> restart

PROBLEMA DE LA CUERDA DE GUITARRA VIBRANDO

> $EcuacionOriginal := diff(y(x,t), t\$2) = c\cdot\cdot2\cdot diff(y(x,t), x\$2)$

$$EcuacionOriginal := \frac{\partial^2}{\partial t^2}\, y(x,t) = c^2 \left(\frac{\partial^2}{\partial x^2}\, y(x,t)\right) \quad \textbf{(1)}$$

> $Ecuacion := subs(c\cdot\cdot2 = 1, EcuacionOriginal)$

$$Ecuacion := \frac{\partial^2}{\partial t^2}\, y(x,t) = \frac{\partial^2}{\partial x^2}\, y(x,t) \quad \textbf{(2)}$$

> $EcuacionSeparable := eval(subs(y(x,t) = F(x)\cdot G(t), Ecuacion))$

$$EcuacionSeparable := F(x)\left(\frac{d^2}{dt^2}\, G(t)\right) = \left(\frac{d^2}{dx^2}\, F(x)\right) G(t) \quad \textbf{(3)}$$

> $EcuacionSeparada := \frac{lhs(EcuacionSeparable)}{(F(x)\cdot G(t))} = \frac{rhs(EcuacionSeparable)}{(F(x)\cdot G(t))}$

$$EcuacionSeparada := \frac{\frac{d^2}{dt^2}\, G(t)}{G(t)} = \frac{\frac{d^2}{dx^2}\, F(x)}{F(x)} \quad \textbf{(4)}$$

> $Ecuacion_x := rhs(EcuacionSeparada) = \mathrm{alpha};\ Ecuacion_t := lhs(EcuacionSeparada) = \mathrm{alpha};$

$$Ecuacion_x := \frac{\frac{d^2}{dx^2}\, F(x)}{F(x)} = \alpha$$

$$Ecuacion_t := \frac{\frac{d^2}{dt^2}\, G(t)}{G(t)} = \alpha \quad \textbf{(5)}$$

> $SolucionCero_x := dsolve(subs(\mathrm{alpha} = 0, Ecuacion_x))$

$$SolucionCero_x := F(x) = \_C1\, x + \_C2 \quad \textbf{(6)}$$

> $sistema := subs(x = 0, rhs(SolucionCero_x) = 0), subs(x = 1, rhs(SolucionCero_x) = 0)$:
> $sistema_1; sistema_2;$

$$\_C2 = 0$$

$$\_C1 + \_C2 = 0 \quad \textbf{(7)}$$

> **_C:**

> $parametros := solve(\{sistema\}, \{\_C1, \_C2\})$

$$parametros := \{\_C1 = 0, \_C2 = 0\} \quad \textbf{(8)}$$

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SUSTITUYENDO EL VALOR DE LOS PARÁMETROS NOS CONDUCE A LA SOLUCION TRIVIAL, POR LO TANTO LA SOLUCIÓN GENERAL PARA ALPHA=0 NO ES APLICABLE

> $SolucionPos_x := dsolve(subs(\mathrm{alpha} = \mathrm{beta}\cdot\cdot2, Ecuacion_x))$

$$SolucionPos_x := F(x) = \_C1\, e^{\beta x} + \_C2\, e^{-\beta x} \quad \textbf{(9)}$$

> $sistemaPos := eval(subs(x = 0, rhs(SolucionPos_x) = 0)), subs(x = 1, rhs(SolucionPos_x) = 0)$:
> $sistemaPos_1; sistemaPos_2;$

$$\_C1 + \_C2 = 0$$
$$\_C1\, e^{\beta} + \_C2\, e^{-\beta} = 0 \qquad \mathbf{(10)}$$

> $parametroPos := solve(\{sistemaPos\}, \{\_C1, \_C2\})$

$$parametroPos := \{\_C1 = 0, \_C2 = 0\} \qquad \mathbf{(11)}$$

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SUSTITUYENDO EL VALOR DE LOS PARÁMETROS NOS CONDUCE A LA SOLUCION TRIVIAL, POR LO TANTO LA SOLUCIÓN GENERAL PARA ALPHA POSITIVA NO ES APLICABLE

> $SolucionNeg_x := dsolve\left(subs\left(\mathrm{alpha} = -\mathrm{beta} \cdot{} \cdot 2, Ecuacion_x\right)\right)$

$$SolucionNeg_x := F(x) = \_C1 \sin(\beta x) + \_C2 \cos(\beta x) \qquad \mathbf{(12)}$$

> $sistemaNeg := subs\left(x = 0, rhs\left(SolucionNeg_x\right) = 0\right), subs\left(x = 1, rhs\left(SolucionNeg_x\right) = 0\right) :$
$sistemaNeg_1; sistemaNeg_2;$

$$\_C2 = 0$$
$$\_C1 \sin(\beta) + \_C2 \cos(\beta) = 0 \qquad \mathbf{(13)}$$

> $parametroNeg := solve\left(\left\{sistemaNeg_1, subs\left(\mathrm{beta} = n \cdot \mathrm{Pi}, \sin(n \cdot \mathrm{Pi}) = 0, sistemaNeg_2\right)\right\}, \{\_C1, \_C2\}\right)$

$$parametroNeg := \{\_C1 = \_C1, \_C2 = 0\} \qquad \mathbf{(14)}$$

> $SolucionNegMod_x := subs\left(\mathrm{beta} = n \cdot \mathrm{Pi}, \_C2 = 0, SolucionNeg_x\right)$

$$SolucionNegMod_x := F(x) = \_C1 \sin(n \pi x) \qquad \mathbf{(15)}$$

> $SolucionNegMod_t := dsolve\left(subs\left(\mathrm{alpha} = -\mathrm{beta} \cdot{} \cdot 2, \mathrm{beta} = n \cdot \mathrm{Pi}, Ecuacion_t\right)\right)$

$$SolucionNegMod_t := G(t) = \_C1 \sin(n \pi t) + \_C2 \cos(n \pi t) \qquad \mathbf{(16)}$$

> $SolucionNeg := y(x, t) = subs\left(\_C1 = 1, rhs\left(SolucionNegMod_x\right)\right) \cdot rhs\left(SolucionNegMod_t\right)$

$$SolucionNeg := y(x, t) = \sin(n \pi x) \left(\_C1 \sin(n \pi t) + \_C2 \cos(n \pi t)\right) \qquad \mathbf{(17)}$$

> $SolucionGeneral := y(x, t) = Sum\left(subs\left(\_C1 = 1, rhs\left(SolucionNegMod_x\right)\right) \cdot subs\left(\_C1 = a_n, \_C2 = b_n, rhs\left(SolucionNegMod_t\right)\right), n = 1 ..infinity\right)$

$$SolucionGeneral := y(x, t) = \sum_{n=1}^{\infty} \sin(n \pi x) \left(a_n \sin(n \pi t) + b_n \cos(n \pi t)\right) \qquad \mathbf{(18)}$$

> $eval(subs(t = 0, SolucionGeneral))$

$$y(x, 0) = \sum_{n=1}^{\infty} \sin(n \pi x)\, b_n \qquad \mathbf{(19)}$$

> $b_n := subs\left(\sin(n \cdot \mathrm{Pi}) = 0, \cos(n \cdot \mathrm{Pi}) = (-1) \cdot{} \cdot n, simplify\left(\left(\frac{1}{\left(\frac{5}{10}\right)}\right) \cdot int\left(\left(\frac{\frac{5}{1000}}{\frac{5}{10}} \cdot x \cdot \sin(n \cdot \mathrm{Pi} \cdot x), x = 0 .. \frac{5}{10}\right)\right) + \left(\frac{1}{\left(\frac{5}{10}\right)}\right) \cdot int\left(\left(-\frac{\left(\frac{5}{1000}\right)}{\frac{5}{10}} \cdot x + \frac{1}{100}\right) \cdot \sin(n \cdot \mathrm{Pi} \cdot x), x\right.\right.\right.$

$= \frac{5}{10} ..1 \Big)\Big)\Big)$

$$b_n := \frac{1}{25} \frac{\sin\left(\frac{1}{2} n \pi\right)}{n^2 \pi^2} \quad \textbf{(20)}$$

> $eval(rhs(subs(t=0, diff(SolucionGeneral, t))) = 0)$

$$\sum_{n=1}^{\infty} \sin(n \pi x)\, a_n\, n \pi = 0 \quad \textbf{(21)}$$

> $a_n := 0;$

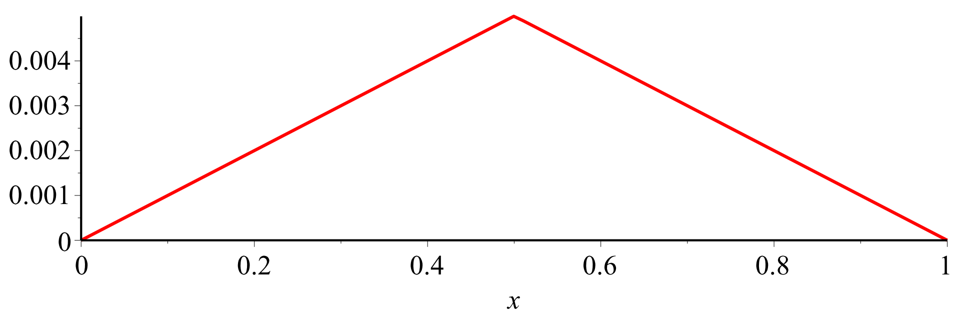
$$a_n := 0 \quad \textbf{(22)}$$

> $SolucionGeneral;$

$$y(x, t) = \sum_{n=1}^{\infty} \frac{1}{25} \frac{\sin(n \pi x) \sin\left(\frac{1}{2} n \pi\right) \cos(n \pi t)}{n^2 \pi^2} \quad \textbf{(23)}$$

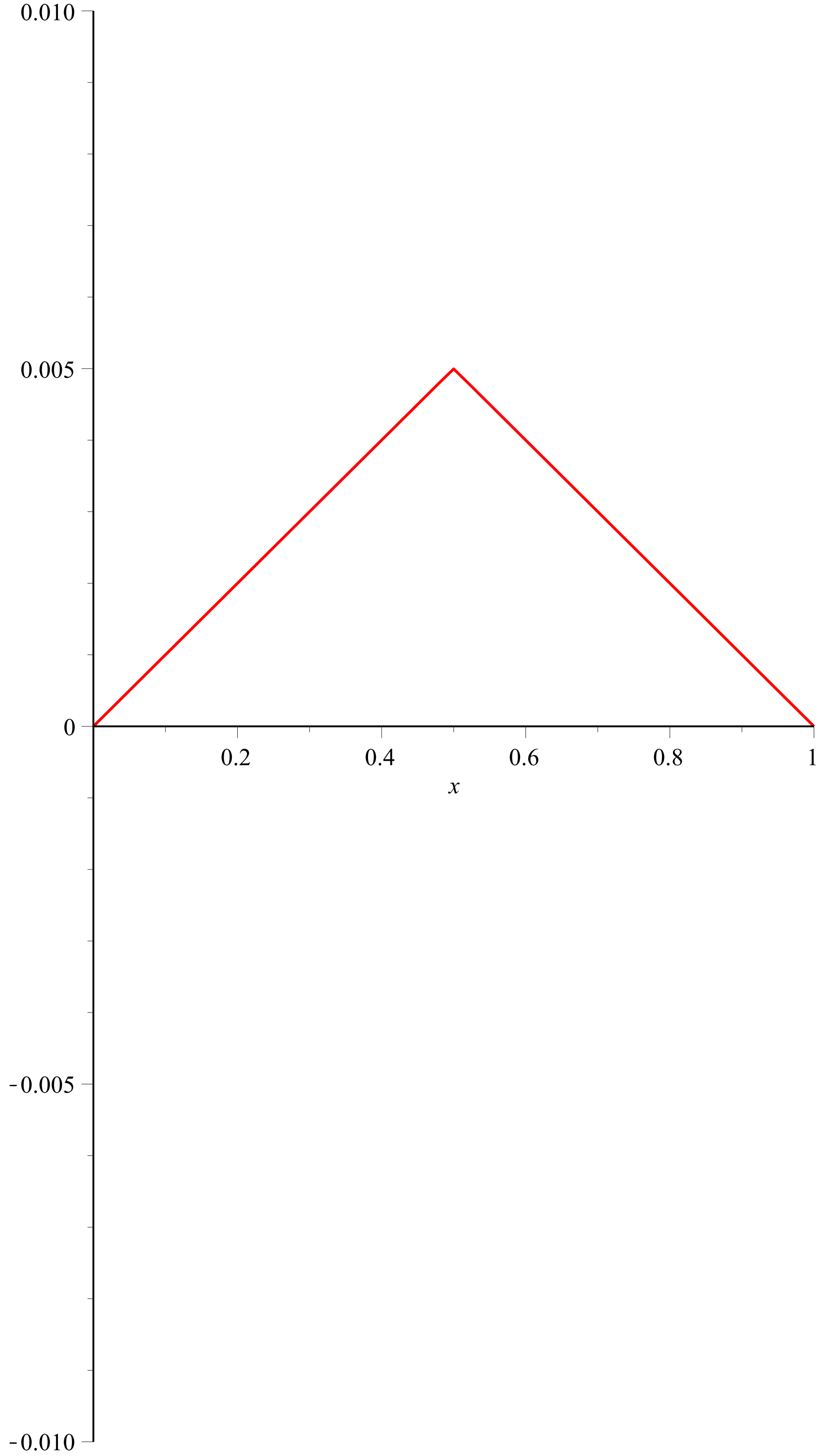
> $SolucionGeneral500 := y(x, t) = sum\left(\frac{\frac{1}{25} \cdot \left(\sin(n \cdot \mathrm{Pi} \cdot x) \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{2}\right) \cdot \cos(n \cdot \mathrm{Pi} \cdot t)\right)}{n \cdot \cdot 2 \cdot \mathrm{Pi} \cdot \cdot 2}, n = 1 ..500\right) :$

> $plot(subs(t=0, rhs(SolucionGeneral500)), x = 0 ..1)$



> $with(plots) :$

> $animate(rhs(SolucionGeneral500), x = 0 ..1, t = 0 ..4, frames = 150, view = [0 ..1, -0.01 ..0.01])$



0.010
0.005
0
-0.005
-0.010
0.2
0.4
0.6
0.8
1
x

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