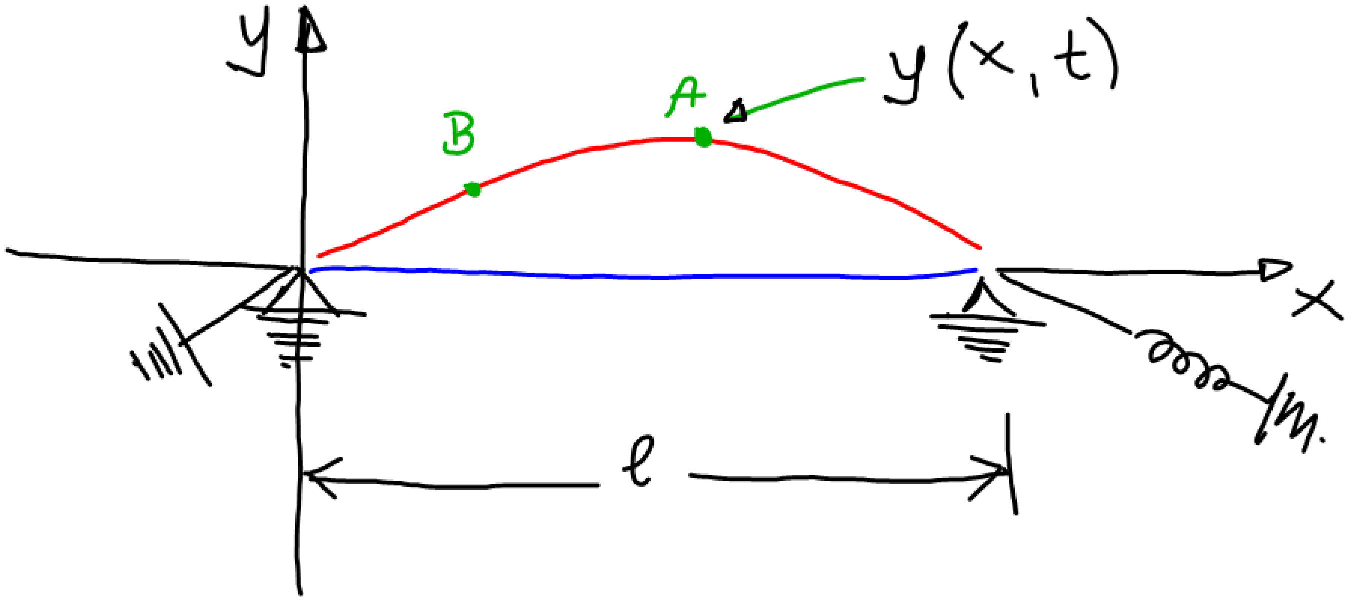
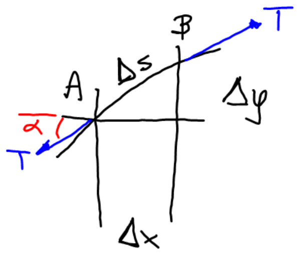
PROBLEMA DE APLICACIÓN EDenDP.

CUERDA DE GUITARRA.



Fuerza de restituir cuerda



$$F_R = m\,a$$

$$a = \frac{\partial^2 y(x,t)}{\partial t^2}$$

ρ: densidad de masa por longitud.

$$m = \rho\,\Delta s$$

$$\operatorname{sen}(\alpha) \doteq \tan(\alpha) = \frac{\Delta y}{\Delta x}$$

$$F_R = T_{V_B} - T_{V_A}$$

$$T_{V_B} = T\frac{\partial y}{\partial x} + \frac{\partial}{\partial x}\left(T\frac{\partial y}{\partial x}\Delta x\right)$$

$$T_{V_B} = T\frac{\partial y}{\partial x} + T\frac{\partial^2 y(x,t)}{\partial x^2}\Delta x$$

$$T_{V_A} = T\frac{\Delta y}{\Delta x}$$

$$\Delta x \to 0$$

$$T_{V_A} = T\frac{\partial y(x,t)}{\partial x}$$

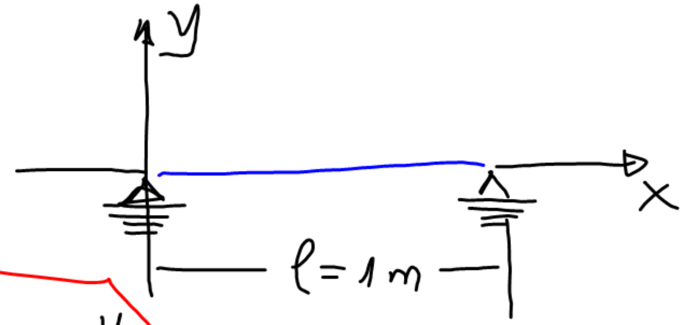
$$T\frac{\partial^2 y(x,t)}{\partial x^2} = \rho\frac{\partial^2 y(x,t)}{\partial t^2}$$

$$\frac{T}{\rho} = c^2$$

$$\boxed{\frac{\partial^2 y(x,t)}{\partial t^2} = c^2\frac{\partial^2 y(x,t)}{\partial x^2}}$$

$$\frac{\partial^2 y(x,t)}{\partial t^2} - c^2\frac{\partial^2 y(x,t)}{\partial x^2} = 0.$$

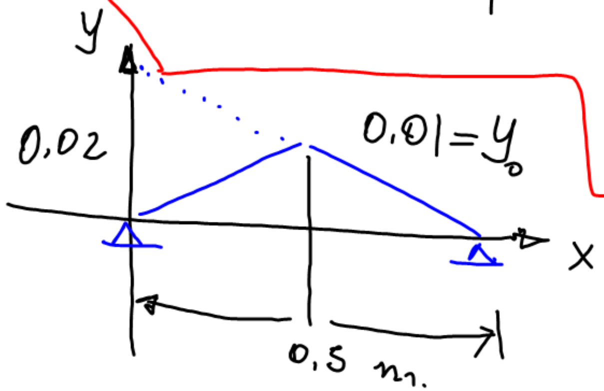
Método de Variables Separables



condiciones de frontera.

$$y(0,t) = 0$$

$$y(1,t) = 0$$



condiciones iniciales $t=0$

$$y(x,0) = f(x)$$

$$f(x) = \begin{cases} \dfrac{0{,}01}{0{,}5}x \; ; & 0 \le x \le 0{,}5 \\ 0{,}02 - \dfrac{0{,}01}{0{,}5}x \; ; & 0{,}5 < x \le 1 \end{cases}$$

$$\left.\frac{\partial y(x,t)}{\partial t}\right|_{t=0} = 0$$

$$\frac{\partial^2 y}{\partial t^2} - c^2 \frac{\partial^2 y}{\partial x^2} = 0$$

$$y(x,t) = F(x) \cdot G(t)$$

$$\frac{\partial y}{\partial t} = F G' \quad \frac{\partial y}{\partial x} = F' G$$

$$\frac{\partial^2 y}{\partial t^2} = F G'' \quad \frac{\partial^2 y}{\partial x^2} = F'' G$$

$$F G'' - c^2 F'' G = 0$$

$$F G'' = c^2 F'' G$$

$$\boxed{\frac{G''}{c^2 G} = \frac{F''}{F}}$$

$$\frac{F''}{F} = \alpha \qquad \frac{G''}{c^2 G} = \alpha$$

para $\alpha = 0$

$$\frac{F''}{F} = 0 \rightarrow \begin{matrix} F'' = 0 \\ F \neq 0 \end{matrix} \rightarrow F' = C_1 \rightarrow \boxed{F = C_1 x + C_2}$$

cond. frontera

$y(0,t) = 0 \rightarrow F(0) \cdot G(t) = 0 \quad F(0) = 0 \quad C_1(0) + C_2 = 0$

$C_2 = 0$

$y(1,t) = 0 \rightarrow F(1) \cdot G(t) = 0 \quad F(1) = 0$

$F = C_1 x$

para $\alpha = 0$ NO ES LA SOL GRAL.

$C_1 = 0$

para $\alpha > 0$ $\quad \alpha = \beta^2 \ \forall \beta \neq 0 \in \mathbb{R}$

$$\frac{F''}{F} = \beta^2 \rightarrow F'' = \beta^2 F \rightarrow F'' - \beta^2 F = 0$$

$$m^2 - \beta^2 = 0 \qquad (m+\beta)(m-\beta) = 0 \qquad m_1 = \beta \quad m_2 = -\beta.$$

$\alpha > 0$ $\quad F(x) = c_1 e^{\beta x} + c_2 e^{-\beta x}$

$$F(0) = 0 \qquad c_1 e^{\beta(0)} + c_2 e^{-\beta(0)} = 0 \qquad c_1 + c_2 = 0$$

$$F(x) = c_1 e^{\beta x} - c_1 e^{-\beta x} \qquad c_2 = -c_1$$

$$F(1) = 0 \qquad c_1 e^{\beta} - c_1 e^{-\beta} = 0 \qquad e^{\beta} = e^{-\beta}$$

$$e^{\beta} = \frac{1}{e^{\beta}} \qquad e^{2\beta} = 1 \qquad \beta = 0$$

$\alpha > 0$ no es solución general cuerda.

para $\alpha < 0$ $\alpha = -\beta^2$ $\forall \beta \neq 0 \in \mathbb{R}$

$$\frac{F''}{F} = -\beta^2 \qquad F'' = -\beta^2 F \qquad F'' + \beta^2 F = 0$$

$$m^2 + \beta^2 = 0 \qquad m_1 = +\beta i \qquad m_2 = -\beta i$$

$$F(x) = C_1 \cos(\beta x) + C_2 \operatorname{sen}(\beta x)$$

$$F(0) = C_1 + C_2 \cdot (0) = 0 \qquad C_1 = 0$$

$$F(1) = C_2 \operatorname{sen}(\beta) = 0 \qquad \operatorname{sen}(\beta) = 0$$

$$C_2 \neq 0$$

$$F(x) = C_2 \operatorname{sen}(n\pi x).$$

$$\operatorname{sen}(n\pi) = 0 \qquad n = 1, 2,$$

$$c^2 \frac{G''}{G} = -n^2\pi^2 \qquad G'' = -c^2 n^2 \pi^2 G \qquad \beta = n\pi$$

$$m^2 + c^2 n^2 \pi^2 = 0$$

$$G(t) = k_1 \cos(cn\pi t) + k_2 \operatorname{sen}(cn\pi t).$$

$$y(x,t) = C_2 \operatorname{sen}(n\pi x)\left(k_1 \cos(cn\pi t) + k_2 \operatorname{sen}(cn\pi t)\right).$$

$$y(x,t) = \operatorname{sen}(n\pi x)\left(C_{10} \cos(cn\pi t) + C_{20} \operatorname{sen}(cn\pi t)\right)$$