seccionalmente continua.

a)

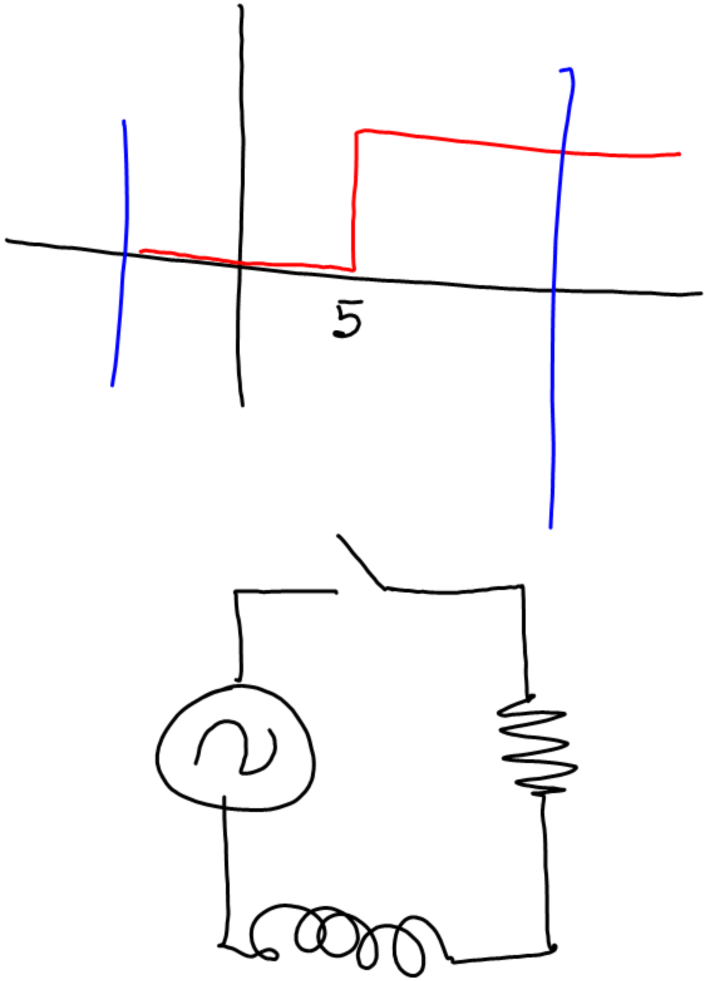
$$|f(t)| \leq Me^{At}$$





$$\left| e^{t^n} \right| \leq Me^{At}$$

b) ser seccionalmente continua



$$\mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s}$$

$$\mathcal{L}\{r(t-a)\} = \frac{e^{-as}}{s^2}$$

$$\mathcal{L}\{f'(t)\} = s\mathcal{L}\{f\} - f(0)$$

$$\mathcal{L}\{r'(t-a)\} = s\mathcal{L}\{r(t-a)\} - (0)$$

$$= s\left[\frac{e^{-as}}{s^2}\right]$$

$$\mathcal{L}\{r'(t-a)\} = \left[\frac{e^{-as}}{s}\right]$$

$$\mathcal{L}\{r'(t-a)\} = \mathcal{L}\{u(t-a)\}$$

$$r'(t-a) = u(t-a)$$