TEMA 4.- EDenDP

MÉTODO VARIABLES SEPARABLES

"prueba y error" $H_1 \Rightarrow z = \frac{F}{G}$

Hipótesis inicial H_0 $z(x,y) = F(x) \cdot G(y)$

$z = F \cdot G$

EDenDP $\longrightarrow \boxed{\frac{\partial^2 z}{\partial x^2} + 8\frac{\partial z}{\partial y} = z}$

$z = F \cdot G$ $\frac{\partial z}{\partial x} = F' \cdot G$ $\frac{\partial^2 z}{\partial x^2} = F'' G$

$\frac{\partial z}{\partial y} = F \cdot G'$

Sust. en la EDenDP $\Rightarrow (F'' \cdot G) + 8(FG') = FG$

$$M(x, F, F', F'') = N(y, G, G')$$

$$F'' \cdot G = -8FG' + FG$$

$$F'' \cdot G = -8F\left(G' - \frac{G}{8}\right)$$

$$\frac{F''}{-8F} = \frac{G' - \frac{G}{8}}{G}$$

H_0 funcionó porque se logró separar las variables

$$F'' \cdot G - FG = -8FG'$$

$$(F'' - F)G = -8FG'$$

$$\frac{F'' - F}{-8F} = \frac{G'}{G}.$$

EDO$_1$

$$\frac{F''}{-8F} = \alpha$$

EDO$_2$

$$\frac{G' - \frac{G}{8}}{G} = \alpha$$

$\alpha = 0$, $\alpha < 0$, $\alpha > 0$

$\alpha = 0$

$$\frac{F''}{-8F} = 0 \qquad F'' = 0 \rightarrow \frac{d^2F}{dx^2} = 0 \qquad \frac{dF}{dx} = k_1$$

$F \neq 0$

$$\boxed{F_{\alpha=0} = k_1 x + k_2}$$

$$\frac{G' - \frac{G}{8}}{G} = 0 \qquad G' - \frac{G}{8} = 0 \qquad \frac{dG}{dy} - \frac{G}{8} = 0$$

$G \neq 0$

$$\frac{dG}{dy} = \frac{G}{8} \Rightarrow \frac{dG}{G} = \frac{dy}{8} \qquad LG + C_1 = \frac{1}{8}y + C_2$$

$$LG = \frac{1}{8}y + (C_2 - C_1) \qquad LG = \frac{y}{8} + L\,C_{10}$$

$$\frac{LG}{LC_{10}} = \frac{y}{8} \qquad \frac{G}{C_{10}} = e^{\frac{y}{8}} \qquad G = C_{10} e^{\frac{y}{8}}$$

$$Z(x,y) = (k_1 x + k_2) \cdot C_{10} e^{\frac{y}{8}}$$

$$\boxed{Z(x,y) = (C_1 x + C_2) e^{y/8}}$$

$\alpha = 0$

para $\alpha < 0$ $\qquad \alpha = -\beta^2 \qquad \forall \beta \neq 0 \in \mathbb{R}$

$$\frac{F''}{-8F} = -\beta^2 \qquad F'' = 8\beta^2 F \Rightarrow \underline{\frac{d^2F}{dx^2} - 8\beta^2 F = 0}$$

$$m^2 - 8\beta^2 = 0 \qquad (m - \sqrt{8}\beta)(m + \sqrt{8}\beta) = 0$$

$$m_1 = \sqrt{8}\beta \qquad m_2 = -\sqrt{8}\beta$$

$$F(x) = k_1 e^{\sqrt{8}\beta x} + k_2 e^{-\sqrt{8}\beta x}$$

$\alpha < 0$

$$\frac{G' - \frac{G}{8}}{G} = -\beta^2 \Rightarrow G' - \frac{G}{8} = -\beta^2 G$$

$$\frac{dG}{dy} - \left(\frac{1}{8} - \beta^2\right) G = 0$$

$$G(y) = C_{10} e^{\left(\frac{1}{8} - \beta^2\right) y}$$

$\alpha < 0$

$$\boxed{Z(x, y) = \left(C_1 e^{\sqrt{8}\beta x} + C_2 e^{-\sqrt{8}\beta x}\right) e^{\left(\frac{1}{8} - \beta^2\right) y}}$$

$\alpha < 0$

$\alpha > 0 \quad \alpha = \beta^2 \quad \forall \beta \neq 0 \in \mathbb{R}$

$$\frac{F''}{-8F} = \beta^2 \qquad F'' = -8\beta^2 F \qquad F'' + 8\beta^2 F = 0$$

$$\frac{d^2F}{dx^2} + 8\beta^2 F = 0 \qquad m^2 + 8\beta^2 = 0 \qquad m_1 = \sqrt{8}\beta i \quad m_2 = -\sqrt{8}\beta i$$

$$F(x) = \left(k_1 \cos(\sqrt{8}\beta x) + k_2 \operatorname{sen}(\sqrt{8}\beta x)\right).$$

$\alpha > 0$

$$\frac{G' - \frac{G}{8}}{G} = \beta^2 \qquad G' - \frac{G}{8} = \beta^2 G \qquad G' - \left(\frac{1}{8} + \beta^2\right)G = 0$$

$$\frac{dG}{dy} - \left(\frac{1}{8} + \beta^2\right)G = 0 \qquad G = G_0 e^{\left(\frac{1}{8} + \beta^2\right)y}$$

$\alpha > 0$

$$Z(x,y) = \left(C_1 \cos(\sqrt{8}\beta x) + C_2 \operatorname{sen}(\sqrt{8}\beta x)\right) e^{\left(\frac{1}{8} + \beta^2\right)y}$$

$$Z(x,y) = C_1 e^{\left(\frac{1}{8} + \beta^2\right)y} \cos(\sqrt{8}\beta x) + C_2 e^{\left(\frac{1}{8} + \beta^2\right)y} \operatorname{sen}(\sqrt{8}\beta x).$$

$\alpha > 0$