$$\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 9y = 3e^{3t} + 5\cos(2t)$$

$$y(0)=1 \quad y'(0)=-2$$

T. Laplace

1ª prop. $$\mathcal{L}\left\{\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 9y\right\} = \mathcal{L}\left\{3e^{3t} + 5\cos(2t)\right\}$$

$$\mathcal{L}\left\{\frac{d^2y}{dt^2}\right\} - 6\mathcal{L}\left\{\frac{dy}{dt}\right\} + 9\mathcal{L}\{y\} = 3\mathcal{L}\{e^{3t}\} + 5\mathcal{L}\{\cos(2t)\}$$

$$\left(s^2\mathcal{L}\{y\} - s(1) - (-2)\right) - 6\left(s\mathcal{L}\{y\} - (1)\right) + 9\mathcal{L}\{y\} =$$

$$= 3\left(\frac{1}{s-3}\right) + 5\left(\frac{s}{s^2+4}\right)$$

$$\left(s^2 - 6s + 9\right)\mathcal{L}\{y\} - (s-8) = \frac{3}{s-3} + \frac{5s}{s^2+4}$$

TRANSFORMADA DE LAPLACE ED con C.I.

... ED con C.I.

$$(s^2-6s+9)\mathcal{L}\{y\}=\frac{3}{s-3}+\frac{5s}{s^2+4}+(s-8)$$

$$=\frac{3(s^2+4)+5s(s-3)+(s-8)(s-3)(s^2+4)}{(s-3)(s^2+4)}$$

$$=\frac{3s^2+12+5s^2-15s+(s-8)(s^3+4s-3s^2-12)}{(s-3)(s^2+4)}$$

$$=\frac{8s^2-15s+12+s^4-3s^3+4s^2-12s-8s^3+24s^2-32s+96}{(s-3)(s^2+4)}$$

$$(s^2-6s+9)\mathcal{L}\{y\}=\frac{s^4-11s^3+36s^2-59s+108}{(s-3)(s^2+4)}$$

$$\mathcal{L}\{y\}=\frac{s^4-11s^3+36s^2-59s+108}{(s-3)^2(s-3)(s^2+4)}$$

FRACCIÓN RACIONAL PROPIA

$$\mathcal{L}\{y\}=\frac{s^4-11s^3+36s^2-59s+108}{(s-3)^3(s^2+4)}\quad \begin{matrix}\Rightarrow \text{EXP}=4\\ \Rightarrow \text{EXP}=5\end{matrix}$$

Solución particular transformada

particular transformada

$$\frac{s^4-11s^3+36s^2-59s+108}{(s-3)^3(s^2+4)} = \frac{A}{s-3} + \frac{B}{(s-3)^2} + \frac{C}{(s-3)^3} + \frac{Ds+E}{s^2+4}$$

$$s^4-11s^3+36s^2-59s+108 = A(s-3)^2(s^2+4) + B(s-3)(s^2+4) +$$

$$+ C(s^2+4) + (Ds+E)(s-3)^3$$

$$= A(s^2-6s+9)(s^2+4) + B(s^3-3s^2+4s-12) +$$

$$+ C(s^2+4) + (Ds+E)(s^3-9s^2+27s-27)$$

$$= A(s^4-6s^3+9s^2+4s^2-24s+36) +$$

$$+ B(s^3-3s^2+4s-12) +$$

$$+ C(s^2+4)$$

$$+ D(s^4-9s^3+27s^2-27s) +$$

$$+ E(s^3-9s^2+27s-27).$$

$$s^4-11s^3+36s^2-59s+108 = s^4(A+D) + s^3(-6A+B-9D+E) +$$

$$+ s^2(13A-3B+C+27D-9E) +$$

$$+ s(-24A+4B-27D+27E) +$$

$$+ (36A-12B+4C-27E).$$

$$\boxed{\begin{aligned} &A+D=1\\ &-6A+B-9D+E=-11\\ &13A-3B+C+27D-9E=36\\ &-24A+4B-27D+27E=-59\\ &36A-12B+4C-27E=108 \end{aligned}}$$

$$\mathcal{L}\{y\}=\left(\frac{936}{1121}\right)\cdot\left(\frac{1}{s-3}\right)+\left(-\frac{4450}{1121}\right)\cdot\left(\frac{1}{(s-3)^2}\right)+\left(\frac{4443}{1121\cdot 2}\right)\cdot\left(\frac{2}{(s-3)^3}\right)+$$

$$+\left(\frac{185}{1121}\right)\left(\frac{s}{s^2+4}\right)+\left(-\frac{600}{1121\cdot 2}\right)\cdot\left(\frac{2}{s^2+4}\right)$$

$$y=\left(\frac{936}{1121}\right)\mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\}-\frac{4450}{1121}\mathcal{L}^{-1}\left\{\frac{1}{(s-3)^2}\right\}+\frac{4443}{2242}\cdot\mathcal{L}^{-1}\left\{\frac{2}{(s-3)^3}\right\}+$$

$$+\frac{185}{1121}\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\}-\frac{300}{1121}\mathcal{L}^{-1}\left\{\frac{2}{s^2+4}\right\}$$

$$y_p=\frac{936}{1121}e^{3t}-\frac{4450}{1121}te^{3t}+\frac{4443}{2242}t^2e^{3t}+\frac{185}{1121}\cos(2t)-\frac{300}{1121}(\operatorname{sen}(2t))$$

$$y(0)=\frac{936}{1121}+\frac{185}{1121}=\frac{1121}{1121}\Rightarrow 1$$

Sistema de EDOL.

$$S(n)\ \text{EDOL}(1) \Longleftrightarrow \text{EDOL}(n)$$

$$\bar{x} = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \qquad \frac{d}{dt}\bar{x} = \begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix}$$

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 4t^2 \\ 2e^{3t} \end{bmatrix}$$

$$\frac{d}{dt}\bar{x} = A\bar{x} + b(t)$$

$$\frac{d}{dt}\bar{x} = A\bar{x} \quad \text{hom.}$$

$$\bar{x} = \left[e^{At}\right]\bar{x}(0)$$

Matriz Exponencial

$$\left[e^{At}\right]_{t=0} = I \qquad \frac{d}{dt}e^{At} = Ae^{At}$$

$$\left[e^{At}\right]^{-1} = e^{A(-t)}$$