TEMA 3.- SISTEMAS EDOL.

- MÉTODO DEL LA MATRIZ EXPONENCIAL

EDO(1)LH $\frac{dy}{dt} = 5y \longrightarrow \frac{dy}{dt} - 5y = 0$

$\longrightarrow y = C_1 e^{5t}$ ✓ $\qquad m - 5 = 0 \qquad m_1 = 5$

$\longrightarrow \frac{dy}{dt} = 5C_1 e^{5t}$

$$\left[5C_1 e^{5t}\right] = 5\left[C_1 e^{5t}\right]$$

$$\left[5C_1 - 5C_1\right] e^{5t} = 0$$

$$(0)e^{5t} = 0$$

$$0 \equiv 0$$

$$\frac{dx_1}{dt} = 2x_1 + 3x_2$$

$$\frac{dx_2}{dt} = x_1 + 4x_2$$

$$\bar{x} = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \qquad \frac{d\bar{x}}{dt} = \begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix}$$

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

$$\frac{d}{dt}\bar{x} = A\bar{x} \qquad A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

$$\left[e^{At}\right]$$

$$e^{At}\Big|_{t=0} = I.$$

$$\frac{d}{dt}e^{At} = Ae^{At}$$

$$\left[e^{At}\right]^{-1} = e^{A(-t)}$$

$$e^{At} = B_0(t)I + B_1(t)A + B_2(t)A^2 + \dots + B_{n-1}(t)A^{n-1}$$

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \qquad \underline{e^{At} = B_0(t)I + B_1(t)A.}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad e^{At} = B_0(t)I + B_1(t)A + B_2(t)A^2$$

$$e^{At} = \beta_0(t)I + \beta_1(t)A$$

"toda matriz A satisface su ecuación característica"

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \longrightarrow \begin{vmatrix} 2-\lambda & 3 \\ 1 & 4-\lambda \end{vmatrix} = 0$$

Ec. carad. $|A - \lambda I| = 0$

$$(2-\lambda)(4-\lambda) - (3)(1) = 0$$

$$\lambda^2 - 6\lambda + 8 - 3 = 0$$

Ecuación característica de A. $\lambda^2 - 6\lambda + 5 = 0$

$$\underbrace{\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}}_{A^2} \underbrace{- 6 \cdot \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}}_{-6A} \underbrace{+ 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{+5I} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = [0]$$

$$\begin{bmatrix} 4+3 & 6+12 \\ 2+4 & 3+16 \end{bmatrix} + \begin{bmatrix} -12 & -18 \\ -6 & -24 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} =$$

$$\overset{A^2}{\begin{bmatrix} 7 & 18 \\ 6 & 19 \end{bmatrix}} + \begin{bmatrix} -12 & -18 \\ -6 & -24 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} =$$

$$\begin{bmatrix} 7-12+5 & 18-18 \\ 6-6 & 19-24+5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$e^{\lambda_i t} = B_0(t) + \lambda_i B_1(t)$$

$$e^{At} = B_0(t) I + B_1(t) A.$$

$$\lambda^2 - 6\lambda + 5 = 0$$

$$(\lambda - 5)(\lambda - 1) = 0 \qquad \lambda_1 = 5 \quad \lambda_2 = 1$$

$$\boxed{\begin{aligned} e^{5t} &= B_0(t)\cdot 1 + 5B_1(t) \\ e^{t} &= B_0(t)\cdot 1 + B_1(t)\cdot 1 \end{aligned}}$$

$$e^{5t} - e^{t} = 4B_1(t) \qquad B_1(t) = \frac{e^{5t} - e^{t}}{4}$$

$$B_0(t) = e^{t} - B_1(t)$$

$$B_0(t) = e^{t} - \left(\frac{e^{5t} - e^{t}}{4}\right)$$

$$= \frac{4e^{t} - e^{5t} + e^{t}}{4}$$

$$B_0(t) = \frac{-e^{5t} + 5e^{t}}{4}$$

$$e^{At} = \left(\frac{-e^{5t} + 5e^{t}}{4}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \left(\frac{e^{5t} - e^{t}}{4}\right)\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

$$4e^{At} = \begin{bmatrix} -1+2 & 3 \\ 1 & -1+4 \end{bmatrix} e^{5t} + \begin{bmatrix} 5-2 & -3 \\ -1 & 5-4 \end{bmatrix} e^{t}$$

$$4e^{At} = \begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix} e^{5t} + \begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix} e^{t}$$

$$\Rightarrow \quad e^{At} = \frac{1}{4}\begin{bmatrix} e^{5t} + 3e^{t} & 3e^{5t} - 3e^{t} \\ e^{5t} - e^{t} & 3e^{5t} + e^{t} \end{bmatrix}$$

$$e^{At}\Big|_{t=0} = \frac{1}{4}\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$e^{At} = \frac{1}{4}\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix} e^{5t} + \frac{1}{4}\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix} e^{t}$$

$$\frac{dx_1}{dt} = 2x_1 + 3x_2$$

$$\frac{dx_2}{dt} = x_1 + 4x_2$$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} e^{5t} + \frac{1}{4}\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} e^{t}$$

$$\begin{cases} x_1(t) = \dfrac{c_1 + 3c_2}{4} e^{5t} + \dfrac{3c_1 - 3c_2}{4} e^{t} \\ x_2(t) = \dfrac{c_1 + 3c_2}{4} e^{5t} + \dfrac{-c_1 + c_2}{4} e^{t} \end{cases}$$

$$\boxed{\begin{aligned} x_1(t) &= c_{10} e^{5t} - 3c_{20} e^{t} \\ x_2(t) &= c_{10} e^{5t} + c_{20} e^{t} \end{aligned}}$$

$$\frac{dx_1}{dt} = x_1 + x_2$$

$$\frac{dx_2}{dt} = 2x_1 + 2x_2$$

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \qquad \text{Ec. Carac} = \begin{vmatrix} 1-\lambda & 1 \\ 2 & 2-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(2-\lambda) - 2 = 0$$

$$\lambda^2 - 3\lambda = 0$$

$$(\lambda - 3)\lambda = 0 \qquad \lambda_1 = 3, \quad \lambda_2 = 0$$

$$e^{At} = B_0 I + B_1 A$$

$$e^{\lambda t} = B_0 + \lambda B_1$$

$$e^{3t} = B_0 + 3B_1$$

$$e^{(0)t} = B_0 + (0)B_1 \qquad B_0 = 1$$

$$e^{3t} = 1 + 3B_1 \qquad B_1 = \frac{e^{3t} - 1}{3}$$

$$e^{At} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \left(\frac{e^{3t} - 1}{3}\right)\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} 1-\frac{1}{3} & -\frac{1}{3} \\ -\frac{2}{3} & 1-\frac{2}{3} \end{bmatrix} + \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{2}{3} \end{bmatrix} e^{3t}$$

$$e^{At} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{2}{3} & \frac{1}{3} \end{bmatrix} + \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{2}{3} \end{bmatrix} e^{3t}$$

$$e^{At}\Big|_{t=0} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\bar{X} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{2}{3} & \frac{1}{3} \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{2}{3} \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} e^{3t}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{2}{3}c_1 - \frac{1}{3}c_2 \\ -\frac{2}{3}c_1 + \frac{1}{3}c_2 \end{bmatrix} + \begin{bmatrix} \frac{c_1}{3} + \frac{c_2}{3} \\ \frac{2c_1}{3} + \frac{2c_2}{3} \end{bmatrix} e^{3t}$$

$$x_1 = c_{10} + c_{20}e^{3t} \qquad \frac{dx_1}{dt} = x_1 + x_2$$

$$x_2 = -c_{10} + 2c_{20}e^{3t} \qquad \frac{dx_2}{dt} = 2x_1 + 2x_2$$